SOME REMARKS ON DANJON'S LAW

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(Received 4 March, 1974)

Abstract. A hypothetical scattering layer in the upper atmosphere is unable to explain quantitatively the variations of lunar eclipses described by Danjon's law.

A new discussion of lunar eclipses since 1900 (Link, 1974) has confirmed the validity of Danjon's law (Danjon, 1920) describing the variations of the luminosity of lunar eclipses during the solar cycle of 11 years. In addition it was found that the internal mechanism of the law is not the absorption of the light in the terrestrial atmosphere as originally assumed Danjon, but an additional light to the normal solar illumination of the eclipsed Moon.

Among possible sources of this additional light, Vassy (1956) proposed the scattering of solar light on some centers in the upper atmosphere. Their production controlled by the solar activity should explain, in consequence, the Danjon's law. We

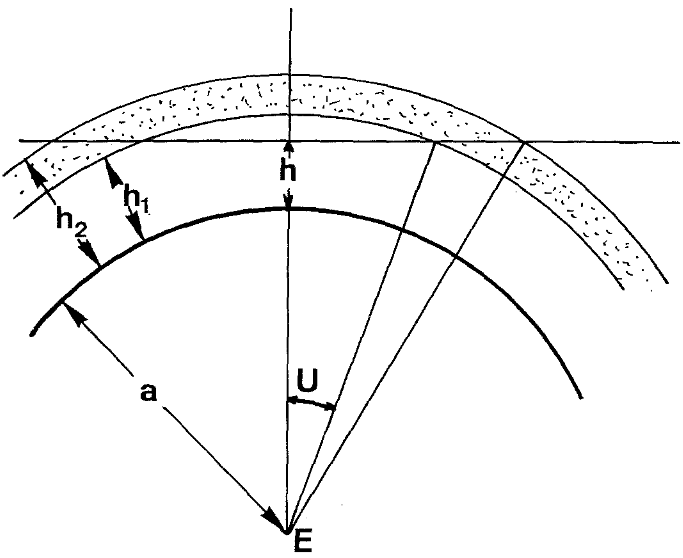

Fig. 1. Scattering layer in the upper atmosphere.

shall examine here the possible implications of this hypothesis for the twilight and daytime sky as observed from the Earth.

Let us assume a homogeneous scattering layer between the heights $h_2 > h_1$ (Figure

1). The luminance of this layer observed along the path x will be ignoring the absorption

$$b = k(\vartheta)\, x\,, \tag{1}$$

where $k(\vartheta)$ is the function of the scattering angle ϑ and the composition of the scattering medium.

The path for the lunar observer (see Figure 1) is given by

$$x_1 = \sqrt{8a}\,[\sqrt{h_2 - h} - \sqrt{h_1 - h}]\,, \quad a = 6370\ \text{km}\,. \tag{2}$$

An elementary ring of the layer between the heights h and $h + \mathrm{d}h$ is seen from the Moon under the solid angle

$$\mathrm{d}\omega = 6.28\pi_{☾}^2 \left(1 + \frac{h}{a}\right) \frac{\mathrm{d}h}{a}\,, \tag{3}$$

and its luminance according to (1) will, therefore, be

$$b_1 = k(\vartheta_1)\sqrt{8a}\,[\sqrt{h_2 - h} - \sqrt{h_1 - h}] \tag{4}$$

The ring produces on the Moon an elementary illumination

$$\mathrm{d}E = b_1\, d\omega = 6.14 \times 10^{-5} k(\vartheta_1)\,[\sqrt{h_2 - h} - \sqrt{h_1 - h}]\, \mathrm{d}h \tag{5}$$

where we neglected

$$\frac{a}{h} < 3\%$$

The illumination by the whole layer is, in consequence,

$$E = \int_0^{h_2} \mathrm{d}E = 4.1 \times 10^{-5} k(\vartheta_1)\,[\sqrt{h_2^3} - \sqrt{h_1^3}]\,, \tag{6}$$

where we assumed the perfect transparence of the terrestrial atmosphere from the surface $h = 0$.

