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Erratum

On the representation of integers by binary cubic forms of positive discriminant

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On page 120 of the above article I derived Theorem 1 from Theorem 3 by showing the following:

Proposition. *Let $F(x, y) \in \mathbb{Z}[x, y]$ be a binary cubic form of positive discriminant which is reduced and irreducible over $\mathbb{Q}$. Then the equation*

$$F(x, y)=1 \quad in \quad x, y \in \mathbb{Z} \tag{1}$$

has at most three solutions with $|x y| \leqq 1$.

Our proof of this proposition was not correct and in fact the proposition is not true. For if F is equal to the reduced form $F_1(x, y)=x^3+x^2 y-2x y^2-y^3$ of discriminant 49 then (1) has the solutions $(x, y)=(1,0),(0,-1),(-1,1),(-1,-1)$. But Baulin showed (cf. p. 117 of the above article) that (1) has exactly nine solutions if $F=F_1$ and this is better than the result of Theorem 1.

We shall now give a correct proof of the proposition, however under the assumptions that F has positive discriminant and is not equivalent to F_1. This suffices to prove Theorem 1.

Similar to the arguments on p. 120 of the above article we may assume that $F(1,0)=1$ and that (1) has two solutions with $|x| \leqq 1$ and $y=1$. Then

$$F(x, y)=(x-p y)(x-q y)(x-r y)+y^3 \quad \text{with } p, q \in\{-1,0,1\}, p>q, r \in \mathbb{Z}. \tag{2}$$

If (1) has three solutions with $y=1, |x| \leqq 1$ then $|r| \leqq 1$, $r \neq p$, $r \neq q$. But then F is a form of discriminant -23 which case was excluded. If (1) has a solution with $|x| \leqq 1, y=-1$ then

$$(x+p)(x+q)(x+r)=2.$$

If $x=-1$ then $p=0$, $q=-1$, $r=2$ and $F(x, y)=F_1(x,-y)$. If $x=0$ then $p=1$, $q=-1$, $r=-2$ and F has discriminant -87. Finally, if $x=1$ then $p=1$, $q=0$, $r=0$ and F has discriminant -23. Hence if F is of type (2) then the only solutions of (1) with $|x y| \leqq 1$ are $(1,0)$, $(p, 1)$, $(q, 1)$. This proves the proposition, under the restrictions that F has positive discriminant and is not equivalent to F_1.