Obituary

L. M. Gluskin in Memoriam

BORIS M. SCHEIN

Lazar Matveyevich Gluskin was born on March 10, 1922, in Artëmovsk (formerly Bakhmut), a town in the Donetsk (formerly Stalino, before that — Yuzovka) province

of the Ukraine. From 1939 he was a student of the Physico-Mathematical Faculty of the University in Kharkov. When German troops occupied Kharkov in 1941, Gluskin managed to escape and continued his studies at the University of Saratov. In 1943 he became an officer of the Red Army, a commander of an anti-aircraft battery.

After the war he continued his studies at Kharkov under the supervision of Professor A. K. Suschkewitsch (Suškevič), a pioneer researcher in the theory of semi-groups from the early twenties. Under his influence Gluskin became interested in semigroups. He received his Diploma (M.A.) in Mathematics in 1949. Although he was one of the best students and a veteran with numerous military decorations, the normal road to mathematics through a graduate school was not possible for him. He continued research on his own. Kindly and bravely, Suschkewitsch agreed to supervise his research on a strictly unofficial basis. In 1952 Gluskin received the Candidate (Ph.D.) degree from Kharkov University.

In 1958 Gluskin was invited to the chair of mathematics at the Institute of Mining and Metallurgy in Voroshilovsk (formerly Alchevsk, now Kommunar'sk), a town in the Voroshilovgrad (formerly Lugansk) province of the Ukraine. Kharkov, (one of the biggest cities in the USSR), has many first-rate mathematicians, and an active cultural life. But Gluskin had no real choice: he had nowhere to live in Kharkov, while his new employer provided him with an apartment to rent.

In 1961 Gluskin received a Doctor of Sciences in Mathematics degree from the Moscow University; the official readers of his new dissertation were Professors A. G. Kurosh (Moscow), E. S. Lyapin (Leningrad) and V. V. Vagner (Saratov). After becoming a full Professor in 1962, Gluskin started to look for ways of moving back to Kharkov. He succeeded in 1965, this time taking the chair of mathematics at the Kharkov Mining Institute (later renamed as Institute for Radioelectronics). Under his guidance the chair became one of the centers of mathematical research. He invited to this chair S. D. Berman, a renowned algebraist from the University of Uzhgorod, a town in the Western Ukraine (formerly Užhorod in Czechoslovakia, before that Ungvár in Hungary). In 1968, when Gluskin's health deteriorated very badly, Berman took over the chair, and Gluskin continued to work as a professor of mathematics. He used to say that he lived on borrowed time.

Research papers of Gluskin have been appearing since 1954. At this time the theory of semigroups emerged as a separate branch of general algebra, in many countries researchers were actively laying down the cornerstones of this theory, and Gluskin joined them. First he considered matrix semigroups, homomorphisms of semigroups, completely $[0]$ -simple semigroups, the structure of monogenic inverse semigroups and congruence-free semigroups with zero. Very early he turned to semigroups of continuous transformations of topological spaces, and remained interested in this topic all his life. Then he published a series of brilliant results on semigroups of linear transformations. He showed that a good deal of the classical theory of rings

of linear transformations could be carried over to semigroups, addition not being really needed. Later he considered semigroups of endomorphisms of modules over rings, thus generalizing his results on semigroups of linear transformations. From his point of view the semigroups of transformations formed the real core of the theory of semigroups, thus justifying the very existence of the whole theory.

His contributions to the theory of ideals in semigroups were no less important. I will mention here only his theory of dense embeddings. In 1953 E. S. Lyapin introduced the concept of a densely embedded ideal of a semigroup. An ideal I of a semigroup S is *densely embedded* if

(i) every noninjective (that is, not one-to-one) homomorphism of S induces a noninjective homomorphism on I (that is, I is a *large* or *essential* ideal of S);

(ii) S is maximal with respect to (i), that is, if S is a proper subsemigroup of a semigroup T and I is an ideal of T , then (i) does not hold for I and T .

From the end of the fifties Gluskin considered densely embedded ideals of semigroups of transformations and general semigroups. One of the problems he attacked was the following: given I , how can S be reconstructed? Clearly, each element s of S induces two translations λ_s and ρ_s of I : for every $i \in I$, $\lambda_s i = si$ and $i\rho_s = is$ (we write λ_s and ρ_s as left and right operators, respectively). The correspondence $s \rightarrow (\lambda_s, \rho_s)$ is a homomorphic embedding of S into $\Omega(I)$, the *translational hull* of I . If this correspondence is one-to-one (in which case I is called “a *weakly reductive semigroup*”), then I is isomorphic to a subsemigroup $\Pi(I)$ of $\Omega(I)$. Obviously, $\Pi(I)$ is an ideal of $\Omega(I)$, and condition (i) for densely embedded ideals holds. It follows that, if I is weakly reductive, then — up to isomorphism — there exists exactly one semigroup, $S = \Omega(I)$, with I being its densely embedded ideal. Gluskin conjectured (correctly) that the converse held. He found various properties of densely embedded ideals. For example, every automorphism of I has a uniquely determined extension to an automorphism of S , and in many situations the automorphism group of S is isomorphic to that of I . Translational hulls play an important role in various parts of the theory of semigroups.

The idea of dense embeddings (and dense extensions — when only property (i) is satisfied) turned out to be very fruitful in the theory of ideals of semigroups. Gluskin applied it not only to ideals but to other classes of subsystems as well, not only to semigroups but to wider classes of mathematical systems, and, in the case of semigroups, not only to the class of all semigroups. For example, if one considers only commutative semigroups and requires all ideal extensions to be commutative, the results about $\Omega(I)$ do not hold, and one has a multitude of interesting problems with no less interesting solutions. Significant results are obtained if, in the class of all commutative semigroups, one considers *normal* subsemigroups, rather than ideals.

