

Erratum

**The Spectral Class
 of the Quantum-Mechanical Oscillator**

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B. Levitan (Moscow State Univ.) has kindly pointed out two places at which the proofs are inadequate. The first occurs on p. 481 where it is stated that the contribution to $\int e_n f_n^0 \Delta q dx$ from $|x| \geq n^{-1/6}$ is rapidly vanishing. The estimates advanced do not support this, but B. Levitan says that $\|e_n f_n^0\|_2 \leq n^{-1/2+}$ ($n \uparrow \infty$), which suffices in view of the rapid vanishing of Δq . The proof is reported to be complicated. The second point occurs on p. 482 where the vanishing of the relative trace $H = \int [p(t, x, x) - p^0(t, x, x)] dx$ is said to prove the vanishing of the relative KDV invariants $J_n: n \geq 0$ via the expansion $H = (4\pi t)^{-1/2} (J_0 t + J_1 t^2 + \dots)$ ($t \downarrow 0$).

The idea is this. Let $Q = \int_0^t q[x(s)] ds$, in which $x(t): t \geq 0$ is the standard Brownian motion with infinitesimal operator $\partial^2/\partial x^2$. Then

$$\sqrt{4\pi t} H = \int E_x [e^{-Q} - e^{-Q^0} | x(t) = x] dx = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \int E_x [(\Delta Q)^n e^{-Q^0} | x(t) = x] dx .$$

The expression is exact, producing an error $\leq \text{constant} \times t^n$ upon breaking it off after $n-1$ terms. The individual terms are now developed in powers of t . The computation is routine, the rapid vanishing of Δq providing the necessary domination to the formal manipulations.

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