CORRECTION

Ram LAL and U. Narayan BHAT, REDUCED SYSTEMS IN MARKOV CHAINS AND THEIR APPLICATIONS IN QUEUEING THEORY.
Queueing Systems, Theory Appl. 2 (1987) 147–172.

It has come to the attention of the authors that the argument, that the unique solution to $xP = x$, $xe = 1$ implies that $\lim_{n\to\infty} P^n$ exists, used in proving theorem 3.5(ii) does not hold true. For example, consider the probability matrix

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Solution to $xP = x$, $xe = 1$ is $(x_1, x_2) = (1/2, 1/2)$ and $\lim_{n\to\infty} P^n$ does not exist. Thus theorem 3.5(ii) does not hold true and thereby theorem 3.6 which uses the result of theorem 3.5(ii) does not hold true as stated.

As we know from the theory of Markov chains $A + B(I - D)^{-1}C$ is aperiodic if and only if there exists an integer N such that for all $n \geqq N$ $(A + B(I - D)^{-1}C)^n > 0$. We may also say that $A + B(I - D)^{-1}C$ is aperiodic if A is aperiodic or $B(I - D)^{-1}C$ is aperiodic. However, note that these conditions are not exhaustive. Therefore, since checking aperiodicity of $B(I - D)^{-1}C$ involves its computation, a way to do a quick check for aperiodicity of $A + B(I - D)^{-1}C$ is to check if A is aperiodic.

Based on these observations, theorem 3.5 and 3.6 can be restated as follows

THEOREM 3.5 (APERIODICITY)

The matrix $A + B(I - D)^{-1}C$ in the reduced system is aperiodic if P is stochastic, A is aperiodic, and either $(I - D)^{-1}$ exists and D is of finite dimension, or D has no traps, or P is irreducible and positive recurrent.

THEOREM 3.6

If P is an aperiodic, irreducible and positive recurrent stochastic matrix and A is aperiodic then the standard reduced matrix $A + B(I - D)^{-1}C$ is also an aperiodic, irreducible and positive recurrent stochastic matrix.

These results do not affect other results in the paper. Moreover, in almost all queueing problems A is aperiodic and thus theorem 3.6 can be used even without checking the aperiodicity of A.

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