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Tutorial article

Mathematical modelling of ventilation mechanics

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Abstract

Routine application of 'rule of thumb' parameter sets in clinical practice pushes model visions to the background, including the complete framework of assumptions, simplifications, suppositions and conditions. But: models can be a very strong tool, when applied selectively – that means, with a clear idea of destination, definition, parameter selection and verification.

This article discusses universal issues of modelling – based on ventilation mechanics models in intensive care medicine.

Abbreviations: AIC – AKAIKE's information theoretic criterion, ARX – auto regressive model with external input, C – compliance, E – elastance, EX – expiratory, FPE – final prediction error, IN – inspiratory, LMS – least mean squares method, R – resistance, s^2 – dispersion, P, V, V' – pressure, volume and flow at airway opening, PCV – pressure controlled ventilation

Introduction

Did you experience similar discussions?

> 'Life is too complex' – says the physician – 'to fit it into mathematical models'.
>
> 'Processes in human organism are predictable' – says the engineer – 'let us find adequate models to design effective clinical equipment'.

And that's what they really mean:

> 'Interdependencies between functions of the human organism are so complex that separating single parts for the purpose of modelling them is a naive attempt. Treating a patient and being successful is a matter of medical science and clinical experience. You never will write that down in a few lines of computer code.'
>
> 'Right, but there are some points, by which you decide, which are favourable ventilator settings and how they depend on C, R and other indices. Can't we join to find a more precise mathematical model of physical, chemical and biological processes during ventilation? When improved computer algorithms can take part of the clinical routine burden you get more time for patient treatment!'

Critical care medicine has to face lots of trouble, and ventilation mechanics is a minor one, compared to others, e.g. gas exchange. Intensivists must rely on minimal sets of parameters displayed, when they decide, which measures to take. They got accustomed to the fact that there are some dependencies between those parameters and the patient's state. And experience forms an intuitive model. But what do monitoring devices display actually?

As a matter of fact, scientists (physiologists/engineers) feel the complexity of processes in human organism and thus they tend to design adequate, i.e. sophisticated, models of those processes. When the affair comes to the design of clinical equipment, all those fine models have to 'be adapted' to the

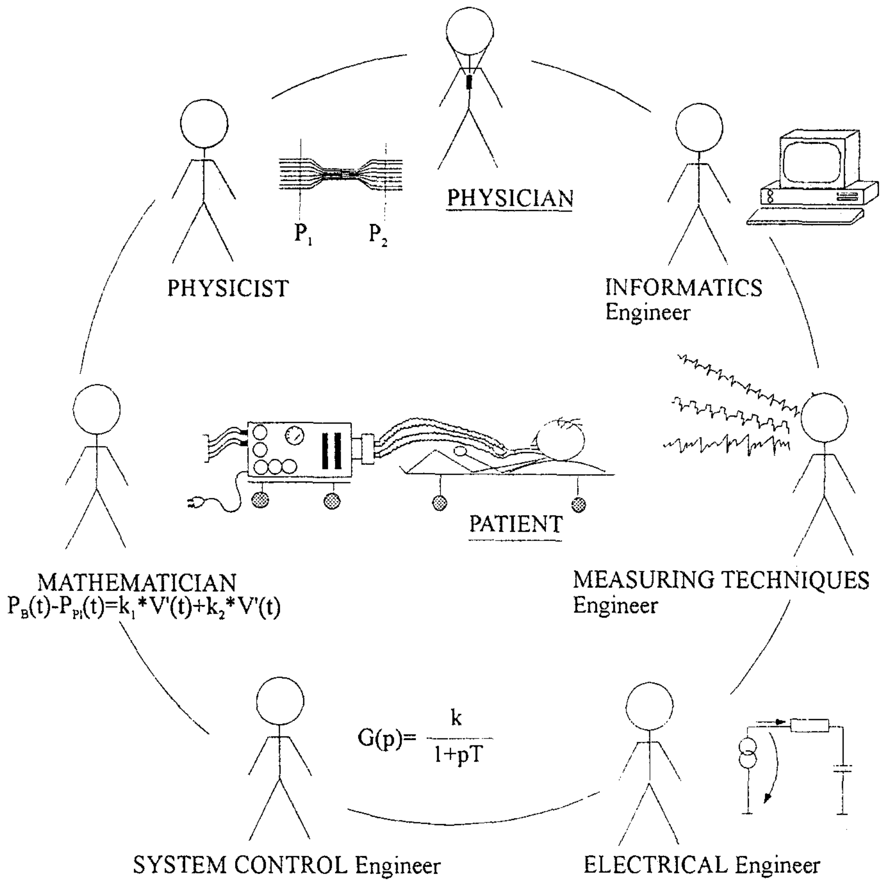


Fig. 1. Models from various fields support clinical processes.

