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Risky Production Processes and Demand for Preventive Safety Measures Under Uncertainty

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Abstract

This paper analyzes the behavior of a competitive risk-averse firm that has to choose the optimal level of labor and preventive safety measures. If the firm's injury function includes a random component, then the firm is characterized by a lower demand for labor and by a higher demand for safety measures with respect to a firm whose injury risk is completely under its control. The comparative statics show that shifts in the parameters of the risk distribution will have effects that depend on the absolute risk-aversion functions. The introduction of safety standards will prompt a reduction in the demand for labor while a tightening of the compensation system will have ambiguous effects on the demand for the two inputs.

Key words: Preventive investment, occupational injuries, compensation, risk-aversion

1. Introduction

In a recent issue of this journal, a paper by Thierry Schneider discussed the labor-hoarding strategies and the demand for preventive safety measures of a risk-neutral firm that runs a risky production process. The interesting result that emerged from that paper was that a firm will not always react to increases in the cost of workers' compensation by purchasing more safety measures in equilibrium.

The issue of the firm's response to changes in regulatory policies is crucial to the success of regulations in the area of health and safety at work. Legislated injury benefits (either fixed according to the occupational risk class or partially experience-rated) together with safety standards are to date the two most common forms of government intervention. The economic literature on these issues, however, which has been mainly empirically orientated, has failed to identify unambiguously whether these regulatory devices achieve a significant reduction in accident rates without inflicting excessive compliance costs on firms.

Models like those of Elder [1985] and Sider [1983], which have analyzed the firm's production decision and factor choice using a deterministic framework, have concluded that the effect of either the introduction of safety standards or an increase in the cost of injury compensation for the firm is to lead to a reduction in the level of firm's activity through a contraction in their demand for labor.

As for the effect of safety standards on accident rates, while the results of some empirical studies—for instance, Viscusi [1986]—point out that safety regulations do achieve a reduction in accident rates, other studies, such as Bartel and Thomas [1985], have found that this effect is insignificant while the indirect costs imposed by safety legislation are high.

The performance of legislated injury benefits, on the other hand, may be more difficult to assess when it is explicitly recognized that accidents are jointly determined by the firm and the worker—the latter contributing through varying levels of care. The introduction of legislated injury benefits increases the costs of accidents for the firm and thus increases the incentive to purchase safety measures, but it also can induce a risk-taking behavior of the worker if it reduces the opportunity cost of injuries. This offsetting effect is one of the possible outcomes predicted in the model by Rea [1981], and some proof of it is empirically found in Butler and Worral [1983] and in Dionne and St-Michel [1991].

The importance of industrial safety policies requires that the issue of the firm's choice of preventive safety measures in a regulated environment be the object of an in-depth theoretical analysis. This analysis should address the robustness of the outcome of the firm's decision making under a larger variety of circumstances than those considered so far in the literature.

We think that one extension of the analysis that is of potential policy relevance is the hypothesis of risk aversion of the firm. The aim of this paper is therefore to allow for an expected utility maximizing competitive firm that runs a risky production process and seeks the optimal amount of inputs—that is, of labor and preventive measures.

This formulation allows us to view the model by Schneider [1992], which assumes risk neutrality, as a special case. The analysis that follows assesses the robustness of the results obtained in that model across different attitudes to risk, while at the same time adding new refinements to it. Although it can be argued that risk aversion is unlikely for a firm whose shareholders can diversify the risk in a portfolio of assets, it is nevertheless true that safety and injuries are a major problem for small firms with single owners and nondiversified assets. At any rate, the assumption of risk aversion has—at least since Sandmo [1971]—long traditions in the theory of the firm.

I proceed in two steps. First, I identify the differences (if any) between the behavior of a firm with a deterministic accident risk function (henceforward the *certainty firm*) and the behavior of a firm with a stochastic accident function. In the first instance, it is assumed that the firm has full control of the hazardousness it produces; in the second case, a random component, independent of the firm's behavior, is added to the deterministic part.

