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Differences of tensile strength distribution between mechanically high-grade and low-grade Japanese larch lumber I: Effect of length on the strength of lumber*

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Abstract An experimental study was conducted to evaluate the effect of length on the parallel-to-grain tensile strength of Japanese larch (*Larix kaempferi*, Carriere) lumber. Six hundred pieces of mechanically graded lumber were tested at gauge lengths of 60, 100, and 180 cm. The lumber was sorted into matched groups according to the dynamic Young's modulus measured by the longitudinal vibration method before the lumber was cut to the particular length. The averages of the dynamic Young's modulus of high-grade (H) and low-grade (L) specimens were 12.8 and 7.5 GPa, respectively. Using nonparametric estimates, the estimated length effect parameters of H and L were 0.268 and 0.304 for the 50th percentile and 0.121 and 0.256 for the 5th percentile, respectively. We then concluded that the different length effect factors between H and L could be used when using the lumber for practical purposes. The parameters of L were larger than those for H, and the parameters for 5th percentiles were smaller than the parameters for 50th percentiles. When two-parameter Weibull distribution functions were fitted to the strength data, the estimated shape parameters of the Weibull distribution by the parametric method were almost identifical to the inverse of nonparametric parameters except the 5th percentiles for H. The influence of defects such as knots on the lower tail of the strength distribution in H may be different from that in L.

Key words Mechanical grading · Tension parallel-to-grain · Weakest link theory

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Introduction

The design of structures using glued laminated timber, wood trusses, and so on may be governed by the strength of the structural lumber in tension parallel to the grain. In the past, the tensile strength properties of lumber were derived by testing small, clear specimens with adjustments for defects such as knots and the slope of the grain. A more recent approach to the development of tensile strength properties involves testing full-size lumber. The ASTM standard D 1990[1] indicates that the property values of all lumber test data should be adjusted to the characteristic size, and the effect of length on tensile strength is 0.14 expressed in an exponential factor. The tensile strength test of full-size lumber as an alternative testing method to the usual bending test added the Japanese agricultural standard for structural glued laminated timber[2] (JAS) revised in 1996. However, the method of length adjustments of tensile strength was not involved in JAS.

Calculating the effect of size was based on the weakest link theory. Bohannan[3] reported the first study in which the Weibull brittle fracture theory was applied to wood. He studied clear wood beams and found that for geometrically similar beams the strength was proportional to the depth of the beam to the power 1/9, this being the result of a depth effect and length effect of equal importance. This power (1/9) is adopted in JAS as the depth effect factor. Many studies have used the weakest link theory, for example the tension perpendicular to the grain strength of Douglas fir reported by Barrett,[4] the length effects in 38 mm spruce-pine-fir (SPF) dimension lumber reported by Madsen,[5] the effect of length on the tensile strength of visually graded kiln-dried nominal 2 × 4 inch SPF lumber reported by Lam and Varoglu,[6] and a comparison of the length effect models for lumber tensile strength reported by Taylor et al.[7] In Japan, Hayashi et al.[8] studied the effect of size on tensile strength using sugi lumber of various lengths and Okohira et al.[9] studied the effect of size on the tensile strength in small clear western hemlock.

At first, we thought that the effect of length on tensile strength of Japanese larch (*Larix kaempferi*, Carriere) lumber would be different from that in other species such as sugi and SPF, because Madsen and Buchanan[10] showed species dependence based on bending test results. We thought also that the length effect in mechanically graded lumbers would be different from the length effect in visually graded lumbers, a theory studied by many researchers. Then we thought that the length effect might be dependent on the mechanical grade of the lumber, as characters such as knots should be different for each grade. There is little information about the length effect in Japanese larch, so we conducted tensile tests[11] of Japanese larch, which is usually used for glued laminated lumber. Here we discuss the differences of the length effect in mechanically high-grade and low-grade Japanese larch lumber.