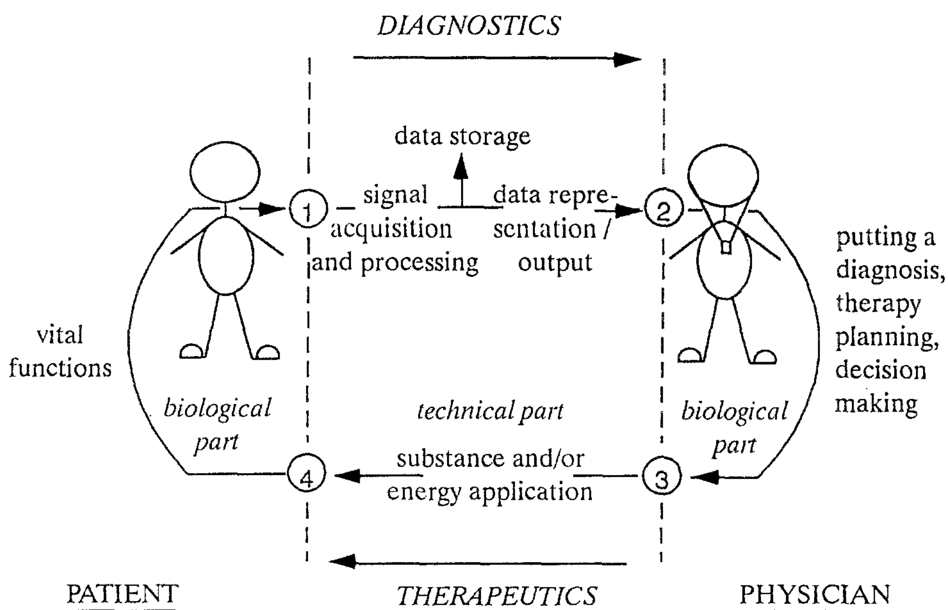


Fig. 2. Four interfaces between biological and technical system in biomedical engineering.

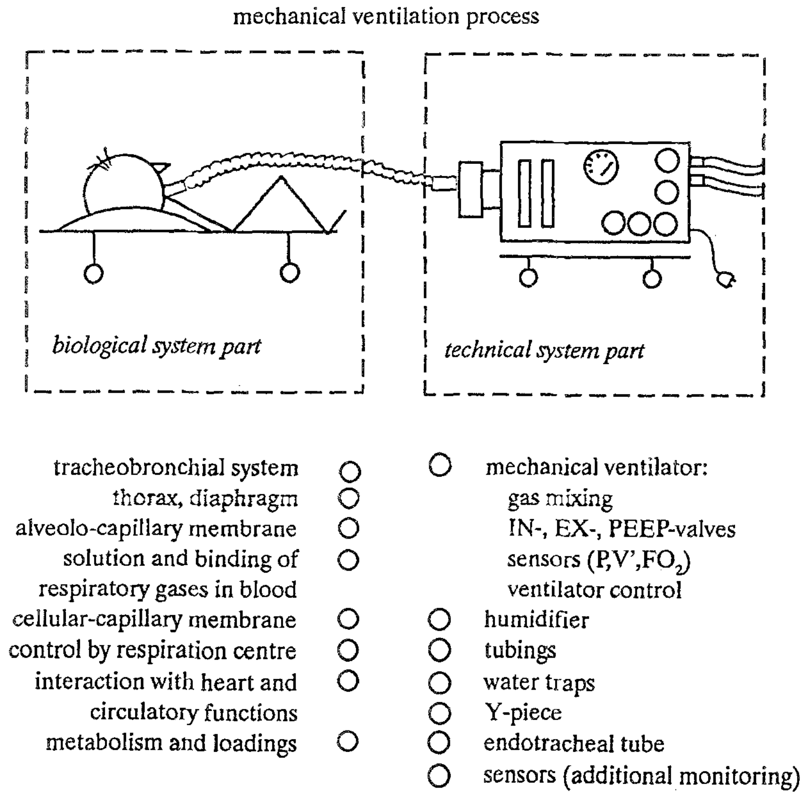


Fig. 3. Biological and technical subsystems of mechanical ventilation process.

critical care environment. That means, they have to be reduced to a minimum ('managable') set of interface parameters.