Then I carry out a series of comparative static exercises to identify

1. The effects of shifts in the parameters of the accident risk distribution on the firm's choice of input levels (this exercise allows us to check, for instance, whether a change in technology that reduces the firm's control of the accident risk induces a shift from one input to another or reduces the investment in safety);

2. The effects of increasing the cost of injury compensation on the choice of preventive safety measures and the demand for labor;
3. The effect of tightening safety standards on the demand for labor (compliance with the standards is assumed);
4. The results under some form of experience rating of the cost of injury protection compared with those obtained in the case in which no experience rating exists.

The paper is structured as follows: Section 2 discusses the accident risk function, while Section 3 presents the model. The comparative statics analysis is carried out in Section 4, and a brief discussion of the effects of experience rating can be found in Section 5. The paper concludes with a discussion of the policy implications of the results obtained.

2. The accident risk function

The hazardousness of a firm may have a random component due to three main factors:

1. For some industries, natural events not predictable by the firm (such as weather conditions or natural disasters) are beyond the firm's control and thus represent a random component of the accident risk.
2. Workers' behavior may influence the determination of accident risk. This amounts to assuming either or both of the following: (1) the worker takes accident-avoiding precautions, but there is a chance that he might make a mistake, so the specification of a stochastic accident risk as random could account for the influence of human error in accidents; (2) workers increase the risk of accidents if they do not take safety precautions. If the level of care taken by workers in accident prevention is not observable by the firm—or if monitoring is too costly—workers' care is a random element to the firm. Hence, the introduction of a random disturbance in the accident function accounts for the influence of the heterogeneity of the workforce on the number of accidents.[1]
3. There might be a random element in the occurrence of injuries due to poor machine design. Technical reports show that for some sectors, such as the construction industry, the distribution of accidents can be only partially affected by safety measures. Similarly, some accidents due to machinery handling have been shown to depend not so much on machine poor maintenance but rather on unpredictable machinery failure.

The three factors listed above give rise to a probability distribution of accident rates that is conditional on the safety measures undertaken by the firm. We assume that this probability distribution is known to the firm.

The uncertainty thus generated has to be accounted for both by a firm that has to decide on the optimal amount of accident-risk-reducing measures and that pays compensation for injuries and also by the regulator who has to monitor the firm's safety behavior. Exact observability of the firm's safety measures might be in general costly to achieve, but lack of a deterministic relation between safety and accident rates makes it difficult to infer safety measures from accident rates.[2]

Define $\tilde{A}$ as the number of injuries of a given kind for each worker employed by the firm in each given period, such as the year. $\tilde{A}$ is a random variable determined by the amount of safety measures undertaken by the firm, S, and by a random disturbance, ε, which enters the function in a multiplicative fashion. The amount of safety measures purchased by the firm can be considered—using a well-known terminology—as the level of self-protection adopted by the firm, given that safety measures can reduce the probability of injury.

$$\tilde{A} = A(S)(1 + \varepsilon) \tag{1}$$
$$\partial\tilde{A}/\partial S < 0 \qquad \partial^2\tilde{A}/\partial S^2 > 0$$

The random disturbance ε has p.d.f. $f(\varepsilon)$, zero mean, $E(\varepsilon) = 0$, and finite variance, $\sigma^2(\varepsilon)$. We also assume that ε lies in the interval $(-1, +1)$. This specification of the injury function implies that the firm can influence, through safety investment, both the expected value and the spread of the risk distribution.[3] With respect to points 2 and 3 above, this implies that the effects of workers' errors and carelessness and of faults in the design of safety machinery are not separated from safety measures undertaken by the firm. Although the firm cannot determine the exact injury risk, nevertheless it can influence the moments of the injury risk distribution by expanding safety investment.

I assume that accidents affect only the firm's cost equation because the risky firm faces ex post accident costs for each worker injured and positive wage differentials with respect to the risk-free sector of the economy. If the number of accidents has a random component, accident and labor costs become random as well.

It is also possible to assume that accidents enter the production function directly besides affecting the firm's costs. In this case the effective output, Q, depends on the injury rate, and therefore output is itself a random variable. This model, however, presents ambiguous results that are difficult to interpret. I therefore restrict the analysis to the model with uncertainty in the cost equation.