Theory

A statistical strength theory has been developed on the basis of the "weakest link theory," which states, "when subjected to tension a chain is as strong as its weakest link." The size effect[12] on the strength of lumber is based on the weakest link theory. The length effect on tensile strength using the brittle fracture theory is described as a relation between the length and strength of two members with the same cross-sectional shape and different lengths. The relation is given by

$$\frac{x_1}{x_2} = \left(\frac{L_2}{L_1}\right)^s = \left(\frac{L_1}{L_2}\right)^{-s} \qquad (1)$$

where x_1 and x_2 are the strengths of members of length L_1 and L_2, respectively, and s is the length effect parameter. The change in strength for doubling the length can be obtained by setting $L_2/L_1 = 0.5$. If s becomes greater, the effect of doubling the length becomes severe, and for $s = 0.3$ only 81% of the strength remains.

Suppose that each member consists of a large number of brittle elements selected at random from a parent population of elements with a cumulative distribution function of strength given by a three-parameter Weibull (3P-Weibull):

$$F(x) = 1 - \exp\left[-\left(\frac{x - x_0}{m}\right)^k\right] \qquad (2)$$

where x is the strength; k, m, and x_0 are parameters of the 3P-Weibull: k is the shape parameter, m is the scale parameter, x_0 is the location parameter.

If the location parameter x_0 is assumed to be zero, as is often done, the 3P-Weibull described above reduces to 2P-Weibull with the parameters k and m. The 2P-Weibull is given:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{m}\right)^k\right] \qquad (3)$$

where x is the strength, k is a shape parameter, and m is a scale parameter.

If a member contains n elements, the cumulative distribution function of this member should be derived from the function of one element. When the function of one element can be assumed in the 2P-Weibull, the function with n elements is given as:

$$1 - F_n(x) = \{1 - F_1(x)\}^n = \exp\left[-n\left(\frac{x}{m}\right)^k\right] \qquad (4)$$

where x is a strength, and $F_n(x)$ and $F_1(x)$ are the 2P-Weibull of n elements and one element, respectively. Equation (4) can be rearranged to give the strength at any quantile q in the distribution:

$$x_q = mn^{-1/k}\left[-\ln(1 - q)\right]^{1/k} \qquad (5)$$

Now consider two members of different sizes containing n_1 and n_2 elements, the ratio of strength of two sizes at any quantile q is:

$$\frac{x_q(n_1)}{x_q(n_2)} = \frac{mn_1^{-1/k}\left[-\ln(1 - q)\right]^{1/k}}{mn_2^{-1/k}\left[-\ln(1 - q)\right]^{1/k}} = \left(\frac{n_1}{n_2}\right)^{-1/k} \qquad (6)$$

When the distribution of the strength follows 2P-Weibull, s in Eq. (1) and $1/k$ in Eq. (5) are the same value at any quantile q.

In general, there are three methods to obtain estimates of the size effects from experiments: the slope method, the shape parameters, and the fracture position shown by Madsen.[12] Because the last method is applicable to a simply supported beam with a concentrated load in the center of the span, we did not deal with it. With the slope method, Eq. (1) can be rearranged to give a linear relation between the logarithm of strength and the logarithm of length:

$$\frac{\ln x_2 - \ln x_1}{\ln L_2 - \ln L_1} = -s \qquad (7)$$

where s is the length effect parameter, which is the slope of the regression line of x on L (disregarding the negative sign). With the shape parameter method, s is the inverse of k of the 2P-Weibull presented in Eq. (6). This method is generally used for estimating not only the length effect parameter but also the depth, width, and volume effect parameters.

The 50th and 5th percentiles of tensile strength distributions were obtained by the nonparametric method according to ASTM Standard D2915-94.[13] The sample nonparametric percent point estimate (NPE) at any quantile q is given by:

$$\text{NPE} = \left[q(n + 1) - (j - 1)\right]\left(x_j - x_{(j-1)}\right) + x_{(j-1)} \qquad (8)$$

where x_j is the j-th value by arranging the test values in ascending order; n is the sample size; j is the smallest order satisfying $j/(n + 1) \geq q$. In the following section, we use NPM and NPL, which denote 50th and 5th percentiles, respectively, estimated by the nonparametric method.

