

Corrigendum to Spaces of Vector-Valued Measurable Functions

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I am greatly indebted to A.V. Buhvalov, who pointed out a mistake in several theorems of [1]. To be precise, in [1] I stated that if f_1, f_2, \dots is a sequence in $L_\rho(E)$ such that $f_n \downarrow 0$, then $f_n(x) \downarrow 0$ (in E) μ -a.e. has to hold. This is in general false. Accordingly the Theorems 3.1, 4.3, 6.1, 7.4 and the Lemmas 4.2, 7.1, 7.2 and 7.3 have to be adjusted.

Definitions. (i) Let $(L_\rho(E))_d^*$ denote the set of all $\varphi \in (L_\rho(E))^*$ such that $f_n \in L_\rho(E)^+$, $f_n(x) \downarrow 0$ μ -a.e. on Δ implies $\varphi(f_n) \downarrow 0$.

(ii) Let $L_\rho^g(E)$ denote the set of all $f \in L_\rho(E)$ with the property that $|f| \geq g_n$, $g_n(x) \downarrow 0$ μ -a.e. on Δ implies $\rho(g_n) \downarrow 0$.

(iii) $L_\rho(E)$ is called a *weak semi-M-space* if $f_1, f_2 \in L_\rho(E)^+$, $\rho(f_1) = \rho(f_2) = 1$ and $f_1 \vee f_2 \geq g_n$, $g_n(x) \downarrow 0$ μ -a.e. on Δ implies $\lim \rho(g_n) \leq 1$.

It is not hard to see that $(L_\rho(E))_d^*$ is a band in $(L_\rho(E))^*$ containing $(L_\rho(E))_c^*$. Moreover, if $(L_\rho(E))_{ds}^*$ is the disjoint complement of $(L_\rho(E))_d^*$ in $(L_\rho(E))^*$, then we have

$${}^o\{(L_\rho(E))_{ds}^*\} = L_\rho^g(E).$$

Also we have that $L_\rho(E)$ is a weak semi-M-space if and only if $(L_\rho(E))_{ds}^*$ is an L -space. The proofs of all above statements are exactly the same as their analogues for the order convergence case. Finally observe that if $L_\rho^g(E) = L_\rho(E)$ or if $L_\rho(E)$ is an M -space, then $L_\rho(E)$ is a weak semi-M-space. Now, a careful rereading of the proofs in [1] shows that we actually have proved the following results:

- 3,1(i) $L_\rho(E_c^*) \subset (L_\rho(E))_d^*$.
- 4.2 Read $G \in (L_\rho(E))_d^*$ instead of $G \in (L_\rho(E))^*$.
- 4.3 If E_c^* has the R-N property, then $(L_\rho(E))_d^* \cong L_\rho(E_c^*)$.
- 6.1 $L_\rho^g(E) = L_\rho(E)$ if and only if $L_\rho^g = L$ and $E^a = E$.
- 7.1, 7.2, 7.3 Read $L_\rho(E)$ is a weak semi-M-space.
- 7.4 For (a) read $L_\rho(E)$ is a weak semi-M-space.

Finally we observe that statement (ii) of Theorem 3.1 remains a conjecture, as the measurability of the functions f_n employed in the proof is not shown.

Reference

1. Jonge, E. de: Spaces of vector-valued measurable functions. *Math. Z.* **149**, 97–107 (1976)

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