3. The model

In this section I examine the behavior of a utility maximizing firm that runs a risky production process and that operates in a perfectly competitive market for inputs and output. Accidents are defined as the occurrence of any dangerous event that causes an injury to a member of the workforce. All accidents are supposed to be of equal severity.

The firm faces positive wage differentials with respect to the risk-free sector of the economy: that is, it is assumed that the number of injuries per worker affects the wage bargaining process between the firm and the workers. Since wages are bargained before the actual number of accidents takes place, it is also assumed that the wage rate depends on the expected injury rate. The wage function of the firm operating in a representative risky sector of the economy is thus $\omega(E(\tilde{A}))$. Since the expected injury risk is determined only by the firm's safety investment—$E(\tilde{A}) = A(S)$—the wage function is not affected by the random component ε. Let ω_0 be the wage rate if the injury function were deterministic. Then $\omega(E(\tilde{A})) = \omega_0$.[4]

In addition to paying compensating wage differentials, the firm pays accident costs after accidents have taken place to compensate the workers for any loss they suffer, so that the total compensation bill depends on the realized accident toll. We assume that the firm pays a benefit equal to c for every injured worker.[5]

As already mentioned, I study how the firm's purchase of safety inputs and labor vary when the accident risk function has a random element. Next, I look at how these purchases are affected by an increase in the mean and in the variability of the accident risk and in the parameters of the wage and compensation costs. To keep the analysis simple, I assume that the only control variable to enter the injury risk function is the firm's preventive safety measures.

Assume the firm has a Von Neumann-Morgestern utility function and maximizes the expected utility of wealth by choosing the optimal amounts of labor and safety measures, which we denote by L^* and S^*, respectively.[6] The analysis applies to a given period of time, such as the year:

$$\max_{L,S \geq 0} E(U(W)) \tag{2}$$

The firm's wealth equation is

$$W = W_0 + pQ(L) - \omega(E(\tilde{A}))L - sS - c\tilde{A}L \tag{3}$$

where

W_0 = initial level of wealth,
$Q(L)$ = a well-behaved production function,
L = labor employed,
S = preventive safety inputs,
$\omega(E(\tilde{A}))$ = wage function fixed before the realization of the random variable $\tilde{A}$, with $\omega' > 0$,
s = price of a unit of safety inputs,

c = cost of ex-post compensation per accident, and
p = price of the market output.

The initial level of wealth, W_0, is assumed to be constant. Since Q is concave and total costs are convex in the labor price, so that an expansion in the amount of labor used (and hence in the volume of output produced) increases the marginal cost of producing Q, uncertainty combined with risk aversion should lead to decreased output.

For a risk-averse firm, the first-order conditions for a maximum are

$$(pQ_L - \omega_0)E[U'(W)] = E[U'(W)c\tilde{A}] \tag{4}$$

$$(-\omega_0' A_s L - s)E[U'(W)] = E[U'(W)cA_s(\varepsilon + 1)L] = 0, \tag{5}$$

where A_s denotes the first derivative of the accident function with respect to preventive safety measures. Using simple manipulations, we are now able to prove two results:

Proposition 1. *Under uncertainty concerning wage and accident costs, the risk-averse firm demands a lower amount of labor than a firm whose accident function is deterministic, as in the standard analysis of a risk-averse firm behavior.*

Proof: Rewrite (4) as

$$(pQ_L - \omega_0 - cA(S)) = \frac{Cov[U'(W), c\tilde{A}]}{E[U'(W)]}. \tag{6}$$

With risk-aversion, $U'(W)$ is decreasing in W. W, in turn is decreasing in $c\tilde{A}$. Thus, the covariance term on the right side is positive, and it must be

$$pQ_L - \omega_0 - cA(S) > 0. \tag{7}$$

Noting that under a deterministic injury function, the firm optimal amount of labor is determined by $pQ_L = \omega_0 + cA(S)$, it follows that under uncertainty concerning wage and accident costs, the risk-averse firm demands a lower amount of labor. ■

Proposition 2. *Under uncertainty, the risk-averse firm selects a higher safety per worker ratio, compared to a firm with a deterministic accident function.*