Experiment

Materials

Japanese larch (*Larix kaempferi*, Carriere) lumber was sampled at a manufacturing factory in Nagano Prefecture in Japan. Most of this lumber is used daily for manufacturing structural glued laminated timber in the sawmill. The dimensions of the lumber specimens were nominally 3 cm thick, 17.5 cm wide, and 400 cm long. After kiln-drying, these rough-sawed lumbers were selected with the Japanese made continuous mechanical grading machine.[14] The machine can measure the localized flat-wise Young's modulus for each lumber specimen and can calculate the average of the measured values within a specimen. The target values of Young's modulus of the two sampled groups were 7 and 11 GPa, respectively. We call the former group the L specimen (low-grade lumber) and the latter group H specimen (high-grade lumber) in the following section. Because the lumber was not planed, the actual values of Young's modulus were higher than the values indicated by the machine. The lumber was then measured with the machine again after planing. The lumber specimen were 2.4 × 15.0 × 400 cm.

The dynamic Young's modulus (E_f) values of the selected lumber were measured by the longitudinal vibration method.[15,16] They were calculated from the resonance frequency of the tap tone with a fast Fourier transform (FFT) spectrum analyzer. The specimens were ranked according to their E_f values in ascending order. In the case of the H specimen, lumber with the three lowest E_f values were selected, and one was assigned to the H100 group. The others were assigned to the H060–H180 group. The specimens with the next three lowest E_f values were then selected and assigned similarly. This process was repeated until all the lumber was assigned to the two groups. This process was similarly done for the L specimens. The 400 cm long lumber in the H100 or L100 group was cut at the center point and two 200 cm long specimens were prepared for each specimen. From the H060–H180 group and the L060–L180 group, one specimen with 140 cm length (H060 or L060) and the other with 260 cm length (H180 or L180) were obtained from each specimen. We measured the dimensions, annual ring width, density, and E_f for each specimen.

Tensile test

Tensile tests were conducted with the tensile test machine (NET-501E) made in Japan in accordance with JAS. The contact of the specimen with the grips of the machine is fixed by indenting the faces of the grips. Test spans are 60, 100 and 180 cm as shown in Table 1. The average moisture content (MC) measured at the rupture location by the oven-dried method was 10.9%, with its standard deviation small (0.8%). No adjustments were made for the MC as all members were tested in the same air-dried conditions, and the average MC was close to equilibrium. Test time to failure was about 3–5 min.

Results and discussion

Characteristics of specimens and distribution of tensile strength

Table 1 shows the sample sizes, dimensions, annual ring widths (ARW), densities, and E_f for each specimen. The differences in the ARWs and densities between H and L were clear, but the differences within each grade were small. The densities of H were higher than that of L, and the ARWs of H were narrower than those of L. These results showed that mechanical grading should be useful for sorting lumber according to Young's modulus and for selecting lumber with various characteristics.

The differences of means and coefficients of variation (CVs) of ARWs, densities, and E_f among varying length specimens within a grade were small, as shown in Table 1; and the differences of E_f distributions among them were

Table 1. Dimension, test span, and properties of specimens

Specimen	No.	Width (cm)	Height (cm)	Length (cm)	Test span (cm)	Annual ring width (mm)	Density (g/cm^3)	E_f(GPa)
Grade H								
H060	101	15.0	2.4	140	60	3.4 (25.0)	0.565 (10.0)	12.64 (11.3)
H100	102	15.0	2.4	200	100	3.3 (23.6)	0.570 (10.2)	12.95 (9.3)
H180	101	15.0	2.4	260	180	3.4 (26.1)	0.565 (9.1)	12.86 (7.2)
Grade L								
L060	100	15.0	2.4	140	60	5.2 (18.6)	0.458 (7.0)	7.54 (14.1)
L100	100	15.0	2.4	200	100	5.4 (18.2)	0.456 (7.5)	7.49 (12.0)
L180	100	15.0	2.4	260	180	5.4 (16.2)	0.460 (7.4)	7.55 (12.2)

E_f, Young's modulus measured by the longitudinal vibration method
Values in parentheses are coefficients of variation (%)

also small, as shown in Fig. 1a. The averages of E_f were 12.8 GPa in H and 7.5 GPa in L. The matching among varying length specimens was successful for estimating the length effect on tensile strength. It might be noted that correlation coefficients of E_f between the coupled 140 cm long specimens from one lumber specimen were 0.23 in H and 0.15 in L. CVs of E_f decreased with the increase in specimen length. It was thought that the correlation among mechanical properties of each lengthwise portion in the L specimens might be stronger than that in H because the estimated powers of regression curve in H and L were −0.72 and −0.25, respectively.

