Book Reviews

Liptser, R.S., and A.N. Shiryayev: Statistics of Random Processes I: General Theory. Statistics of Random Processes II: Applications. Translated by A.B. Aries. Springer-Verlag, New York–Heidelberg–Berlin 1977–1978. Volume I, X, 394 p., DM 79.–. Volume II, X, 339 p., DM 73.–.

This comprehensive presentation of the statistics of stochastic processes, conceived by the authors in Russian in 1974 and translated in revised and expanded form into English by A.B. Aries in 1977 is overdue for review. The two volumes of "Statistics of Random Processes" have become in the meantime a standard reference for research mathematicians in the field, less appropriate for beginners, although the authors have tried to make their book selfcontained. In fact, most of the material contained in the first volume (roughly chapters 1 to 7) can be considered to be of preliminary nature for the subject proper whose treatment starts with chapter 8. In these preliminary chapters the authors review essentials of probability and mathematical statistics, give the main ideas of martingale theory, stochastic differential equations and the Wiener process. Highlights of this introduction are the topics on Girsanov's theorem (chapter 6) and on the Cameron-Martin theory (chapter 7). Chapter 8 offers the first results of an elaborate discussion of the filtering problem which can be described as follows: Let (Ω, F, P) be a complete probability space and let (F_t), $0 \leqslant t \leqslant T$, be a non-decreasing family of right continuous σ-algebras containing the P-null sets. Let (θ, ξ) be a 2-dimensional stochastic process, where $\theta = (\theta_t, F_t)$, $0 \leqslant t \leqslant T$, is an unobservable component, and $\xi = (\xi_t, F_t)$, $0 \leqslant t \leqslant T$, is an observable component. Optimal filtering for the partially observable process (θ, ξ) means the construction for each $0 < t \leqslant T$ of an optimal mean square estimate of some F_t-measurable function h_t of (θ, ξ), given the results ξ_s, $s \leqslant t$, of the observation. In the special case of $Eh_t^2 < \infty$ the optimal estimate is the conditional expectation $\pi_t(h) := E(h_t / F_t^\xi)$ where $F_t^\xi := A(\{\xi_s : s \leqslant t\})$. Without the special assumptions on h and ξ, $\pi_t(h)$ is difficult to compute. Less complicated are situations in which (h, ξ) and therefore $\pi_t(h)$ satisfy certain stochastic differential equations. Such hypothesis yields to a thorough analysis of optimal nonlinear filtering, optimal non-linear extrapolation and optimal linear nonstationary filtering. The latter method was suggested by R.E. Kalman and R.S. Bucy: Let the process θ be inaccessible for observation such that only the process ξ with insufficient information on θ can be observed. The problem solved by Kalman and Bucy is to estimate (or filter) in an optimal way the values θ_t on the basis of the process $\xi_0^t = \{\xi_s, 0 \leqslant s \leqslant t\}$.

Starting with chapter II the authors continue the discussion of the Kalman-Bucy scheme within the framework of Gaussian processes and sequences. Here we find applications of filtering equations to problems in statistics (linear estimation) and in control (incomplete data) and information theory (mutual information, optimal coding). There is a whole chapter (chapter 17) devoted to parametric estimation and testing statistical hypotheses for diffusion type processes. Special emphasis deserve the last two chapters 18 and 19 which deal with point (counting) processes. In analogy to the treatment of Itô and diffusion type processes the following topics are taken up: structure of martingales of point processes, of related innovation processes, and the structure of Radon-Nikodym derivatives. The theoretical development is applied to problems of filtering and estimation of unknown parameters from the observations of point processes.

The monograph under review is a valuable contribution to the literature of stochastic processes, written with great competence and skill, on the basis of an extended research experience in the field. The authors cover a remarkable portion of the available literature (217 references) including their own important results. Despite an enormous quantity of useful technical details the main streamline of ideas is preserved. It seems as if the printing was done without the necessary attention: numerous misprints keep the reader irritated.

H. Heyer, Tübingen