Proof: The proof follows that of proposition 1. Rewriting (5) as

$$(-\omega_0' A_s L - s - cA_s L) = \frac{Cov[U'(W), cA_s \varepsilon L]}{E[U'(W)]} \tag{8}$$

and noting that the covariance is negative, comparison with the FOC under a deterministic accident function

$$-(\omega_0' + c)A_s L - s = 0 \tag{9}$$

proves that the risk-averse firm will demand a greater amount of safety measures than the firm with a deterministic injury risk. ■

It follows trivially that since $U''(W) = 0$ under risk neutrality, the behavior of the risk-neutral firm coincides with the behavior of the firm operating with a deterministic injury function. At the optimum, therefore, the risk-averse firm is characterized by a higher safety per worker ratio than the risk-neutral firm. If this is true, one would expect to find that industries whose accident rate is known to be partially determined by noncontrollable variables (such as weather conditions or workers' on-the-job errors) have a higher safety per worker ratio than industries in the same risk class and whose accident function is closer to the deterministic one, subject to the assumption that these firms are risk averse. A shift in the technology of production or in the technology of abatement that reduces the firms' control on the accident risk function will thus affect the input decision of a risk-averse firm but not of a risk-neutral one.

The results of this section show that, as in many other cases, uncertainty corresponds to an extra cost of foregoing safety investment: the random disturbance ε may lead to a realized number of accidents higher than that expected. As such, it increases the incentive to purchase safety measures. In addition, the existence of random compensation costs reduces the amount of labor employed, thus leading to a lower level of productive activity.

4. Comparative statics

4.1. Comparative statics of the model with respect to the moments of the distribution of ε

We turn now to investigate the effects of an increase in the expected value of the accident function and the effects of an increase in the variability of ε. To do this, we carry out a comparative static exercise, totally differentiating the first order conditions and adopting the procedure found in Sandmo [1971] to identify (1) the effects of a bodily rightward shift of the distribution of ε, without changing its spread; (2) the effects of a mean-preserving increase in the spread of the distribution of ε—that is, a *Sandmo increase in risk.*

We endeavor to identify conditions under which the direction of the change in the equilibrium demand for labor and preventive safety—L^* and S^*—can be predetermined according to the general type of utility function under analysis. Whenever the results using multiplicative risk are ambiguous, we refer to the results that can be obtained in a simpler formulation in which the random variable ε is an additive disturbance to the deterministic component $A(S)$.

We start by rewriting the accident function $\tilde{A}$ according to the methodology first introduced by Sandmo. Define

$$A^* = \gamma\,[A(S)(1 + \varepsilon)] + \beta \qquad (10)$$

if $\gamma = 1$ and $\beta = 0$ this function is the one considered earlier in the model. An increase in β accounts for an increase in the mean of the distribution of $\tilde{A}$, so that the sign of the derivatives $\partial L^*/\partial\beta$ and $\partial S^*/\partial\beta$ gives information on the effect of an increase in the mean of the injury rate distribution on the optimal amount of labor and of safety, respectively. The investigation of the effects of an increase in the mean of $\tilde{A}$ is relevant, since an increase in the mean of $\tilde{A}$ determines an increase in the wage rate paid, $\omega(E(\tilde{A}))$, and in the expected compensation bill paid, $cE(\tilde{A})L$, and hence, a fall in expected wealth.

The change in the optimal amounts of labor and safety, respectively, is given by

$$\partial L^*/\partial\beta = (1/D)[-U_{L\beta}U_{SS} + U_{S\beta}U_{LS}] \qquad (11)$$

$$\partial S^*/\partial\beta = (1/D)[-U_{S\beta}U_{LL} + U_{SL}U_{L\beta}], \qquad (12)$$

where D is the determinant of the matrix of second-order derivatives, which is positive if the second-order conditions are satisfied. The Uij terms ($i = L, S; j = L, S, \beta$) are derivatives of the first-order conditions. Details of the derivations can be found in the mathematical appendix.