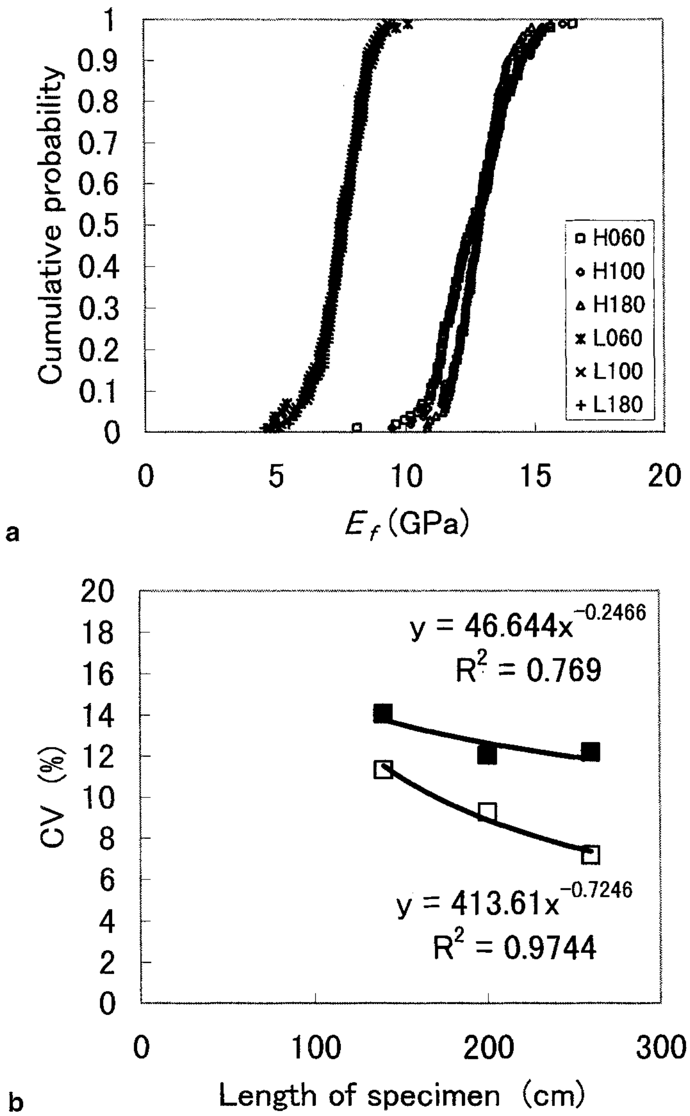

Fig. 1. Distributions and coefficients of variation (*CVs*) of Young's modulus (E_f) by the longitudinal vibration method. Lines are regression curves. **a** Distributions of E_f. *Curves* on the left are comprised of L060, L100, L180. *Curves* on the right are comprised of H060, H100, H180. **b** CV of E_f. *Open squares*, H; *filled squares*, L

The basic statistics of tensile strength data are shown in Table 2. In this table, we distinguished the data from lumber that failed within the span to estimate the length effect on tensile strength. When a specimen failed without the span, the strength of the any portion within the span should be larger than the strength of the failed portion faced to the grips, whose failure mode was valid for practical use. Then we decided to address the strength data of specimens failed within each span.

The distributions of tensile strength (TS) data are shown in Fig. 2. It is clear that increasing the specimen length lowers the TS, whereas the differences between H100 and H180 were small.

