Corrections · Berichtigungen

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Additions and Corrections to Algorithms for motions with constant stretch history-II.

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In a series of papers (1, 2, 3) the author has discussed motions with constant stretch history of various types. As is well known, in these flows, the strain history has the form

$$C_t(t-s) = e^{-sL_1(t)^{\mathrm{T}}} e^{-sL_1(t)}, \qquad [1]$$

where L_1^T is the transpose of L_1. Now, suppose that L_1 commutes with L_1^T, i.e.,

$$L_1 L_1^T = L_1^T L_1 . \qquad [2]$$

Then

$$C_t(t-s) = e^{-sA_1(t)}, \qquad [3]$$

where

$$A_1 = L_1 + L_1^T \qquad [4]$$

is the first Rivlin-Ericksen tensor (4). Therefore, the condition [2] is sufficient that a given motion with constant stretch history be a simple extensional flow (5).

Now, let a velocity field v be such that its gradient L obeys $\dot{L} = 0$. Then, this motion has constant stretch history (2, 3). If, in addition, L obeys [2], i.e., $LL^T = L^T L$, then v is a simple extensional flow from what we have just seen.

In the literature, the example of a simple extensional flow is usually that due to *Coleman* and *Noll* (5):

$$\dot{x}_i = a_i x_i, \quad i = 1,2,3; \quad \text{no sum}; \qquad [5]$$

$$a_1 + a_2 + a_3 = 0 . \qquad [6]$$

Whether [6] is satisfied or not, the velocity gradient of [5] is symmetric. We now show that

$$v = Qx, \quad QQ^T = 1, \quad \dot{Q} = 0, \qquad [7]$$

is a simple extensional flow. The proof is trivial because the velocity gradient Q obeys $\dot{Q} = 0$ and $QQ^T = Q^T Q$, i.e., Q commutes with Q^T. It is clear that Q is an orthogonal tensor and hence tr $Q \neq 0$, i.e., the velocity field in [7] is not isochoric. For an isochoric, unsymmetric velocity gradient example, see (1).

This note provides the author with an opportunity to point out that the correct solution to the differential equation

$$\dot{Q}Q^T = Z, \quad Z = -Z^T, \quad Q(0) = 1, \qquad [8]$$

is given by (6)

$$Q(t) = 1 + \int_0^t Z(\tau)\, Q(\tau)\, d\tau . \qquad [9]$$

In eqs. [8] of (1), [4.14] and [6.10] of (2) and [12] of (3), it was incorrectly stated that

$$Q(t) = \exp\left(\int_0^t Z(\tau)\, d\tau\right).$$

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