For utility functions characterised by constant absolute risk-aversion, $Ra(W) = -U''(W)/U'(W) = \alpha$, and if the wage function is linear, the term $U_{S\beta} = 0$. Thus, the change in the mean of the injury rate does not alter the marginal utility of a unit of safety investment. At the same time, given that $U_{L\beta}$ is negative, the increase in the mean of $\tilde{A}$ increases the marginal cost of injury compensation per worker by $(\omega_0' + c)$. It follows that the effect of an exogenous increase in the mean of the risk distribution is a reduction in both the demand for labor and for safety measures, so that (11) and (12) have a negative sign.

With CARA, but strictly convex wages, the effect of a change in the expected value of the distribution is ambiguous. One of the effects of the increase in the mean is now to increase the utility of an extra unit of safety by an amount that depends on the magnitude by which total wage costs will be reduced by the purchase of an extra unit of safety measures, $\omega_0''A_sL$. According to the value of $\omega_0''(A(S))$, it might be rational for the firm to increase or decrease the optimal amount of labor and safety measures.

Under decreasing absolute risk aversion, the marginal increase in the cost of the compensation package ($\omega' + c$) is reinforced by a term with the same sign that depends on the absolute risk-aversion function, so that $U_{L\beta}$ is again positive. This joint effect tends to reduce the amount of labor and safety employed at the optimum. It is, however, counterbalanced by the positive sign of $U_{S\beta}$. As a consequence of an increase in the expected injury rate, wealth falls, and the marginal utility of an extra unit of safety rises. Since the firm is able to counteract the exogenous increase in β by investing in safety, this effect pushes in the direction of an expansion of the equilibrium amount of safety. Which of the two effects prevails will, of course, depend on the relative magnitudes of $U_{L\beta}$ and $U_{S\beta}$.

It is straightforward to check using the derivations in the mathematical appendix that if uncertainty entered the risk function $\tilde{A}$ in an additive fashion (that is, if the firm was able to control the expected value of the injury distribution but not its spread), the result that would obtain is that linear wages ($\omega'' = 0$) would be a sufficient condition for $\partial L/\partial\beta < 0$ and $\partial S/\partial\beta < 0$, regardless of whether the absolute risk-aversion function is constant or decreasing. Therefore, the comparative static results hinge not only on the class of utility functions considered but also on the way uncertainty is defined.

To identify the effects of an increase in the variability of the distribution of ε we differentiate β with respect to γ subject to the condition that $\partial\beta/\partial\gamma = -A(S)$—that is, subject to the preservation of the same mean of the distribution of $\tilde{A}$. We evaluate the derivative at $\gamma = 1$ and $\beta = 0$.

Because the model is static, it is plausible to assume that the reservation wages of workers are not revised following changes in the spread of the distribution of injuries. The revision of workers' reservation wages should be considered if the model was a multiperiod one and learning occurred.

$$\partial L^*/\partial\gamma = (1/D)[-U_{L\gamma} U_{SS}) + (U_{S\gamma} U_{LS})] \tag{13}$$

$$\partial S^*/\partial\gamma = (1/D)[(-U_{LL} U_{S\gamma}) + (U_{LS} U_{L\gamma})] \tag{14}$$

In the mathematical appendix it is shown that if the risk-aversion function $R_A(W)$ is nonincreasing, $U_{L\gamma} < 0$ and $U_{S\gamma} > 0$. Hence, under multiplicative risk, the sign of (13) and (14) is ambiguous. However, if the disturbance ε enters the injury risk function additively, then $U_{S\gamma} = 0$. It follows that the two expressions above are unambiguously negative. An increase in the spread of the distribution will cause a decrease in the amount of safety measures purchased by the firm and also in the optimal level of labor. The explanation for this behavior lies in the additive specification of the accident risk function $\tilde{A}$, which implies that the firm cannot undo the effects of the increase in the spread of the injury rate by increasing safety expenditure. Once the firm chooses to reduce the equilibrium level of safety measures, the increase in the wage and compensation bills will also determine a fall in the amount of labor employed.

4.2. Other comparative static results

We investigate three more comparative static effects that are of some importance—namely, the increase in the cost of injury compensation, c; the effect of imposition of safety standards mandated by law S_R and such that $S_R \geq S^*$; and the increase in the price of the product marketed by the firm, p.