Estimating length effect by the slope method

To estimate length effect parameters for the 50th percentile we calculated the NPM (50th percentile) for each specimen and then plotted each NPM in Fig. 3a. We then estimated the length effect parameter s in Eq. (1) by the least-squares method as shown in Fig. 3a. The estimated values of s were 0.268 (1/3.73) in H and 0.304 (1/3.29) in L. The length effect on TS in H was slightly smaller than in L.

Similarly, we calculated the NPL (5th percentile) for each specimen and estimated the length effect parameter s as shown in Fig. 3b. The s values were 0.121 (1/8.26) in H and 0.256 (1/3.91) in L, respectively. The s value for NPL in H was small compared to s for NPL in L and s for NPM in H.

We believed that the length effect on TS should be weaker in H compared to that in L, as the physical and mechanical properties of H were better than those in L, and H involved fewer strength-reducing factors than did L. It should be noted that the length effect on TS for NPL was significantly smaller in H.

Table 2. Basic statistics of tensile strength data

Specimen	All specimens				Specimens failed within the span				N′/N (%)
	N	Tensile strength			N′	Tensile strength			
		Mean (MPa)	SD (MPa)	Skewness		Mean (MPa)	SD (MPa)	Skewness	
H060	101	36.83	12.07	0.85	70	35.15	10.37	0.38	69.3
H100	102	34.40	11.43	0.30	87	34.00	11.13	0.23	85.3
H180	101	27.99	9.86	1.31	96	27.39	9.26	1.14	95.0
L060	100	20.66	7.60	0.79	67	19.63	7.44	0.74	67.0
L100	100	16.90	5.92	1.03	81	16.82	6.10	1.03	81.0
L180	100	14.43	5.41	1.01	92	14.21	5.27	1.04	92.0

SD, standard deviation; N, and N′, numbers of all specimen and specimens failed within the span, respectively

