The Geneva Papers on Risk and Insurance Theory, 19: 153–156 (1994)

Review Article of Devolder's *Finance Stochastique*

FRANÇOIS QUITTARD-PINON
Ecole Normale Supérieure de Cachan, 61, Avenue du Président Wilson, 94235 Cachan Cedex, France

Finance Stochastique by Pierre Devolder is a textbook dealing with the foundations of financial theory. Originally designed for students in actuarial sciences, this book will be of interest for doctoral students and researchers in finance. For a long time the French-speaking community did not find French books covering this field in the same way as *Theory of Financial Decision Making* by Jonathan Ingersoll, *Foundations for Financial Economics* by Chi-Fu-Huang and Robert Litzenberger, *Finance Theory* by Robert Jarrow, *Prices in Financial Markets* by Michael Dothan, *Continuous-Time Finance* by Robert Merton, *Dynamic Asset Pricing Theory* by Darell Duffie, now available in French. Devolder's work joins a small list of French books including *Méthodes mathématiques de la finance* by Gabrielle Demange and Jean-Charles Rochet, *Introduction au calcul stochastique appliqué à la finance* by Damien Lamberton and Bernard Lapeyre, *Marchés des capitaux et théorie financière* by François Quittard-Pinon. Students need to have many presentations and several points of view in financial theory. Devolder's goals are to give a rigorous treatment of the financial subjects he chooses and to link financial theory with stochastic processes theory. To achieve his goals, the author claims that only two ways are available: either to present mathematical tools needed to solve financial problems, (the way chosen by Demange and Rochet) or to rally financial instruments round mathematical tools. Pierre Devolder uses this solution to work out the plan of his book, which gives him the following design: stocks, bonds, options. I disagree with this organization because it implies too many constraints. Other choices are, in my opinion, more appropriate—for example, those that privilege financial methods independently of particular financial instruments. To deal with financial theory one has essentially two ways to proceed—either use the headings of arbitrage, optimality, and equilibrium as done in Darell Duffie's book or use as guidelines financial risk in mono-periodic, discrete time and continuous time analysis, as I did in my book.

Pierre Devolder's three well-balanced chapters present the same features: a key idea is developed with a simple question and then with increasing complexity and realism, and the chapter ends generally with an application that illustrates the methods analyzed.

The first chapter is devoted to stocks and is built on the stochastic representation of stock prices and capital-asset pricing models. In the first part, the key idea is the compounding of stocks prices. It begins with the well-known Gordon-Shapiro model and then progressively expresses the prices dynamic with more and more realistic assumptions: First, in a riskless world in discrete and continuous time and then in a risky universe with successively a binomial process, a Poisson process, and finally diffusions. This material paves the way for a general representation of stocks prices through semi-martingales. It is, in my opinion, the most interesting part of the chapter and probably of the entire book. In nineteen dense pages the author convinces his reader that this representation is the definitive tool and shows

that all previous attempts to model prices are only particular cases. This section justifies the writing of the book. The author goes, however, well beyond it and uses this development to make a fine analysis of actualization under risk, and his approach is, as far as I know, new. It should be noted that it could be seen as an original contribution, but there is no free lunch and the rather abstract level reached demands efforts that could prove expensive. So the appendix on semi-martingales is welcome. Nevertheless, I think many will be uncomfortable with this. Fortunately, a full understanding of the topic covered here is not necessary for reading the other parts. A beginner in finance can go back to the flavor of this paradigm after he becomes comfortable with stochastic finance. After this flight over high peaks the remaining of the chapter seems a mere promenade. The portfolio choice and the capital-asset pricing model are treated in the standard fashion in a monoperiodic environment. The key idea of the beginning is lost, and there is a breakdown in sophistication. To get in touch with the preceding paragraph the author could have presented, for instance, the work of Hindy and Huang [1992]. The chapter ends with a brief discussion of utility theory and a too-short presentation of the intertemporal capital-asset pricing model.

In the second chapter, entitled "Bonds," the key idea is, of course, the management of interest-rate risk. The only relative difficulty lies in Heath, Jarrow, and Morton's model, although treated in discrete time. The application chosen is asset-liabilities management. After the usual and necessary definitions on bonds, Devolder illustrates with a simple but relevant example the interest-rate risk. Devolder develops then the classical models of the term structure of interest rates: the work of Vasicek, Brennan, and Schwartz then the new methodology of Ho and Lee and of Heath, Jarrow, and Morton are fully scrutinized and discussed. What is new in this approach is that no information is lost and the actual yield curve is integrated as an exogeneous element in these models. It should, however, be remarked that they are arbitrage models and that arbitrage in general is studied only in Chapter 3: it is one of my objections to the organization of this book. After these second-generation models, come duration analysis and its corollary, immunization. An application to ALM follows, and Redington's model is presented. The chapter ends with a discussion in discrete time of floating-rate notes.