The effect of the first exercise is to tighten the compensation system, for instance, following increasing public concern for the losses connected with injuries. The second gives the effect of fixing the level of safety measures exogenously. The third provides information on the effects of changes in economic conditions on the safety per employee ratio.

Going back to our comparative static exercise, we find that in the case of a risk-averse firm,

$$\partial L^*/\partial c = (1/D)[(-U_{Lc}U_{SS}) + U_{Sc}U_{SL})] \tag{15}$$

$$\partial S^*/\partial c = (1/D)[(-U_{Sc}U_{LL}) + (U_{LS}U_{Lc})]. \tag{16}$$

Since it can be shown that $U_{Lc} < 0$ and $U_{Sc} > 0$, the effect of an increase in the cost of protection has an ambiguous effect on the demand for labor and preventive safety measures, whereas one would expect the increase in c to lead to a fall in the demand for labor and an increase in the amount of safety equipment purchased. This counterintuitive result is explained by the fact that two opposite effects are at play. On the one hand, the increase in the cost of compensation raises the expected cost of a unit of labor and therefore enhances the incentive to purchase safety. On the other hand, the increase in the cost of compensation, by reducing the level of activity of the firm at the optimum (if L^* falls), causes the safety per worker ratio to rise and thus reduces the need for expanding the investment in safety equipment. These results confirm those obtained by Schneider [1992] for the case of a risk-neutral firm.

While the effects of changes in the compensation system are not obvious, the introduction of regulations that fix the safety stock of the firm by law have the unambiguous effect of depressing the demand for labor of the firm. To simplify matters consider the case of a firm that complies with the mandated standards, so that $S^* = S_R$. A marginal increase in S_R reduces L at the optimum:

$$\partial L^*/\partial S_R = -(1/D)(\partial U/\partial L\partial S_R) < 0. \tag{17}$$

Turning to the effect of an increase in the price of the market output, p, we find that under constant absolute risk aversion,

$$\partial L^*/\partial p = (|D|)^{-1}[(-U_{Lp}U_{SS}) + (U_{SL}U_{Sp})] > 0 \tag{18}$$

$$\partial S^*/\partial p = (|D|)^{-1}[-U_{LL}U_{Sp}) + (U_{Lp}U_{SL}) > 0. \tag{19}$$

The demand for labor and safety equipment increases following an increase in the price of the output marketed by the firm. This result accords with the pattern often observed in accident rates over time: although one might be tempted to think that increases in p, by stimulating production, increase the number of accidents, the rise in the injury rate (and therefore the fall in safety standards) is more often observed in periods of economic downturns. Under DARA results are ambiguous, and it cannot be ruled out that when p rises, safety standards fall.

5. Experience rating in accident costs

Let us now redefine total wealth as

$$W = pQ(L) - \omega_0 L - sS - c(1 + \theta(\tilde{A}))\tilde{A}L, \tag{20}$$

with $\theta(\tilde{A}) > 0$, $\theta'(\tilde{A}) > 0$, $\theta''(\tilde{A}) > 0$. The unit cost of injury compensation now increases with the realized number of accidents; it is experience rated according to the firm's own injury record. The maximization of the Von Neuman-Morgestern utility function $U(W)$ now yields FOCs:

$$(pQ_L - \omega_0)\, E[U'(W)] = E\{U'(W)[c(1 + \theta(\tilde{A}))\tilde{A}]\} \tag{21}$$

$$\begin{aligned}(-\,\omega_0' A_s L - s)E[U'(W)] &= \\ = E\{U'(W)[c(\theta'(\tilde{A})\tilde{A} + (1 + \theta(\tilde{A})))\, A_s(1 + \varepsilon)L]. \end{aligned} \tag{22}$$

Comparing (21) and (22) with (4) and (5) shows that the firm that is experience rated employs less labor and purchases more safety equipment than the firm that is not experience rated. When compensation costs are positively related to the firm's accident rate, therefore, the firm does respond to the incentive of experience rating by purchasing more safety equipment and not simply by reducing labor employed. If a greater proportion of accident costs are *internalized* by investing in safety measures, we should find that the response of the experience rated firm to an increase in c is smaller than that of the nonexperience-rated firm. This hypothesis cannot be shown mathematically given the ambiguous sign of the comparative static exercise with respect to c. It has, however, received some empirical support in the study by Ruser [1985], who finds that in the United States there was a smaller relationship between benefits and injuries in more experience-rated establishments.