Which is the way out of that 'state of the art'? May we design equipment, both sophisticated *and* managable? We say YES – and doing so, we have to answer some questions.

What is the system/process to be described?

Every application of technical devices on patients forms a biological – technical system of interacting components. We have to consider four interfaces (Fig. 2) which stand for an interaction between biological subjects (patient/physician) and technical objects (devices). Depending on the direction of flow of information, substance or energy we distinguish between diagnostical and therapeutical activities and corresponding device functions.

For mechanical ventilation of intensive care patients, diagnostical and therapeutical functions are integrated into the ventilator and monitoring subsystems (see Fig. 3).

Smart models will describe this ventilation process complexity with homogeneous tools. Ventilators with valves, tubes, humidifier, Y-piece, same as the patient's respiratory system are networks of gas filled tubes with branchings, junctions and bends, driving flow and resisting against a flow of gas of variable properties (temperature, humidity, concentrations).

Obviously the engineer knows the 'anatomy and physiology' of technical parts designed by himself better than the seemingly very noisy biological part, difficult to be isolated and obstinate against attempts to reveal its design secrets (Grodins [1]).

A possible approach, separating features of the technical part, may be the mathematical description of tube characteristics [2]. This allows to compensate additional resistance during pressure assisted spontaneous breathing [3]. At the same time all the other technical components are assumed to be ideal.

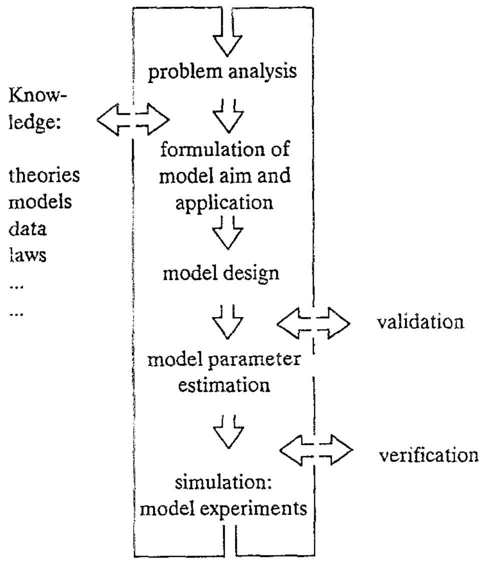


Fig. 4. Steps for model generating.

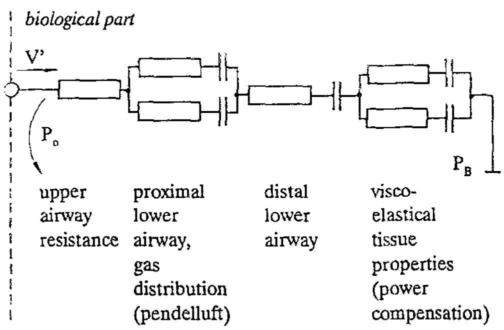


Fig. 5. Passive gas dynamics and respiratory mechanics model.

The genesis of models

Intensivists look at the system 'from outside'. The system is assumed to be a black box. Conclusions on properties are drawn from the system behaviour. Physiologists analyse structure and function of system parts and then create models, reflecting the behaviour of respiratory systems.

Both intensivists and physiologists meet at the crucial point: when defining ventilation indices in connection with model parameters and structure (step 1 . . . 3, Fig. 4). They use available engineering tools to do so.

Estimation of model parameters/indices (step 4, Fig. 4) is based on:

- input/output data,
- a set of candidate models to choose the model structure,
- criteria for selecting a particular model from the set: the identification and validation methods.

Cyclically, in several iterations (Fig. 4) model structure, estimation methods/options and type of experimental data are adapted to meet an optimization goal, a certain model accuracy. Model parameters (i.e. mathematical coefficients) a_i, b_j in Table 1 do not necessarily correspond to physically or physiologically interpretable parameters or indices – the functional model elements R_n, E_m in Table 1. After describing the input-output model, e.g. by differential equation, coefficients have to be estimated and then calculated 'back' to the model elements (which is difficult or nearly impossible for greater systems), Table 1.

But remember: in intensive care, all characteristics of the respiratory and technical system are usually concentrated into two parameters (Resistance R and Compliance C), both of them depend on V'(t), V(t), P(t) . . .

