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Erratum

Quasi-static mode III fracture in a nonhomogeneous viscoelastic body

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J. M. Herrmann, College Station, and **L. Schovanec,** Lubbock, Texas

If α is negative, the transfer function $T(p)$, (2.13), has a zero of order p^2 at $p = 0$. This necessitates a different factorization of $T(p)$ than in the case of α positive so that $\log(T(p))$ is well defined. The correct factorization of this case is $T(p) = T_1(p)\, T_2(p)\, T_3(p)$ where

$$T_1(p) = -\tilde{\mu}(ivp), \qquad T_2(p) = \frac{2p^2}{\sqrt{\alpha^2 + 4p^2}},$$

$$T_3(p) = \left(\alpha + \sqrt{\alpha^2 + 4p^2}\right) \frac{\sqrt{\alpha^2 + 4p^2}}{4p^2} = 1 + \frac{\alpha^2}{4p^2}\left[1 - \sqrt{1 + \frac{4p^2}{\alpha^2}}\right].$$

Note that $1/2 \leqq T_3(p) \leqq 1$ for all real p.

The homogeneous solutions $X_i^{+}(p) = T_i(p)\, X_i^{-}(p)$ are then given by

$$X_1^{+}(z) = 1, \qquad X_1^{-}(z) = \frac{-1}{\tilde{\mu}(ivz)},$$

$$X_2^{+}(z) = \frac{z}{\omega^{+}(z + i\,|\alpha/2|)}, \qquad X_2^{-}(z) = \frac{\omega^{-}(z - i\,|\alpha/2|)}{z},$$

and

$$X_3^{\pm}(z) = \exp\left(\Gamma^{\pm}(z)\right) \quad \text{where} \quad \Gamma^{\pm}(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\log\left(T_3(\tau)\right)}{\tau - z}\, d\tau$$

where

$$T_3(p) = 1 + \frac{\alpha^2}{4p^2}\left[1 - \frac{2}{|\alpha|}\, \omega^{+}(p + i\,|\alpha/2|)\, \omega^{-}(p - i\,|\alpha/2|)\right]$$

is analytic in the lower half plane except along the branch cut $B = \{z = -iq$ where $|\alpha/2| \leqq q < \infty\}$. Furthermore, it can be shown that $\operatorname{Re}[T_3(z)] > 0$ for all z in the lower half plane except at $z = -i\alpha/2$ where $\operatorname{Re}[T_3(-i\alpha/2)] = 0$. Thus an analytic branch of $\log(T_3(z))$ can be formed for z in the lower half plane except along the branch cut B. Then $\Gamma^{\pm}(z)$ can be simplified by residues in a similar manner as α positive and the solution to the homogeneous R-H problem for α negative is determined to be

$$X^{+}(z) = \frac{z \exp\left(I(z)\right)}{\omega^{+}(z + i\,|\alpha/2|)}, \qquad X^{-}(z) = \frac{-1}{\tilde{\mu}(ivz)}\, \frac{\omega^{-}(z - i\,|\alpha/2|)}{z}\, \frac{\exp\left(I(z)\right)}{T_3(z)} \qquad \text{for } z \notin B,$$

$$X_3^-(-iq) = \frac{-e^{i\pi/4}}{\tilde{\mu}(vq)} \frac{\sqrt{q + |\alpha/2|}}{q} \frac{\exp\left(I(-iq)\right)}{\sqrt{1 - \alpha^2/(4q^2)}} \qquad \text{for } |\alpha/2| < q < \infty$$

where

$$I(z) = \frac{-1}{\pi} \int_1^\infty \arctan\left(\frac{1}{\sqrt{u^2 - 1}}\right) \frac{du}{u - |2/\alpha|\, iz} \qquad \text{for } z \notin B$$

and

$$I(-iq) = PV \frac{-1}{\pi} \int_1^\infty \arctan\left(\frac{1}{\sqrt{u^2 - 1}}\right) \frac{du}{u - |2/\alpha|\, q} \qquad \text{for } |\alpha/2| < q < \infty .$$

