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Erratum

On the Euclidean Version of Haag's Theorem in $P(\varphi)_2$ Theories

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As communicated to the author by Newman, estimate (7) does not seem to follow from the results of [7], or at least not easily. (We use the literature quoted in our paper and continue the numeration). As Newman pointed out, however, the weaker result holds

$$E(|E_l(X_{2l,\lambda} - X_{l',\lambda})|) \leqq O(e^{-\varepsilon l}) \tag{12}$$

for $l' \geqq 2l$ and some $\varepsilon > 0$.

With this we may indeed prove

$$E(X_{l',\lambda} A_l) \leqq O(e^{-\varepsilon l}) \qquad (l' \geqq 2l) \tag{13a}$$

and by the $\lambda \leftrightarrow \lambda'$ symmetry

$$E(X_{l',\lambda'} C A_l) \leqq O(e^{-\varepsilon l}) \qquad (l' \geqq 2l)\,. \tag{13b}$$

By the previous arguments, these estimates are of course sufficient for the proof of the theorem. To prove estimate (13a), by Newman's estimate (12), it is sufficient to consider $E(X_{2l,\lambda} A_l)$. Let $V_D = V_{2l,\lambda} - V_{l,\lambda}$ then by the definition of A_l:

$$E(X_{2l,\lambda} A_l) \leqq E(e^{-V_{2l,\lambda}})^{-1} \left(E(e^{-V_{l,\lambda}})\, E(e^{-V_{l,\lambda'}})^{-1}\right)^{\delta} E(e^{-V_D - V_{l,\lambda(\delta)}}) \tag{14}$$

for any $0 < \delta < 1$ with $\lambda(\delta) = \delta\lambda' + (1-\delta)\,\lambda$.

By the Feynman-Kac-Nelson formula, the last factor on the r.h.s. of inequality (14) is

$$K = (\Psi_0, e^{-\frac{1}{2}H_{2l,\lambda}} e^{-l\hat{H}} e^{-\frac{1}{2}H_{2l,\lambda}} \Psi_0) \tag{15}$$

in the standard notation with

$$\hat{H} = H_0 + \int_{X \in \mathbb{R}} g(x) : P(\varphi(0,x)) : dx$$

$$g(x) = \begin{cases} \lambda & -l \leqq x \leqq -\frac{l}{2}; \quad \frac{l}{2} \leqq x \leqq l \\ \lambda(\delta) & |x| \leqq \frac{l}{2} \\ 0 & \text{otherwise}. \end{cases}$$

Let $\hat{E}$ be the groundstate energy of $\hat{H}$ and $E_{2l,\lambda}$ the groundstate energy of $H_{2l,\lambda}$. Then

$$\begin{aligned} K &\leqq (\Psi_0, e^{-lH_{2l,\lambda}}\Psi_0)\, e^{-l\hat{E}} \\ &\leqq e^{-lE_{2l,\lambda} - l\hat{E}}. \end{aligned} \tag{16}$$

By Theorem 1.7 in [3]

$$-\hat{E} \leqq l(\alpha_\infty(\lambda) + \alpha_\infty(\lambda(\delta))) \tag{17}$$

and

$$-E_{2l,\lambda} \leqq 2l\alpha_\infty(\lambda). \tag{18}$$

Combining these estimates with estimate (1) and choosing δ as in (2) gives

$$E(X_{2l,\lambda}A_l) \leqq O(e^{-\varepsilon l^2 + O(l)})$$

q.e.d.

Meanwhile a simpler proof exists, giving essentially the same but not identical results. It is based on the strong mixing (i.e. cluster) and hence ergodicity property of the measure. It has or will have appeared in preprints by Fröhlich, J. Rosen and Simon and by Hegerfeldt.

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