Physical parameters of system components, e.g. tubes, may be calculated theoretically (applying laws of gas dynamics to the component geometry), or may be defined on an experimental basis (measured data). Various mathematical algorithms are applicable:

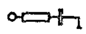
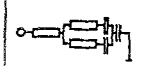
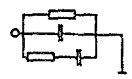
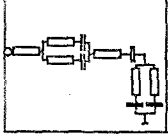
a. methods of isolated values [4] or methods considering or not considering disturbances: parameter estimation/-identification
b. algorithms operating in time or frequency domain: differential equations/spectral analysis
c. methods for continuous or discrete systems: L-/Z-transfer function
d. parametric or nonparametric description: state-space model/impulse response with direct (e.g. ARX) or recursive (e.g. RARX) algorithms and various adaptation principles like LMS (on-line) or forgetting factor analysis [5, 6, 8].

Nonlinearities (e.g. [7]) may be implemented as a (formal) n input – 1 output model. Computer software tools facilitate the handling of mathematical algorithms [8]. Parameters gained by different methods are mutually corresponding, they may be transformed into each other.

At which accuracy may the model describe the process?

Certain physicians tend to interpret mathematical structures 'into' the physiological process, e.g. [9].

Table 1. Mathematical descriptions of the same respiratory system varying with model structure. The indices of model elements are incremented left to right inside the model. Pressure P(t) and Volume V(t) with their derivations represent the measurable dynamical behaviour of the respiratory system mechanics and gas dynamics according to P- or V'-source signal pattern.

model	$a_1P''(t)+$	$a_2P'(t)+$	$a_3P(t) =$	$b_1V'''(t)+$	$b_2V''(t)+$	$b_3V'(t)+$	$b_4V(t)$
			1			R	E
		R_2	E_2		R_1R_2	$R_1E_2 + R_2(E_1+E_2)$	E_1E_2
		R_2+R_3	E_1+E_2		$R_1(R_2+R_3) + R_2R_3$	$R_1(E_1+E_2) + R_2(E_2+E_3) + R_3(E_1+E_3)$	$E_1(E_2+E_3) + E_2E_3$
	R_1R_2	$R_1(E_1+E_2) + R_2E_1$	E_1E_2		$R_1R_2E_1$	$R_1E_1E_2$	
	"$\sum$R*R"	"$\sum$R*E"	"$\sum$E*E"	"$\sum$R*R*R"	"$\sum$R*R*E"	"$\sum$R*E*E"	"$\sum$E*E*E"

In terms of the 'black box' model, they try to light the inner of the box. Thus model 5 from Table 1 may be interpreted as follows:

Viscoelastical tissue properties are described together with gas dynamics by P-V-relations. Differences in behaviour result from different time constants. Such highly parametric models could be a good basis for simulation, but elements R_n, C_m are hardly to be calculated from model coefficients a_i, b_j.

For a chosen model structure parameters vary corresponding to the estimation algorithm. Figure 6 shows R_{IN}, R_{EX}, C, calculated by a method of isolated values (useful only with volume controlled ventilation and inspiratory pause) [4] in comparison to R, C estimated by least squares method for ARX model (usable with every ventilation signal form/ventilation pattern). Both 2-parameter-models seem to be insufficiently exact in describing the ventilation process, Fig. 6. Hess [10] tested six methods of calculating airway resistance. Notice: unfortunately, in *this* case values estimated by *different methods* should not be simply compared to each other: Mathematical model structure and parameter estimation procedures influence the model accuracy, and input/output data is measured incorrectly.

Model accuracy is indicated by validation criteria like dispersion s^2, information theoretic criterion AIC, final prediction error FPE.

Research institutes provide an environment for highly accurate measurings under stable conditions. In clinical use, equipment has to be applied economically. Device characteristics and sensor location influence parameter estimation too, Fig. 7, Table 2.

Especially V'(t)-measurings are influenced by the sensor error (up to 15% relative error already under static conditions).

Consequently, a trend analysis gives more useful results than comparing absolute measurement results to normative values.

Which phenomenons may be investigated by which models?

