Integr. equ. oper. theory 34 (1999) 500-501
0378-620X/99/040500-02 $ 1.50+0.20/0

Integral Equations and Operator Theory

ERRATUM TO "COMPACT TOEPLITZ OPERATORS VIA THE BEREZIN TRANSFORM ON BOUNDED SYMMETRIC DOMAINS"

MIROSLAV ENGLIŠ

Integral Equations and Operator Theory Vol. 33, No. 4, 1999

The paragraph before eq. (18) in Section 2 of the paper [1] contains an error: it is not true that $f \in \mathcal{P}_{\mathbf{m}}$ transforms under $k \in K$ into $\chi_1(k)^{m_1} \dots \chi_r(k)^{m_r} f$ for certain characters $\chi_j(\exp(X)) = \exp(-\gamma_j(X))$, but only that the representation of K on $\mathcal{P}_{\mathbf{m}}$ has highest weight $-(m_1\gamma_1 + \cdots + m_r\gamma_r)$. Consequently, (18) does not hold and instead of it we have just the statement

$$f \in \mathcal{P}_{\mathbf{m}}, g \in \mathcal{P}_{\mathbf{n}} \implies fg \in \sum_{|\mathbf{k}|=|\mathbf{m}|+|\mathbf{n}|} \mathcal{P}_{\mathbf{k}}, \tag{18'}$$

which follows directly from the fact that each $\mathcal{P}_{\mathbf{m}}$ consists of polynomials homogeneous of degree $|\mathbf{m}|$, and which is weaker than (18) if $r \neq 1$.

The only places where (18) (and the above transformation property of f under the action of K) were used are the proofs of the implications (c) $\implies$ (d) of Theorem A and Theorem B in Section 4 and Section 5, respectively. In the latter case (the Fock space) $\mathbf{m} = (|\mathbf{m}|)$, i.e. $r = 1$, so (18') coincides with (18) and there is no problem. In the former case, the corresponding argument in the proof of (c) $\implies$ (d) in Section 4 has to be modified as follows. Instead of (37), the sum (36) is equal to

$$\varrho^{2d+|\mathbf{n}|} \sum_{\substack{\mathbf{j},\mathbf{m} \\ |\mathbf{m}|-|\mathbf{j}|=|\mathbf{n}|}} (\nu)_{\mathbf{j}}(\nu)_{\mathbf{m}} \varrho^{|\mathbf{m}|+|\mathbf{j}|} \int_\Omega \langle S_z K_x^{\mathbf{j}}, K_x^{\mathbf{m}} \rangle_\nu \overline{p_{\mathbf{n}}(x)}\, dx. \tag{37'}$$

The contribution from the summands with $\mathbf{j} = (0, 0, \dots, 0)$ is again

$$\begin{aligned} \sum_{|\mathbf{m}|=|\mathbf{n}|} (\nu)_{\mathbf{m}} \varrho^{|\mathbf{m}|} \int_\Omega \langle S_z 1, K_x^{\mathbf{m}} \rangle_\nu \overline{p_{\mathbf{n}}(x)}\, dx = \\ = \sum_{|\mathbf{m}|=|\mathbf{n}|} \frac{(\nu)_{\mathbf{m}} \varrho^{|\mathbf{m}|}}{(p)_{\mathbf{m}}} \int_\Omega (S_z 1)(y) \overline{\langle p_{\mathbf{n}}, K_y^{\mathbf{m}} \rangle_F}\, d\mu_\nu(y) \\ = \frac{(\nu)_{\mathbf{n}} \varrho^{|\mathbf{n}|}}{(p)_{\mathbf{n}}} \langle S_z 1, p_{\mathbf{n}} \rangle_\nu, \end{aligned} \tag{38'}$$

since $\langle p_{\mathbf{n}}, K_y^{\mathbf{m}}\rangle_F = 0$ if $\mathbf{m} \neq \mathbf{n}$. To estimate the sum over the remaining $\mathbf{j}$, proceed as before to obtain

$$\Bigg| \sum_{\substack{\mathbf{j},\mathbf{m}:\, |\mathbf{j}|>0, \\ |\mathbf{m}|-|\mathbf{j}|=|\mathbf{n}|}} (\nu)_{\mathbf{j}} (\nu)_{\mathbf{m}} \varrho^{|\mathbf{m}|+|\mathbf{j}|} \int_\Omega \langle S_z K_x^{\mathbf{j}}, K_x^{\mathbf{m}}\rangle_\nu \overline{p_{\mathbf{n}}(x)}\, dx \Bigg| \leq$$

$$= \|p\|_\infty \|S\| \underbrace{\Big(\sum_{\substack{|\mathbf{j}|>0 \\ |\mathbf{m}|-|\mathbf{j}|=|\mathbf{n}|}} (\nu)_{\mathbf{j}} K^{\mathbf{j}}(\varrho^2 e, e) \Big)}_{:= F(\varrho^2)}^{1/2} \underbrace{\Big(\sum_{\substack{|\mathbf{j}|>0 \\ |\mathbf{m}|-|\mathbf{j}|=|\mathbf{n}|}} (\nu)_{\mathbf{m}} K^{\mathbf{m}}(\varrho^2 e, e) \Big)}_{:= G(\varrho^2)}^{1/2}.$$

Observe that the number $N(k)$ of signatures of modulus k is $\leq (k+1)^r$. Thus for any ζ in the unit disc $\mathbf{D}$

$$\begin{aligned} |F(\zeta)| &= \Big| \sum_{|\mathbf{j}|>0} N(|\mathbf{j}| + |\mathbf{n}|)(\nu)_{\mathbf{j}} \zeta^{|\mathbf{j}|} K^{\mathbf{j}}(e,e) \Big| \\ &\leq \sum_{|\mathbf{j}|>0} (|\mathbf{j}| + |\mathbf{n}| + 1)^r (\nu)_{\mathbf{j}} |\zeta|^{|\mathbf{j}|} K^{\mathbf{j}}(e,e) \\ &\leq (|\mathbf{n}| + 2)^r \sum_{|\mathbf{j}|>0} |\mathbf{j}|^r (\nu)_{\mathbf{j}} |\zeta|^{|\mathbf{j}|} K^{\mathbf{j}}(e,e) \\ &= (|\mathbf{n}| + 2)^r \Big(x \frac{d}{dx} \Big)^r \Big(h(xe,e)^{-\nu} - 1 \Big) \Big|_{x=|\zeta|} < +\infty, \end{aligned}$$

and similarly for G. It follows that F and G are holomorphic functions in $\mathbf{D}$, by the Weierstrass M-test. As again $F(0) = 0$ and G has a zero of order $\geq |\mathbf{n}|$ at the origin, the argument can be finished in the same way as before, and the implication (c) $\Longrightarrow$ (d) follows.

The author thanks Jonathan Arazy for pointing out to him this mistake.

Mathematical Institute
Academy of Sciences
Žitná 25
11567 Prague 1, Czech Republic

email: `englis@math.cas.cz`

AMS Subject Classification: 47B35, 32M15