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Correction to "Two Generator Fuchsian Groups of Genus One"

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It was assumed in [1] that a Fuchsian group (i.e. a discrete subgroup of $LF(2, R)$) with presentation

$$\langle a_1, b_1, \ldots, a_n, b_n, c_1, \ldots, c_t \mid c_1^{\alpha_1} = \cdots = c_t^{\alpha_t} = c_1^{-1} \ldots c_t^{-1} [a_1, b_1] \ldots [a_n, b_n] = 1 \rangle,$$

where $n, t \geqq 0$, $\alpha_i > 1$ $(1 \leqq i \leqq t)$ and $[a_i, b_i] = a_i b_i a_i^{-1} b_i^{-1}$ $(1 \leqq i \leqq n)$, has rank $2n + t - 1$ (i.e. cannot be generated by fewer than this number of generators); a proof of this statement had been offered in [2]. The following examples, which arose in conversations with R.G. Burns, A. Karrass, and A. Pietrowski, show that the statement is false in some instances. The simplest of our examples is the Fuchsian group

$$G = \langle c_1, c_2, c_3, c_4 \mid c_1^2 = c_2^2 = c_3^2 = c_4^3 = c_1^{-1} c_2^{-1} c_3^{-1} c_4^{-1} = 1 \rangle,$$

which, we claim, is generated by the two elements $c_1 c_2$ and $c_1 c_3$. This can be seen as follows. Write H for the subgroup generated by $c_1 c_2$ and $c_1 c_3$. Then since $c_4 = c_1 c_2 c_3$, we have that $c_4^2 = c_1 c_2 c_3 c_1 c_2 c_3 = (c_1 c_2)(c_1 c_3)^{-1} (c_1 c_2)^{-1} (c_1 c_3) \in H$, whence $c_4 \in H$. Thus $c_1 c_2 c_3 \in H$, and therefore, in succession, $c_3 \in H$, $c_1 \in H$, $c_2 \in H$. Hence $H = G$. A similar argument shows that the group

$$\langle c_1, \ldots, c_{2k-1}, c_{2k} \mid c_1^2 = \cdots = c_{2k-1}^2 = c_{2k}^l = c_1^{-1} \ldots c_{2k-1}^{-1} c_{2k}^{-1} = 1 \rangle$$

where $k \geqq 1$ and l is odd, has rank at most $2k - 2$.

Professor Zieschang has informed us that he is correcting his paper [2]. As to [1], the assumption that $\{A, B\}$ has genus 1 should be added to the hypotheses of Theorems 1 and 2. (The group G above is a counterexample to both theorems as they stand in [1].) With these modifications the proofs are as before. (This leaves open the question of when two hyperbolic elements of $LF(2, R)$ whose commutator is elliptic, generate a Fuchsian group.) In addition the last paragraph of the introduction of [1] is false; counterexamples are provided by certain triangle groups of the type $(2, 4, q)$ and $(2, 3, q)$.

References

1. Purzitsky, N., Rosenberger, G.: Two generator Fuchsian groups of genus one. Math. Z. **128**, 245–251 (1972)
2. Zieschang, H.: Über die Nielsensche Kürzungsmethode in freien Produkten mit Amalgam. Inventiones math. **10**, 4–37 (1970)

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