Another direction of Gluskin's research was “position operatives”. He defined them as algebras with a single n -ary operation satisfying conditions analogous to associativity. In the case $n = 3$, two types of such systems have been considered:

3-semigroups which satisfy the identities $[[xyz]uv] = [x[yzu]v] = [xy[zuv]]$, and Vagner semigroups (= semiheaps) which satisfy the identities $[[xyz]uv] = [x[uzv]v] = [xy[zuv]]$. Gluskin considered both semigroups and semigroups. This led him to the concept of a position operative: if the multiplication is n -ary, the position operative satisfies associativity identities of the form

$$[a_1 \dots a_{k-1} [a_k \dots a_{k+n-1}] a_{k+n} \dots a_{2n-1}] = [[a_1 \dots a_{n\pi}] a_{(n+1)\pi} \dots a_{(2n-1)\pi}],$$

where π is a permutation of $\{1, 2, \dots, 2n-1\}$ which, in general, depends on k . If $k = 1$ or $k = n$, we assume that π is the identity permutation. It turns out that position operatives can be subdivided into a few well-defined classes. Quite often they may be reduced to m -operatives with $m < n$. Many basic results in the theory of semigroups have analogs for position operatives. This helps to understand better both position operatives and semigroups. Gluskin published a series of papers on position operatives. His results in this direction developed and complemented results of previous researchers, and his students continued his research. In the sixties Gluskin was interested in Menger algebras which are sets with a single n -ary operation subject to the identity of “superassociativity”

$$[[xy_1 \dots y_{n-1}] z_1 \dots z_{n-1}] = [x[y_1 z_1 \dots z_{n-1}] \dots [y_{n-1} z_1 \dots z_{n-1}]].$$

If $n = 1$, ordinary semigroups are obtained. Menger algebras have nice isomorphic representations by n -place functions with a natural composition of functions as a multiplication. Gluskin considered ideals of Menger algebras.

Gluskin supervised about twenty Ph.D. dissertations in algebra. Many more young people are grateful to him for his unselfish advice and help. The times were complicated, and many people had difficulties in publishing their results and getting academic degrees or jobs for reasons which had nothing to do with the quality of their work. If Gluskin was sure that the mathematical substance of the results was sound, he was there to help — even if this could hurt his own career. He could not live in any other way.

Gluskin was very active at mathematical conferences. In thirty years he gave scores of talks at the conferences, colloquium lectures, and courses on invited lectures at universities. He had many invitations from foreign universities and mathematical conventions and wanted very much to attend, but he was never allowed to go abroad (if one does not count the visits he had to pay during the war). The only exception was his short visit to a semigroup conference in Czechoslovakia in June, 1968. From 1982 he served as an editor of *Aequationes Mathematicae*, from 1970 he was as an editor of *Semigroup Forum*, and from 1974 an editor of *Theory of Semigroups and Its Application*, a semiperiodical collection of papers published in Russian by Saratov

University. Gluskin played an important role in making algebraic research in the Soviet Union more cohesive and purposeful. Not only was he an active reviewer of all three major mathematical reviewing journals, but, since 1962, he wrote and published four successive surveys of all semigroup papers reviewed in *Referativnyi Zhurnal Matematika* in the USSR between 1953 and 1970 (the last two surveys were written by Gluskin with co-authors). One of Gluskin's last papers is the first part of a survey of various results on "polyadic semigroups" (which are systems with an n -ary associative operation.)

In 1968 Gluskin published an article about main directions and achievements of the theory of semigroups in the USSR, an analysis which was very helpful for the further development of the theory. In 1970 an analogous article about algebraic research in Saratov appeared. In 1970 he was able to find one of the very few surviving copies of the doctoral dissertation (1922) of A. K. Suschkewitsch. In its 80 pages quite a few results and ideas about semigroups and quasigroups appeared for the first time. In many a case these ideas were developed by other people many decades later: usually they were not aware of the contribution of Suschkewitsch. We wrote a short survey of this dissertation and had plans for a detailed paper about this remarkable man who died in 1961.

Gluskin's interest in and appreciation of the history of mathematics was a part of his general interest in history, ancient and new, in the development of human civilization and related moral questions. He loved books. If only he could find a good book, which could be a difficult if not impossible enterprise in the surrounding general atmosphere, he would read it and, if possible, recommend or lend it to his friends. He read books in Russian, Yiddish, and Ukrainian as well as in English, French, German, and Polish.

I met him first in September of 1960 at the Third All-Union Algebraic Colloquium in Sverdlovsk organized by Professor P. G. Kontorovich. We had been corresponding since 1957, and he used to make fun of me telling selected companies of mathematicians of my first letter to him: "Imagine me getting a letter from a 19-year-old boy I never heard of which read 'Dear Lazar Matveyevich! Let D be a semigroup with commuting idempotents which is not embeddable in a generalized group.' Then three pages of formulas and 'What do you think about that?' That was love at first sight."

After that we met many times at various conferences, visiting each other, lecturing at the same places, or just spending vacations with our families at the same place. We corresponded quite often. Each one of our joint papers and talks at conferences was prepared in a series of letters. His last letter was about his completing a proof of a theorem on normal extensions of commutative semigroups which had eluded him for fifteen years. He added that he wanted to ask me, when I was leaving Russia and giving my friends various books and mementos, to give him my copy of "Le dernier de Justes" by André Schwartz-Bart. "I always wanted to have it, but now it is too

late," wrote he. "It is never too late," I replied at once, promising to send him the book. My letter was mailed two days after he became free to read any books he liked. He died suddenly, in the morning of April 15, 1985, just about to leave his apartment for his classes.

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