6. Conclusions

This paper has presented a behavioral model illustrating the firm's choice of labor and of safety investment when the injury risk function has a random component.

The model is general enough to apply both to the case of risk-averse and risk-neutral firms, so that it has been possible to check whether different attitudes to risk give rise to different demands for the two inputs considered and to different responses to changes in the model's parameters.

The model suggests that a hazardous firm whose utility function displays risk aversion has a lower demand for labor and a higher demand for safety measures than a risk-neutral firm. These results accord with those obtained in the cases in which uncertainty in general enters the firm's costs. Its interest, in our specific case, lies in the fact that safety measures are an extra and "special" input to the production process and thus uncertainty affects the firm's strategy concerning preventive action.

If we consider the deterministic versus the random injury functions to be determined by different technologies of production or of risk abatement, our findings should lead to the (possibly) empirically observable fact that industries whose accident rate is known to be partially determined by noncontrollable variables (such as weather conditions and workers' on-the-job errors) have a higher safety per worker ratio than industries in the same risk class and whose accident is deterministic.

Worthy of notice, the comparative static exercise with respect to the cost of injury compensation shows that the effect of higher unit costs of compensation is ambiguous and depends on how much the marginal utility of extra safety is increased by the rise in the unit cost of compensation c. Thus, regulation through intervention in the compensation system may not always achieve its targets of an improvement in safety standards and thus a reduction in the accident rate. These results confirm those obtained by Schneider in the case of a risk-neutral firm and point out that workers' moral hazard is not the only explanation for the ambiguous effect of legislated injury compensation benefits.

The introduction of experience rating in the compensation system, however, provides the firm with an incentive to adopt a higher ratio of safety per worker than the firm that is not experience rated, and therefore it represents a better alternative than an increase in the fixed compensation cost, c.

Changes in the mean and spread of the injury rate that occur as a consequence of a change in the technology of production (or possibly as a consequence of changes in workers' care) have effects on the demand for labor and safety that depend on the way in which uncertainty is defined and on the class of utility function adopted. In general, firms with constant absolute risk aversion will reduce their level of employment and safety investment as a consequence of a shift in the parameters of the injury distribution function.

The general conclusion to be drawn from the results above is that without exact knowledge of the risk attitude of individual firms and of their "injury production function," $\tilde{A}$, it is difficult to predict their behavior as far as the provision of safety is concerned and hence to devise an optimal regulatory policy. The experience rating of compensation costs seems to be a way to induce the firm to undertake unambiguously more safety investment, but perfect experience rating is difficult to implement because of monitoring costs.

It should be noted that the above discussion relies on the exact knowledge of the technology of prevention—that is, of the rate at which the increase in safety measures reduces the accident rate. An interesting extension of the model could consider the possibility of random returns to injury abatement along the lines of Briys, Schlesinger, and Schulenburg [1991]. This would introduce an extra source of uncertainty that may alter the firm's behavior in directions different than those predicted by the model.

Also, it would be interesting to investigate how the results of the model change when it is assumed that the firm insures compensation costs on the market. With such a formulation we could try to determine the optimal degree of insurance coverage.

Finally, what does emerge unambiguously from the analysis is that a randomized accident risk brings about a fall in the usage of labor. Hence, when uncertainty brings about an increase in the safety per worker ratio, it is always at the cost of a fall in production and a loss in profit.[7] The evaluation of these effects requires a welfare analysis that weighs the benefits to workers against the costs borne by the firms.

