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ERRATA CORRIGE

Stochastic Differential Games: Occupation Measure Based Approach

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Abstract. An error in the proof of Theorem 4.1 of Ref. 1 is corrected.

Key Words. Occupation measure, Markov strategy, invariant measure, Isaacs equation, equilibrium.

The proof of Theorem 4.1 in Ref. 1 is incorrect. The crux of the proof is based on a minimax theorem due to Fan (Theorem 3 of Ref. 2). To use Theorem 3 of Ref. 2, we need to show that the map

$$(v_1, v_2) \in M_1 \times M_2 \rightarrow R_\lambda[v_1, v_2](\pi) \tag{1}$$

(see Ref. 1 for notation and other details) is continuous. But in Ref. 1 we have only shown that the above map is continuous in each argument. Thus, there was an error in using Theorem 3 of Ref. 2. The proof can be salvaged if we make the following assumption:

$$\bar{m}(x, u_1, u_2) = \bar{m}^1(x, u_1) + \bar{m}^2(x, u_2), \tag{2a}$$

$$\bar{r}(x, u_1, u_2) = \bar{r}_1(x, u_1) + \bar{r}_2(x, u_2), \tag{2b}$$

where $\bar{m}^i$: $\mathbb{R}^d \times V_i \rightarrow \mathbb{R}^d$, $\bar{r}_i$: $\mathbb{R}^d \times V_i \rightarrow \mathbb{R}$, $i = 1, 2$ satisfy the same conditions as $\bar{m}$ and $\bar{r}$. Under this assumption, the map (1) is continuous as shown in Section 5 of Ref. 1. Thus, the proof of Theorem 4.1 of Ref. 1 would be valid.

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We would like to mention that Theorem 4.1 in Ref. 1 is true without Assumption (2) above. Here, we indicate a proof very briefly. Using the techniques involving quasilinear p.d.e. (see Chapter 4, Ref. 3; see also the Appendix of Ref. 4, where analogous arguments are used) and a minimax theorem due to Fan (Theorem 1, Ref. 5), one can show that the Isaacs equation [Eq. (43) in Ref. 1],

$$\begin{aligned}\lambda\phi(x) &= \min_{v_2}\max_{v_1}[L_{v_1,v_2}\phi(x)+r(x,v_1,v_2)]\\ &= \max_{v_1}\min_{v_2}[L_{v_1,v_2}\phi(x)+r(x,v_1,v_2)],\end{aligned} \tag{3}$$

has a unique solution in $C^2(\mathbb{R}^d)\cap C_b(\mathbb{R}^d)$. The unique solution of this equation will be the value of the game with the discounted payoff criterion. Using this solution, Theorem 4.3 of Ref. 1 is valid, which would in turn provide a proof of Theorem 4.1 of Ref. 1.

Again for the same reason, the proof of Theorem 4.4 in Ref. 1 is incorrect. The proof can be salvaged by Assumption (2) above. However, all the results in Section 4.2 in Ref. 1 are valid without this extra assumption. Using the unique solution of Eq. (3), we can modify the arguments in Section 4.2 in Ref. 1 to derive all the desired results.

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