Models may be of practical use in either of two ways:

- After defining *one* fixed application oriented model *structure*, model parameters are estimated and normative values are established: i.e. classifying patient characteristics under determined conditions – model parameters are compared to normative (including pathological) values. Traditionally, since 1915 (Rohrer) the simpliest RC-model has been in use!
- Model *structure* may be *varied* until a model behaves like the reflected system: i.e. comparing *behaviour*. Consider that the nature of parameters of differently structured models varies with the type of model. Sets of parameters of different models

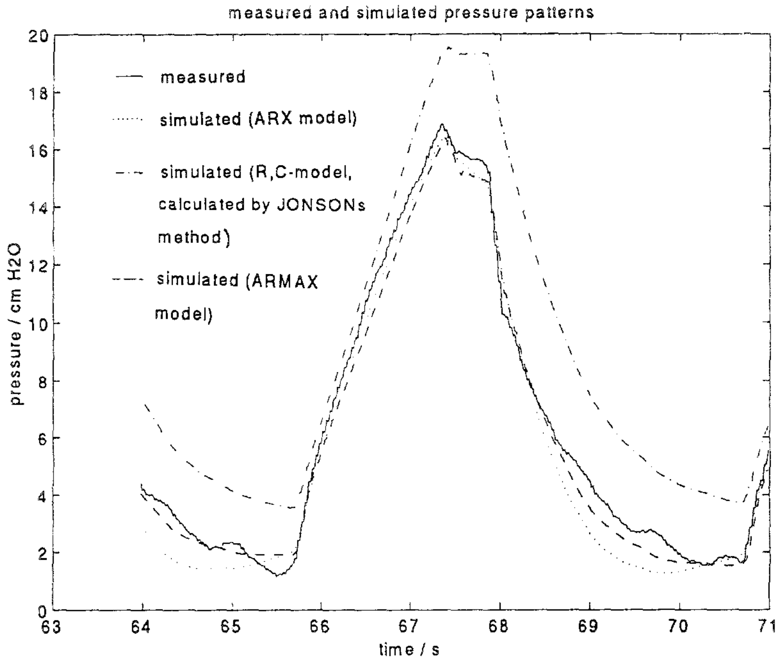


Fig. 6. Model accuracy of R-C model, for data set see Fig. 7.

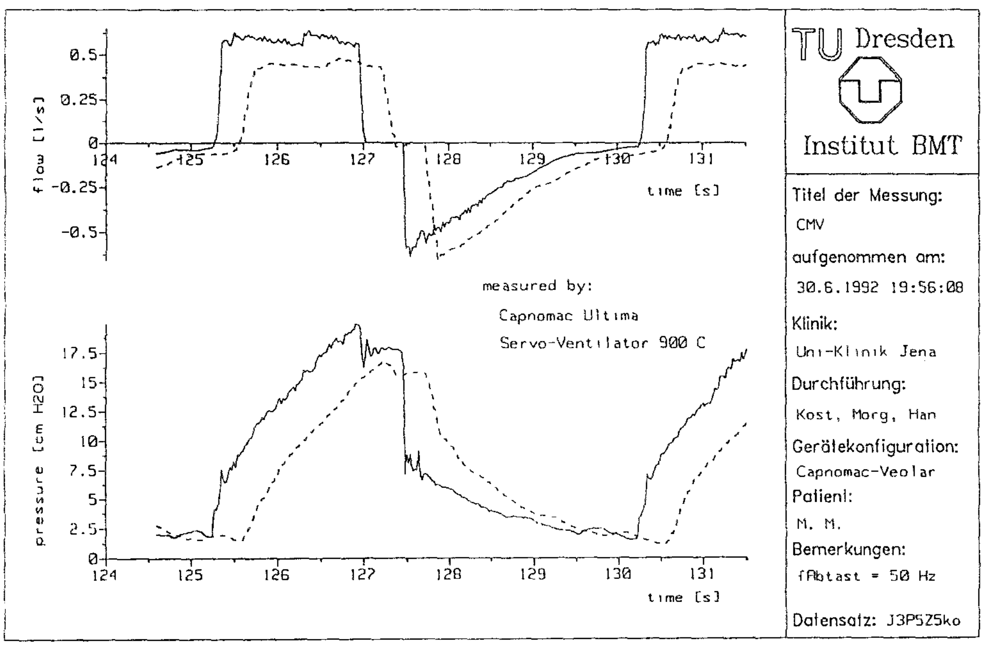


Fig. 7. P-V'-data set recorded by different devices coupled with data acquisition system [11].

are not comparable to each other. There is no information with respect to normatives.