The observed excess of the luminosity of an eclipse can be expressed by

$$E = \omega_{\odot} b_{\odot} \varepsilon = 6.8\ 10^{-5} b_{\odot} \varepsilon\,, \quad \varepsilon < 1\,. \tag{7}$$

Comparing both expressions for E (6) and (7) we get for

$$k(\vartheta_1) = 1.65 \frac{b_{\odot}\varepsilon}{\sqrt{h_2^3} - \sqrt{h_1^3}}\,. \tag{8}$$

As the twillight sky is concerned we have for the path $x_2 = h_2 - h_1$ if we observe in zenith and, in consequence,

$$\frac{b_2}{b_{\odot}} = 1.65\varepsilon \frac{h_2 - h_1}{\sqrt{h_2^3} - \sqrt{h_1^3}} \times \frac{k(\vartheta_2)}{k(\vartheta_1)} = 1.65\varepsilon f(h_1, h_2) \frac{k(\vartheta_2)}{k(\vartheta_1)}\,. \tag{9}$$

Some values of the function $f(h_1, h_2)$ are given below:

h_2	h_1	$f(h_1, h_2)$	U	
84 km	76 km	7.4×10^{-2}	8.9°	
105	95	6.6	9.9	$\bar{f}(h_1, h_2) = 6 \times 10^{-2}$.
210	190	4.8	13.1	

The value of the function depends but little on the width $h_2 - h_1$ of the layer. The luminance of the twilight sky given by (9) can be observed until the solar depression U (Figure 1), constrained by the inequality

$$U < \arccos\left(1 - \frac{h - 20}{a}\right) \tag{10}$$

if we assume the perfect transparence of the atmosphere for $h > 20$ km.

In order to confront our theory with observations we adopt following extreme values of the shadow density measured by Dubois (Link, 1960) at 10′ from the edge in green light during the 18th solar cycle

Eclipse	Density	Illumination
1950 IX 26	3.00	0.001
1954 I 18	4.20	0.00006
		$\varepsilon = 0.001$

and we consider the difference $\varepsilon = 0.001$ as the consequence of the Danjon's law. Putting it in (9) we get

$$\frac{b_2}{b_\odot} > 10^{-9} \quad \text{for} \quad U < 9°, \tag{11}$$

where according to calculations by Giese (1971) for different kinds of dust

$$\frac{k(\vartheta_2)}{k(\vartheta_1)} > 10^{-5}, \quad \begin{array}{l} \vartheta_2 = 100° \\ \vartheta_1 = 0° \end{array} \tag{12}$$

The observations at $U = 9°$ gave for the twilight luminance the limits (Link, 1973)

$$10^{-12} < \frac{b_2}{b_\odot} < 10^{-11}. \tag{13}$$

In other words the proposed hypothesis of the scattering layer would have as consequence exceedingly bright twilight sky which never has been observed.

Let us turn now toward the daytime sky. For its luminance we find from (9) that

$$\frac{b_3}{b_\odot} = 1.65\varepsilon \frac{k(\vartheta_3)}{k(\vartheta_1)} f(h_1, h_2), \tag{14}$$

where according to Giese (1971) we put

$$\frac{k(2.5°)}{k(0°)} > 10^{-2}; \tag{15}$$

so that, finally,

$$\frac{b_3}{b_\odot} > 10^{-6}. \tag{16}$$

Dollfus measured from stratospheric ballons the sky luminance at 2.5° from the Sun center at 31–32 km in infrared 8400 Å. Four series between 1967 and 1971 gave (Dollfus, 1974) values between 4 and $6 \times 10^{-9} b_\odot$. Due to the instrumental parasitic light the value 5×10^{-9} should be considered as upper limit of the genuine sky brightness. In other words the daytime sky too is far from the luminance as it would be needed for the proposed explanation of Danjon's law.

In 1947 we proposed (Link, 1947) for the explanation of Danjon's law the luminescence of lunar surface. However, this explanation which fits also for other lunar phenomena (Link, 1972) is in conflict with recent results of laboratory examinations of lunar samples which gave to low luminescence in comparison with lunar phenomena. Unless supposing some drastic differences between the lunar and laboratory conditions we are obliged to abandon the luminescence hypothesis.

Our actual situation is, therefore, an uncomfortable one. Both explanations of Danjon's law – i.e., the scattering layer or the luminescence governed both by solar corpuscular radiations – describe well the general features of Danjon's law but are both unsatisfactory from the quantitative point of view.

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