The key idea of the third chapter, entitled "Options," is arbitrage. Devolder introduces here one of the most important discoveries in financial theory: discounted gain processes are martingales under a suitable probability measure, the risk-neutral one. The change of universe, the historical one for a valuation universe, is a concept of first magnitude and can be considered as revolutionary. In a certain way the forward-neutral measure (not treated here) is a clever use of this concept. These two probability measures are powerful tools for pricing financial assets and derivatives. To help the reader grasp the fundamental methods of finance, Devolder begins with a simple example of the foreign currency option. To stay as simple as possible the example is oversimplified and the problem of interest rates, which is crucial in this domain, is ignored. A simplified example of an option on stock would have been sufficient. Then the Black and Scholes model is developed in full. Two complete and very detailed proofs of the famous formula are given, the traditional one and the solution through martingale. This will certainly help students understand how to tackle this kind of problem. It could have been interesting to follow the same path for Cox and Ross's 1976 option's formula. The problem of pricing options on bonds is treated too briefly. Then comes the analysis of arbitrage in the rigorous and fruitful manner initiated by

Harrison and Kreps and Harrison and Pliska. It's the only difficult part of this chapter, and it stays at a reasonable level, for the question is dealt with, for the main part, in discrete time. The chapter ends with a clever application to the life insurance problem.

The book's five appendices concern results in probability theory used in this book. They are useful and will be much appreciated. The fifth, which introduces the reader to semi-martingales, is only for those who are convinced of the need to invest in this abstract field of probability theory. It's rewarding, but to master this domain is a challenge.

References are organized by chapter and cover the main works on each subject but are far from exhaustive. I think that for each part, important contributions such as Merton's paper "Optimum Consumption and Portfolio Rules in Continuous Time," Hull and White's article "One Factor Interest Rate Model *s* and the Valuation of Interest-Rate Derivative Securities," and Siegried Müller's work *Arbitrage Pricing of Contingent Claims* should have been included.

No major theme in financial theory has been omitted, although Ross's arbitrage pricing theory could have been treated in chapter one after the CAPM or at least mentioned. In Chapter 2, a paragraph to introduce the forward-neutral probability measure with an application to the valuation of floating rate notes could have been advantageously included. In Chapter 3 traditional arbitrage pricing in continuous time using the fundamental partial differential equation is not present and could have been given. Similarly, Girsanov's theorem and the Feynman-Kac representation, which are more and more used in financial theory and which are, in my opinion, of greater help than semi-martingales for pricing problems, should figure in such a 1994 textbook in financial theory. But all these points are mere suggestions for improvements in a book meeting high standards.

In this book Pierre Devolder achieves this goal: he gives a rigorous treatment of the financial subjects he chooses to discuss and he does it carefully. In this respect, he is near perfection. But mathematical rigor is like the roman god Janus: it has two faces. Rigor warrants what is advanced and makes the reader confident in what the author says, but rigor can lead him to technical roads he does not want to take, and I surmise it will be the case for some parts of this book, at least at first reading. I advise the reader not to give up, for the entire book is worth reading. It is well written. The editor's work joins with the author's search for quality. I would add just a remark for French readers: the author should be congratulated on avoiding English terms when not necessary and his suggestion to translate *pricing* into the French word *tarification* is a good one, so I wonder why he uses the English *capital asset pricing model* in stead of *modèle d'evaluation des actifs financiers*.

I read with a great interest and pleasure Devolder's book. It is excellent, I am sure of its success, and I highly recommend it.

References

DEVOLDER, P. [1993]: *Finance Stochastique*. Université de Bruxelles.

DEMANGE, G. AND ROCHET, J.C. [1992]: *Méthodes mathématiques de la finance*. Economica.

HINDY, A. and HUANG, C. [1992]: "Intertemporal preferences for uncertain consumptions: a continuous time approach," *Econometrica*, 60, 781–801.

HULL, J. and WHITE, A. [1993]: "One-factor interest rate models and the valuation of interest-rate derivative securities," *Journal of Financial and Quantitative Analysis*, 28, 235-254.

LAMBERTON, D. and LAPEYRE, B. [1991]: *Introduction au calcul stochastique appliquée à la finance*. Ellipses.

MULLER, S. [1985]: *Arbitrage Pricing of contingent claims*. Lectures Notes in Economics and Mathematical systems, Springer Verlag.

QUITTARD-PINON, F. [1993]: *Marchés des capitaux et théorie financière*. Economica.