

SHORT NOTES

ORDERS OF SUBSEMIGROUPS OF T_n

B. M. Schein in Shevrin [1] posed the problem of determining the orders of all subsemigroups of the semigroup T_n of all transformations on a set of n elements.

THEOREM 1. There exist subsemigroups of T_n of every order from 1 to $n + 2^n - 2$.

PROOF. For $n = 1, 2$ this can readily be checked. For $n = 3$ orders 1 to n can be realized by semigroups of rank 1 functions. Let m be a positive integer such that $n + 2 \leq m \leq 2^n - 2 + n$ and $m \equiv n \pmod{2}$. Then we can obtain a semigroup of order m by taking all rank 1 functions together with a union of rank 2 H -classes lying in a single L -class. (H -classes of rank 2 have order 2 and L -classes have order $2^n - 2$). Let $n + 1 \leq m \leq 2^n - 3 + n$ and $m \equiv n + 1 \pmod{2}$. Then we can obtain a subsemigroup of order m by taking a union of H -classes having image set $\{1, 2\}$ together with all constant functions except that with image set $\{3\}$.

REFERENCE

1. Shevrin, L. N. , The Sverdlovsk Tetrad, Semigroup Forum, 4 (1972), 274-280.

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