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A PROOF OF AXIOMATIZABILITY OF CERTAIN n-VALUED SENTENTIAL CALCULI

(Summary)

The paper is concerned with n-valued sentential calculi (M_n^k) with k distinguished values (where $1 \leqslant k < n$). The primitive terms of those systems are: Łukasiewicz's many-valued implication (C) and the functors T_1 and T_{n-1}.

A system M_n^k has two rules: the rule of substitution (analogous to that adopted in the two-valued sentential calculus based on implication and negation), and the rule of detachment, formulated as follows:

If $C\alpha\beta$ is a thesis in the system M_n^1 and α is a thesis in the system M_n^k, then β is a thesis in the system M_n^k.

When $k = 1$ the adopted rule of detachment is analogous to the rule of detachment in the two-valued sentential calculus based on implication and negation.

A proof of axiomatizability of the system M_n^1 is given first. The proof is by induction with respect to diversiform variables occurring in tautologies. An important role in that proof is played by

Lemma 3. *Every formula of the form*

$$CB\alpha(q/T_1 p)\, CB\alpha(q/T_2 p) \ldots CB\alpha(q/T_n p)\, \alpha$$

is a thesis in M_n^1.

Here are tables of values for the functors B and T_i.

$$Bj = \begin{cases} n \text{ when } j = n \\ 1 \text{ when } j \neq n \end{cases} \qquad T_i j = i \text{ for every } i \text{ and } j.$$

The variables i and j range over the set of natural numbers $\leqslant n$; n is the distinguished value in every system M_n^k. The symbol α stands for any well-formed formula built of primitive functors of M_n^1 and of sentential variables.

Lemma 3 is used in the second part of the proof by induction of axiomatizability of the system M_n^1.

A proof of axiomatizability of the systems M_n^k for $k > 1$ is given next.

The systems $M_n^1, M_n^2, M_n^3, \ldots, M_n^{n-1}$ have the property that each of them contains the preceding one. Those systems are definitionally complete, and two of them (M_n^1 and M_n^{n-1}) are complete in the ordinary sense of the term.