From the expression for the ERR (3.9) and the homogeneous solutions above, it is found that the ERR has the nondimensional form for α negative of

$$G = \frac{L_e{}^2 a_e}{2\mu_\infty} \frac{1 - \varepsilon}{1 + \varepsilon} \frac{1}{\tilde{m}(\gamma/\varepsilon)} (1 + \varkappa/2)\, h(\varkappa, \varepsilon)\, e^{-2I_1 + I_2 + I_3}$$

where

$$h(\varkappa, \varepsilon) = \frac{1}{\sqrt{1 - (\varkappa\varepsilon/2)^2}} \qquad \text{for} \qquad \varkappa\varepsilon < 2,$$

$$h(\varkappa, \varepsilon) = \frac{1}{\sqrt{(\varkappa\varepsilon/2)^2 - 1}\left((\varkappa\varepsilon/2) + \sqrt{(\varkappa\varepsilon/2)^2 - 1}\right)} \qquad \text{for} \qquad \varkappa\varepsilon > 2,$$

with I_1, I_2, and I_3 as in (3.12).

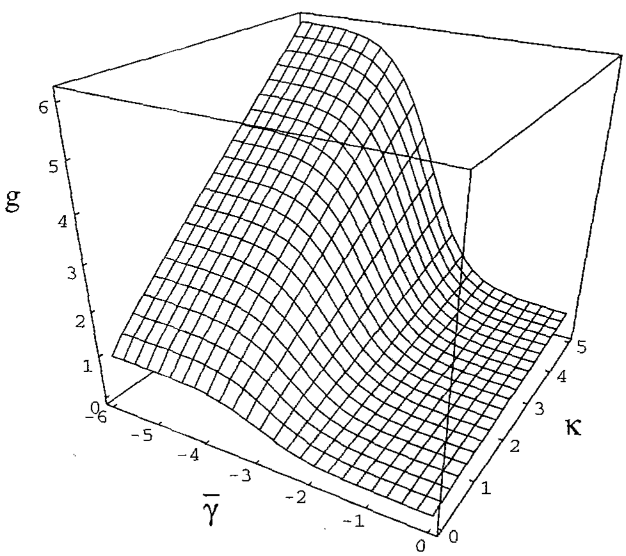

Fig. 1. The normalized ERR g for $\alpha < 0$ versus the inhomogeneity parameter $\varkappa = |\alpha|\, a_e$ and $\bar{\gamma} = \log(\gamma) = \log(\tau v/a_e)$

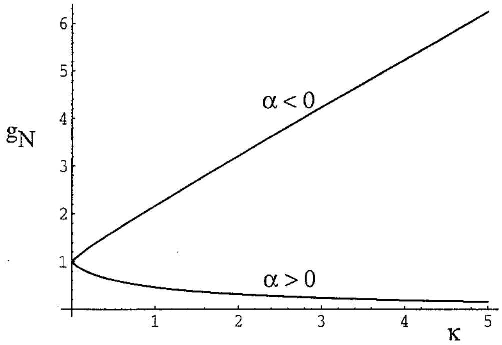

Fig. 2. The normalized ERR with no failure zone versus the inhomogeneity parameter $\varkappa = |\alpha| \, a_e$

The ERR if α is negative for the special cases of no failure zone and an elastic material are

$$G_{nf} = \frac{L_e^2 a_e}{2\mu_\infty} \frac{1}{m(0)} (1 + \varkappa/2)\, e^{-2I_1} = \frac{K_e^2}{2\mu(0)}$$

$$G_e = \frac{L_e^2 a_e}{2\mu} \frac{1-\varepsilon}{1+\varepsilon} (1 + \varkappa/2)\, h(\varkappa, \varepsilon)\, e^{-2I_1+I_2+I_3}.$$

The asymptotic expansion for the ERR as $\varkappa \to 0$ for α negative has the form

$$G = G_h \left[1 + \frac{-1}{\pi} \varkappa \ln(\varkappa) + k_0 \varkappa + o(\varkappa) \right] \quad \text{as} \quad k \to 0$$

where $k_0 = \frac{1}{2} + \frac{1}{\pi} (\ln(2) + c) = .759\,58$. Thus it can be seen that for small values of $\varkappa$ the effect of material inhomogeneity is to increase the ERR for a material which is more rigid near the plane of the crack.

The corrected Figs. 3 and 4 are given above (Figs. 1 and 2 here).

Author's addresses: Dr. J. M. Herrmann, Department of Mathematics, Texas A & M University, College Station, TX 77843, and Dr. L. Schovanec, Department of Mathematics, Texas Tech University, Lubbock, TX 79409, U.S.A.

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