On the mutual noncompatibility of homogeneous analytic non-power means

MARCIN E. KUCZMA

Homogeneous symmetric means μ and $\tilde{\mu}$, defined on $\mathbb{R}_+^n$ and $\mathbb{R}_+^{n+1}$, respectively, are called *compatible* if the value of $\tilde{\mu}$ remains unchanged upon replacing n of its arguments by their μ-mean. Power means (of a common exponent) are a model example, which turns out to be unique, given analyticity of at least one of the two means considered. This is proved by fixing all but one argument in both μ and $\tilde{\mu}$, which leads to a functional equation with two unknown functions, involving their mutual superpositions. The equation is solved in the class of analytic functions by comparing the power series coefficients.