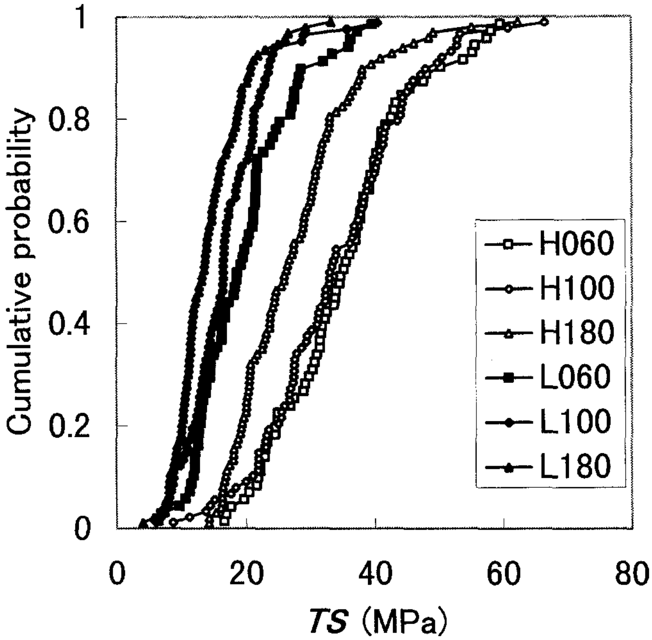

Fig. 2. Distributions of tensile strengths (*TS*) failed within the test span

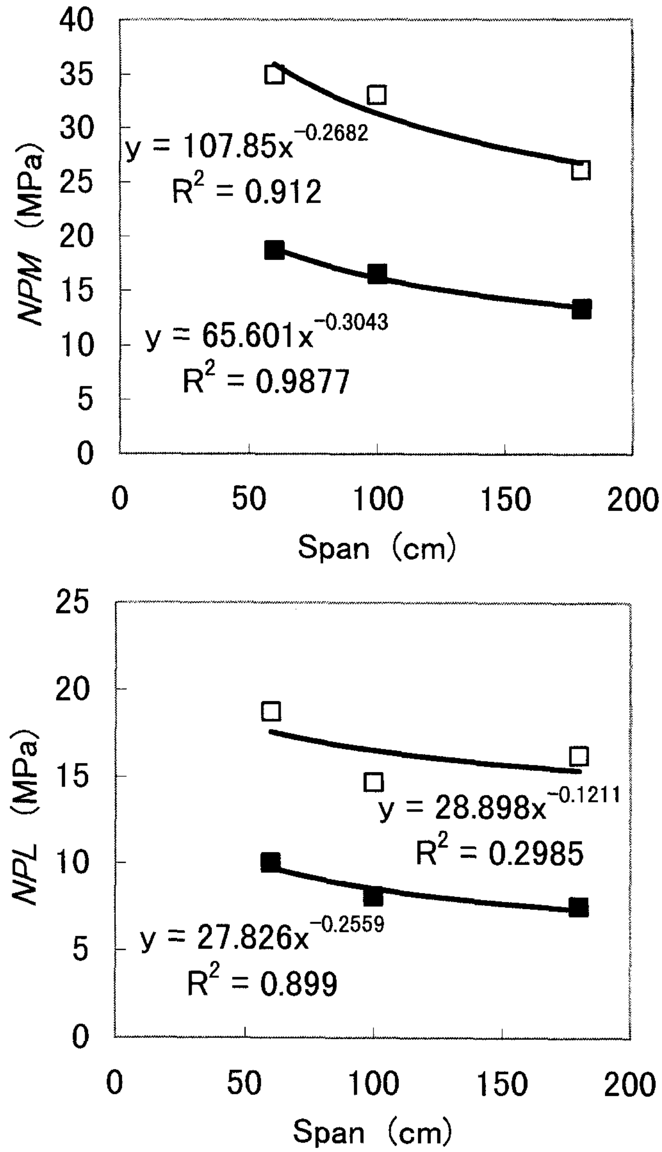

Fig. 3. Relation between span and TS. *H* (*open squares*) denotes high-grade lumber (H060, H100, H180). *L* (*filled squares*) denotes low-grade lumber (L060, L100, L180). The 50th-percentiles (**a**) and 5th-percentiles (**b**) are nonparametric percent point estimates

Estimating the length effect by parametric method

When the distribution of TS may be assumed to be the two-parameter Weibull (2P-Weibull), the shape parameter (k) of the 2P-Weibull should be the inverse of length effect parameter s, as shown in Eqs. (1) and (6). There are various fitting methods of distribution function to estimate the parameters of 2P-Weibull, such as the moment method, the regression method, and the maximum likelihood method. We compared the length effect parameter k determined by these three methods and then compared these values with the inverse of s obtained by the nonparametric method mentioned above.

The parameter k of the 2P-Weibull fitted by the moment method (2PW-M) can be obtained by

$$\frac{\mathrm{SD}}{\mathrm{Mean}} = \frac{\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}}{\Gamma\left(1+\frac{1}{k}\right)} \tag{9}$$

where $\Gamma(x)$ is the gamma function, and Mean and SD are the mean and standard deviation of the TS distribution as shown in Table 2, respectively.

With the k obtained by Eq. (9), the parameter m of the 2P-Weibull can be obtained by

$$\mathrm{SD} = m\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)} \tag{10}$$

The regression method of fitting the 2P-Weibull is applied by first sorting the data, in ascending order, as x_1, $x_2 \ldots x_n$. To each of these values is assigned a plotting position $p_i = 1/(n+1)$. Coordinate pairs (t_i, y_i) are then computed using the transformations $t_i = \ln[-\ln(1-p_i)]$ and $y_i = \ln x_i$. Once the coordinate pairs (t_i, y_i) have been computed, one can use linear regression to estimate the intercept and slope parameters a and b of a straight line of the form $y = a + b\,t$. The parameters k and m of the 2P-Weibull are finally obtained as $k = 1/b$ and $m = \exp(a)$. This is called 2PW-R in the subsequent discussion.

