## Convexity-like inequalities for averages in a convex set

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For a function $f: X \to R \cup \{+\infty\}$, convex and finite over an algebraically open convex subset $C$ of a linear space $X$, we provide a simple necessary and sufficient condition for the "convexity inequality"

$$f\left(\sum \lambda_i x_i\right) \leqslant \sum \lambda_i f(x_i) \qquad (\mathbf{C}_n)$$

to be satisfied for every integer $n \geqslant 2$ and all $x_1, \ldots, x_n \in X$ and $\lambda_1, \ldots, \lambda_n \in R_+$ such that $\sum \lambda_i = 1$ *and*

$$\sum \lambda_i x_i \in C$$

(i.e. if *just the average* $\sum \lambda_i x_i$ lies in the set $C$ of convexity, *not* the points $x_1, \ldots, x_n$ themselves).

Our condition is simply that $f$ *majorizes* a certain convex function, depending only on the restriction $f_C$ of $f$ to $C$, namely the *smallest convex extension* of $f_C$, which is given explicitly.

We apply this result to *enlarge* the set of $n$-tuples $(x_1, \ldots, x_n) \in X^n$ satisfying $(\mathbf{C}_n)$, beyond $C^n$, the domain ensured by the convexity of $f$ on $C$. A number of examples show that such an *extension of the domain of validity* of $(\mathbf{C}_n)$ can be easily obtained when $X = R$.

## Sur les solutions globales de l'équation des cocycles

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Let $\Phi: \mathbb{R}^+ \times X \mapsto X$ be a semidynamical system and $f: \mathbb{R}^+ \times X \mapsto \mathbb{R}^+$ be a cocycle of $(X, \Phi)$, i.e. $f$ verifies the following functional equation:

$$\forall x \in X; s, t \geqslant 0: \qquad f(s + t, x) = f(t, x) f(s, \Phi(t, x)). \qquad \text{(E)}$$

Under convenient conditions, we give an explicit form of the globally measurable or continuous solutions of (E) and we study the particular case when $X = \mathbb{R}$.