Bharucha-Reid (Proc. Nat. Acad. Sci., U.S.A., 60 (4), 1170–1174, 1968). It is found that the representation $S(t)$, $U(t) = (V(t) \pm W(t))/2$, where $V(t)$, $W(t) = S(t) \pm U(t)$, holds in the strong operator topology. Further if $A = s\text{-}\lim_{t \rightarrow 0} (S(t) - I)(t)$, $B = s\text{-}\lim_{t \rightarrow 0} U(t)/t$, then $A + B$ and $A - B$ are infinitesimal generators of semigroups $\{V(t)\}$ and $\{W(t)\}$ respectively. If the intersection of the domains of A and B is dense in $\mathcal{X}$, then A and B are closed linear operators, and a necessary and sufficient condition (of Hille-Yosida type) for the above results to hold is that there exist constants $M > 0$ and β , such that for all real $\lambda > \beta$, λ belongs to resolvent sets of $A + B$ and $A - B$, and $\|R(\lambda; A \pm B)\| \leq M(\lambda - \beta)^{-n}$, $n = 1, 2, \dots$, where $R(\lambda; A \pm B)$ are the resolvent operators, (cf. Hille and Phillips, Functional Analysis and Semi-groups, Amer. Math. Soc. Colloq. Pub., 31, Providence, R.I., 1957). The following is a concrete example on $C[-\infty, \infty]$: $S(t)X(u) = \{\exp(-\lambda t)\} \sum_{k=0}^{\infty} ((\lambda t)^{2k}/(2k)!) X(u - 2k\alpha)$, $U(t)X(u) = \{\exp(-\lambda t)\} \sum_{k=0}^{\infty} ((\lambda t)^{2k+1}/(2k+1)!) X(u - (2k+1)\alpha)$, $X \in C[-\infty, \infty]$, $\lambda, \alpha > 0$.

Regularity Properties of Functional Equations*)

JOHN A. BAKER

The following result is representative of the kind of regularity properties considered in this paper.

Suppose G is a locally compact topological group, a_i is a bicontinuous automorphism of G and $b_i: G \rightarrow G$ is continuous for $1 \leq i \leq n$. Let X_i ($0 \leq i \leq n$) be Banach spaces and let $F: \prod_{i=1}^n X_i \rightarrow X_0$ be continuous. Suppose $f_i: G \rightarrow X_i$ ($1 \leq i \leq n$) and $f: G \rightarrow X_0$ such that

$$f(y) = F[f_1(a_1(x) b_1(y)), \dots, f_n(a_n(x) b_n(y))]$$

for all $x, y \in G$. If f_i is strongly measurable (with respect to Haar measure) for $1 \leq i \leq n$ then f is continuous.

Zur Linearität verallgemeinerter Modulismetrien**)

JÜRGEN RÄTZ

Die bekannte Tatsache, dass eine isometrische Abbildung φ zwischen zwei euklidischen Vektorräumen M , N notwendigerweise eine affine Abbildung ist, wurde nach verschiedenen Richtungen hin verallgemeinert (für reelle lokalkonvexe Vektorräume vgl. z.B. [1], p. 110–111, für Vektorräume über Skalarkörpern der Charakteristik $\neq 2$ vgl. etwa [2], [3]).

In der vorliegenden Note werden die Resultate von [2] wie folgt auf Moduln über geeigneten Skalarringen ausgedehnt:

*) Cf. pp. 243–248.

**) Vgl. S. 249–255.