SPECIALIA

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A Note on the Relevancy of Behaviour of Solutions of the LANE-EMDEN Equation in the Immediate Neighbourhood of the Origin

In this note, the region in a complete polytrope to which the known behaviour of the solutions of the LANE-EMDEN equation, for different values of n, in the immediate neighbourhood of $\xi = 0$, will be relevant, has been pointed out.

The behaviour of solutions of the LANE-EMDEN equation near $\xi = 0$, for $n < 3$[1], $n = 3$[2] and $n > 3$[3] have been studied with a view to finding the arrangement of solutions near the origin of the polytropes; but recently, the author has shown[4] that in a polytrope, whatever be the index, solutions for n tending to zero and to minus one only are relevant. He has also shown[5] that the immediate neighbourhood of the origin is an interfacial region in which 2 sets of equations: one governing the origin and the other governing the rest of the parts of the configuration are relevant simultaneously. This gives an impression that probably solutions near $\xi = 0$ in (ξ; θ)-plane, referred to above, have no physical meaning. The author wishes to point out that, although θ is not defined at the origin, θ is defined in the immediate neighbourhood of the interfacial region. The interfacial region is sufficiently close to the origin; therefore the known behaviour of solutions near $\xi = 0$ in (ξ; θ)-plane, for different values of n, can be taken to be relevant to the immediate neighbourhood of the interfacial region of a polytrope[6].

Zusammenfassung. Untersuchungen über Lösungen der LANE-EMDEN-Gleichung in der Nähe des kritischen Punktes.

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A Series Solution of the LANE-EMDEN Equation of Index 5

In this note, a series solution from which all classes of E-, F- and M-solutions of the LANE-EMDEN equation of index 5 can be derived, has been given.

A series solution of the LANE-EMDEN equation of index n, near the origin, giving E-solutions is known[1] but no such solution giving F- and M-solutions has yet been presented. The reason being that F- and M-solutions have a singularity at the origin; hence an expansion in the form of a series giving F- and M-solutions at the origin will not be possible.

In terms of z and t, defined as

$$z^2 = 2\,\vartheta^2\,\xi;\ \xi = e^{-t}, \tag{1}$$

LANE-EMDEN equation of index 5 after the first integration can be expressed as

$$\frac{dz}{dt} = \left[2D + \frac{z^2}{4} - \frac{z^6}{12}\right]^{1/2}. \tag{2}$$

$t = 0$ corresponds to $\xi = 1$ at which θ is finite for all classes of solutions. The homology theorem shows that we can take $z = 1$ at $t = 0$ without any loss of generality. At $t = 0$ therefore, we can assume a TAYLOR's expansion of the form

$$z = 1 + z_0^{(1)}\,t + \frac{1}{\underline{|2}}\,z_0^{(2)}\,t^2 + \frac{1}{\underline{|3}}\,z_0^{(3)}\,t^3 + \frac{1}{\underline{|4}}\,z_0^{(4)}\,t^4 + \cdots\cdots \tag{3}$$

where $z_0^{(n)}$ is the value of nth derivative at $t = 0$. Calculating different derivatives and substituting in (3), we get

$$z = 1 + \left(2D + \frac{1}{6}\right)^{1/2} t - \frac{1}{6}\left(2D + \frac{1}{6}\right)^{1/2} t^3 + \frac{5}{24}\left(2D + \frac{1}{6}\right) t^4 + \cdots\cdots \tag{4}$$

Reverting to (ξ;θ)-variables, the above series can be written as

$$\vartheta = \frac{1}{(2\,\xi)^{1/2}}\left[1 - \left(2D + \frac{1}{6}\right)^{1/2} \log\xi + \frac{1}{6}\left(2D + \frac{1}{6}\right)^{1/2} (\log\xi)^3 - \frac{5}{24}\left(2D + \frac{1}{6}\right)(\log\xi)^4\right]. \tag{5}$$

If the series in (4) is terminated with t^4 then for the approximation to be good to 4 decimal places we must have[2]

$$\frac{5}{24}\left(2D + \frac{1}{6}\right) t^4 \leqq 0.00005\,. \tag{6}$$

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