By the maximum likelihood method, the log-likelihood function for the 2P-Weibull can be written as

$$\begin{aligned}\ln L &= \sum_{i=1}^{n} \ln f(x_i) \\ &= n\ln k - nk\ln m + (k-1)\sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}\left(\frac{x_i}{m}\right)^k\end{aligned} \tag{11}$$

Table 3. Estimated parameters of 2P-Weibull

Specimen	Moment: 2PW-M		Regression: 2PW-R		Likelihood: 2PW-L	
	k	m	k	m	k	m
H060	3.79	38.9	3.95	38.7	3.68	38.9
H100	3.37	37.9	3.29	37.9	3.34	37.9
H180	3.25	30.6	3.97	29.9	3.05	30.6
L060	2.86	22.0	3.16	21.8	2.83	22.1
L100	3.01	18.8	3.33	18.6	2.87	18.8
L180	2.94	15.9	3.40	15.7	2.83	15.9

k and m, shape and scale parameters of 2P-Weibull, respectively

where $f(x)$ is the probability density function of the 2P-Weibull written as

$$f(x) = \frac{k}{m^k} x^{k-1} \exp\left[-\left(\frac{x}{m}\right)^k\right] \tag{12}$$

Then the parameters k and m were sought to equate partial derivatives of Eq. (11) with respect to each parameter to zero simultaneously by the asymptotic method.

The parameters k and m estimated by the above-mentioned three methods are shown in Table 3. Both k and m values in H were higher than the values in L. This tendency was identified with the results obtained by a nonparametric method. Though there were small differences among the m values estimated by the three methods, the k values estimated by the regression method were slightly higher than the k values estimated by the other two methods. The harmonic averages of k for each grade were calculated, and we compared these averages with the inverse of the length effect parameter s estimated by the nonparametric method. This comparison is shown in Fig. 4. The shape parameter k was almost equal to the inverse of s except the inverse of s for NPL in H. We believe that the parameter method can be useful for estimating length effect parameters, except NPL in H.

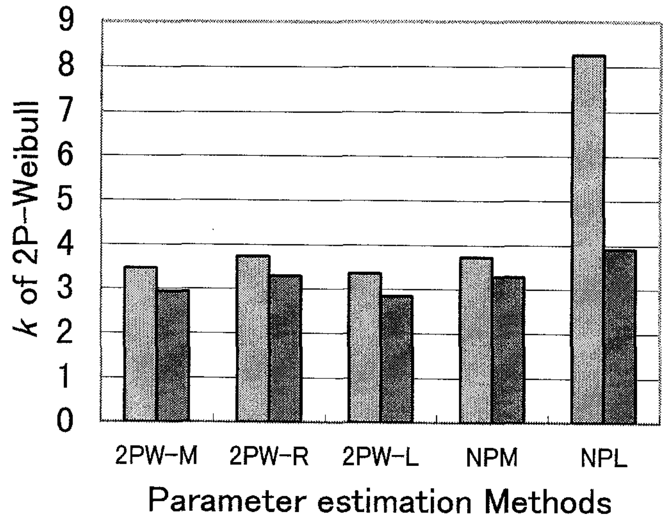

Fig. 4. Shape parameters (k) of 2P-Weibull compared with the inverse of the length effect coefficient s obtained by the nonparametric method. NPM and NPL are the inverse of s for the 50th and 5th percentiles, respectively. 2PW-M, 2PW-R, and 2PW-L, see Table 3. *Lightly shaded bars*, H average; *heavily shaded bars*, L average

Conclusions

An experimental study was conducted to evaluate the effect of length on the parallel-to-grain tensile strength of Japanese larch. The tensile test was conducted for each of three lengths (gauge lengths 60, 100, and 180cm) and for two grades (H and L). We obtained the following results.

1. The percentage of specimen that failed within the span increased with the length of the span in both H and L.
2. The length effect on tensile strength in H was smaller than that in L, and the length effect of NPL (5th percentile) was smaller than that of NPM (50th percentile) for H and L. The size effect factor – defined as the ratio of the strengths when the length has been doubled – were 0.92 in H and 0.84 in L. We believe that these length effect factors for H and L should be used for practical designs.
3. The inverse of s (length effect parameter) of each NPL and NPM was almost equal to the shape parameter estimated by the parametric method except NPL in H. The influence of defects such as knots on the lower tail of the strength distribution in H may be different than that in L.

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