Various model structures give room to flexible and resultative simulations:

Effectiveness of ventilation forms and patterns (also spontaneous activities) can be studied under varying patient's characteristics. Model structure has to be adjusted to the context examined: inhomo-

Table 2. Parameters calculated by Jonson's method (monitored value at ICU devices, 1: Servo Ventilator 900C, 2: Capnomac Ultima) and estimated ARX-model based values from the patient's data set, Figs 6, 7.

Ventilation period recorded with		Calculated by Jonson's method n		Calculated by Jonson's method n + 1		Estimated from ARX n
device number		1	2	1	2	1
R	$cmH_2O/l/s$					10.02
R_{IN}	$cmH_2O/l/s$	3.57	2.22	3.62	2.04	5.05
R_{EX}	$cmH_2O/l/s$	20.46	9.53	20.10	9.62	10.02
C	l/cmH_2O	0.036	0.049	0.041	0.050	0.060
C_{IN}	l/cmH_2O					0.030
C_{EX}	l/cmH_2O					0.060

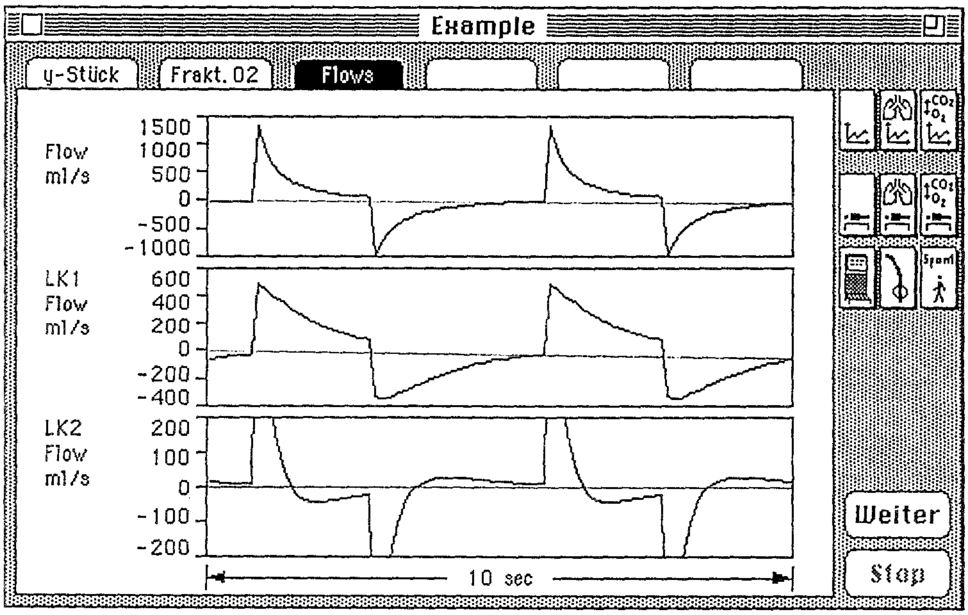


Fig. 8. Simulated pendelluft under PCV mode, simulation system: [12–14].

geneities/pendelluft are realized in two-compartment models only, and a so called thorax or chest wall compliance is necessary for realistic modelling of pressure changes in lower lung compartments.

Among others, pendelluft between two unequally ventilated compartments can be simulated by mathematical techniques for calculating all state-space functions (P_i, V'_i, Vi, . . .) at the inner of the 'black box'. But: searching for a measurable analogue to simulated signals internal to the model, only oesophagus pressure (following pleural pressure) may be used.

If pendelluft is desirable to promote gas exchange in underventilated regions, certain pressure has to be applied by the ventilator source to manage for pressure drops between compartments – pressure drops which are necessary for volume movement. By the help of a model of at least type 3, Table 1, pendelluft may be simulated also in the early expiration phase, in pressure controlled ventilation mode, Fig. 8. In case a pressure compensation in pleural space takes place, internal pressure in the slowest of inhomogeneous compartments may exceed ventilation pressure for a short time.

Models and tools are waiting for application!

Simulation is driven forward by two impulses:

(1) Strong universal engineering tools for identification and simulation are at hand for using them in combination with data acquisition hardware and software [11, 8].

(2) There is an urgent need for simulation software for spontaneous and artificial ventilation and interaction between them [12–14], easy to handle for intensivists.

Theoretically founded ventilation mechanic models and processing tools wait for application. Let us think the usually calculated indices over and provide for better lung function analysis under mechanical ventilation.

Acknowledgements

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