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Erratum to Localization on singular varieties

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There is an error in the proof of Theorem 3.3 (p. 447). Retaining the notations of that proof, we incorrectly claimed that a variety of the form $\mathbf{A}^n/G$ is a rational variety. There are counterexamples to this, due to Saltman [S]. The theorem remains correct as stated however, and the main thread of the argument is the same. The following should replace paragraph 3 of page 448, "Since G acts linearly...".

Let W be the blowup of $\mathbf{A}^n$ at the origin 0, P the exceptional divisor. The inclusion of P in W is split by the map π: $W \to P$ sending a line through the origin to the associated point on $P = \mathbf{P}T_0(\mathbf{A}^n)$. G acts on W and on P, and since the action is linear, the projection π: $W \to P$ induces a map p: $W/G \to P/G$. Each fiber of p is an affine line, and we have a morphism h: $W/G \to X$ which is an isomorphism over U. We may assume that the map f: $\bar{Z} \to \bar{X}$ factors through some projective closure of W/G.

If V is a Noetherian scheme, we have the Quillen spectral sequence

$$E_1^{p,q}(V) = \bigoplus_{x \in V^p} K_{-p-q}(k(x)) \Rightarrow K'_{-p-q}(V).$$

For Z a closed subscheme of V of pure codimension d, $U = V - Z$, there is an exact localization sequence for the E_2 terms:

$$\to E_2^{p-d,q-d}(Z) \to E_2^{p,q}(V) \to E_2^{p,q}(U) \to E_2^{p-d+1,q-d}(Z) \to.$$

If V is a regular scheme over a field, then Quillen [Q] shows there is a natural isomorphism $H^p(V, \mathcal{K}_q) \cong E_2^{p,-q}(V)$.

Lemma. *Let V be an n-dimensional variety over an algebraically closed field k. Suppose V is a finite union of disjoint, locally closed subsets V_i, where each V_i is a locally trivial $\mathbf{A}^1$ bundle, p_i: $V_i \to B_i$. Then the map $CH_1(V) \otimes k^* \to E_2^{n-1,-n}(V)$ is surjective.*

Proof. Sherman [Sh] shows that the E_2 term is a homotopy invariant. Thus, for a variety B, the map $p_1{}^*$: $E_2^{r-1,-r}(B) \to E_2^{r-1,-r}(B \times \mathbf{A}^1)$ is an isomorphism.

If B has dimension $r-1$, then $E_2^{r-1,-r}(B)$ is generated by $CH_0(B)\otimes k^*$, so $E_2^{r-1,-r}(B\times \mathbf{A}^1)$ is generated by $CH_1(B\times\mathbf{A}^1)\otimes k^*$. The result then follows from localization. □

We can stratify P/G by locally closed subsets B_i so that $p: p^{-1}(B_i)\to B_i$ is a locally trivial $\mathbf{A}^1$ bundle. From the lemma, it follows that $E_2^{n-1,-n}(W/G)$ is generated by $CH_1(W/G)\otimes k^*$. Factoring the map $H^{n-1}(\bar{Z}, \mathscr{K}_n)\to H^{n-1}(U, \mathscr{K}_n)$ through $E_2^{n-1,-n}(W/G)$, it follows that the image of $H^{n-1}(\bar{Z}, \mathscr{K}_n)$ in $H^{n-1}(U, \mathscr{K}_n)$ is generated by $CH^{n-1}(U)\otimes k^*$. Since $H^{n-1}(\bar{Z}, \mathscr{K}_n)\to H^{n-1}(Z, \mathscr{K}_n)$ is surjective, this implies that the image $\mathrm{Im}H$ of $H^{n-1}(Z, \mathscr{K}_n)$ in $H^{n-1}(U, \mathscr{K}_n)$ is generated by $CH^{n-1}(U)\otimes k^*$. The surjective map $H^{n-1}(Z, \mathscr{K}_n)\to F^n SK_0$ factors through $\mathrm{Im}H$, hence $F^n SK_0$ is divisible. $F^n SK_0$ is $|G|$-torsion by Theorem 2.7, hence zero.

In addition, there is an incorrect reference on page 449, line 3: the injectivity of the Bloch map was shown by Merkurjev and Suslin in [M-S]. Finally, the paper [Q2] "Higher algebraic K-theory II" was written by D. Grayson, after notes of D. Quillen.

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References

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