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Department of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

Some comments on a paper by V. V. Gokhale

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R. A. Brown

With 1 figure

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The paper by V. V. Gokhale (1) attempts to explain the longitudinal striations observed on a planar jet issuing from a rectangular slit as being related to capillary waves on a stationary liquid sheet. Gokhale applied linear stability theory to analyse for waves on an infinite planar sheet of viscous liquid when the liquid is unaffected by body forces. The coordinate system for the analysis is shown in figure 1. Gokhale's results predicted that some disturbances to an initially quiescent liquid sheet grow in time.

Fig. 1. Geometry of a liquid sheet showing a symmetric perturbation

These results are counter to physical intuition and contradict previous work on the stability of planar films and sheets (2–4). The source of energy for Gokhale's growing disturbances is a mystery. The energy in a system comprised of liquid held between planar interfaces is stored in the form of surface energy due to surface tension and the amount of this energy is proportional to the total surface area of the interfaces. Any perturbation to the shape of the interfaces increases the interfacial area and hence the surface energy. Kinetic energy from the disturbance must be converted to surface energy to account for this increase. This argument implies that surface

tension *stabilizes* disturbances to a planar interface and is consistent with other works (2–6) with the exception of Gokhale's.

In the remainder of this note it is shown that Gokhale's results are artifacts of two errors in his analysis. The notation used in the following comments is identical to that in (1). The first error in Gokhale's analysis occurred in his formulation of the balance of normal stress at the gas/liquid interfaces. For an almost planar interface the appropriate form of this boundary condition is

$$2\mu \frac{\partial^2 \Psi}{\partial x \partial y} - p = \frac{\sigma}{s} \frac{\partial^3 \Psi}{\partial x^3}, \quad \text{at } y = \pm a, \quad [1]$$

where Ψ is the Stokes stream function and p is the dynamic contribution to the pressure in the liquid. Gokhale omitted the pressure from the balance of normal stress (see (1), eq. [5]). This omission is partially responsible for his erroneous conclusions. The other mistake by Gokhale is his mathematical description of a symmetric or varicose interface disturbance, like that shown in figure 1. For such a disturbance the components of velocity in the liquid caused by the perturbation must satisfy the conditions $v_y(x, y, t) = -v_y(x, -y, t)$ and $v_x(x, y, t) = v_x(x, -y, t)$. The solution of the time-dependent Stokes equation ((1), eq. [4]) that has these symmetries is

$$\Psi(x, y) = 2[c_1 \sinh(ky) + c_3 \sinh(Ky)] e^{-ikx} e^{st} \quad [2]$$

with $K \equiv (s\rho/\mu + k^2)^{1/2}$. Eq. [2] is not the same as Gokhale's eq. [6] with $c_1 = c_2$ and $c_3 = c_4$ as he suggests.

A characteristic equation governing the time constant s is easily derived using eq. [1] and [2] in place of Gokhale's results. The dynamic pressure in eq. [1] is evaluated from eq. [2] and the x -component of the momentum equation. The correct characteristic equation is

$$\left\{ 4\mu kK + \frac{2}{k}(sk\rho - \mu k^2 K + \mu K^3) \right\} \sinh(ka) \cosh(Ka) \\ - \left(2\mu + \frac{s\rho}{k^2} \right) \left(\frac{s\rho}{\mu} + 2k^2 \right) \cosh(ka) \sinh(Ka) \\ - \frac{\sigma k}{\nu} \sinh(ka) \sinh(Ka) = 0. \quad [3]$$

To analyse for the stability ($Re(s) \leq 0$) or instability ($Re(s) > 0$) of the film it is sufficient to consider an inviscid ($\mu = 0$) liquid since for initially static fluids viscosity always attenuates any disturbance but does not alter the value of k at the inception of an instability caused by an infinitesimal perturbation (7). Letting μ approach zero in eq. [3] yields

$$s^2 = - \frac{\sigma k^3}{\rho} \tanh(ka), \quad [4]$$

which in the limit of a thin film ($a \ll 1$) agrees with the work of Vrij et al. (see eq. [16] in (3)). Clearly, inviscid disturbances of any wavenumber k oscillate in time, but none grow. Viscosity damps out these disturbances.

The wavy meniscus shapes observed by Gokhale cannot be attributed entirely to a capillary instability

as suggested in (1). These undulations are more likely caused by an interaction between the rearranging velocity field near the edge of the die and the meniscus.

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Author's address:

Professor R. A. Brown
Department of Chemical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139 (U.S.A.)