Appendix

a. Second-order conditions

Satisfaction of the second-order conditions requires that

$$U_{LL} < 0, \; U_{LL}U_{SS} - (U_{LS})^2 > 0$$

where

$$\begin{aligned} U_{LL} &= \partial^2 E[U(W)]/\partial L^2 = E[U''(W)(W_L)^2 + U'(W)pQ_{LL}] = \\ &= E\{U'(W)[-R_A(W)W_L^2 + pQ_{LL}]\} \end{aligned} \quad \text{(A1)}$$

$$\begin{aligned} U_{SS} &= \partial^2 E[U(W)]/\partial S^2 = E\{U''(W)(W_S)^2 - U'(W)[\omega_0' A_{ss} + \omega_0'' A_s^2 + \\ &\quad + cA_{ss}(\varepsilon + 1)]L\} = \\ &= E\{U'(W)[-R_A(W)W_S^2 - (\omega_0' A_{ss} + \omega_0'' A_s^2 + cA_{ss}(\varepsilon + 1))L]\}. \end{aligned} \quad \text{(A2)}$$

In (A1) and (A2), $R_A(W) = -U''(W)/U'(W)$ denotes the Arrow-Pratt absolute risk-aversion index.

If $R_A(W)$ is constant, the sign of (A1) and (A2) is determined by the sign of the second part of these expressions, which are negative, including sign. If $R_A(W)$ is decreasing, $E[U'(W)(-R_A(W)W_S^2)]$ and $E[U'(W)(-R_A(W)W_L^2)]$ are negative as well, so that the sign of U_{LL} and U_{SS} is still negative.

$$U_{LS} = \partial^2 E[U(W)]/\partial L\partial S = E[U''(W)(W_L W_S) - U'(W)(\omega' + c(1 + \varepsilon))A_s] = E\{U'(W)[-R_A(W)W_L W_S - (\omega' + c(1 + \varepsilon))A_s]\}. \quad (A3)$$

Under CARA, the sign of U_{LS} is that of $-(\omega' + c(+ \varepsilon))A_s$—that is, positive. Under DARA, U_{LS} remains positive since $U'(W)W_L W_S$ is decreasing in $\tilde{A}$ and so is $R_A(W)$.

Satisfaction of the second-order conditions can be easily proved for the case of risk neutrality, but it is not easy to assess under risk aversion. With risk-neutral firms,

$$U_{LL}U_{SS} - (U_{LS})^2 = pQ_{LL}E(U'(W))[E(U'(W))W_{SS}] - - \{E[U'(W)(W_S - s)L^{-1}]\}^2. \quad (A4)$$

This will be positive if

$$pQ_{LL}\, W_{SS} > [(W_S - s)L^{-1}]^2.$$

b. Comparative statics with respect to the moments of Ã

Total differentiation of the first-order conditions with respect to β yields the system of equations:

$$\begin{bmatrix} U_{LL} & U_{LS} \\ U_{SL} & U_{SS} \end{bmatrix} \begin{bmatrix} \partial L^*/\partial\beta \\ \partial S^*/\partial\beta \end{bmatrix} = \begin{bmatrix} -U_{L\beta} \\ -U_{S\beta} \end{bmatrix}, \quad (A5)$$

where

$$U_{L\beta} = -E\{[U'(W)(\omega_0' + c)] + [U''(W)W_L(\omega_0' + c)L]\} = = -(w_0' + c)E\{U'(W)[1 - R_A(W)W_L L]\} \quad (A6)$$

$$U_{S\beta} = -E\{[U'(W)(\omega_0'' A_s L)] + [U''(W)W_S(\omega_0' + c)L]\} = = -E\{U'(W)[(\omega_0'' A_s L) - R_A(W)W_S(\omega_0' + c)L]\}. \quad (A7)$$

As for (A1) and (A2), we can examine cases of absolute risk aversion as follows:

1. If CARA applies, the sign of (A6) and (A7) is determined by their first term. Hence, under $R_A(W)$ constant $U_{L\beta}$ is negative, while $U_{S\beta} < 0$ if the wage function is strictly convex and $U_{S\beta} = 0$ if the wage function is linear in the risk of injury.

2. If the utility function displays DARA, then, by the sign of the first derivative of $R_A(W)$, $U_{L\beta} < 0$ and $U_{S\beta} > 0$ (no matter whether the second derivative of the wage function is positive or zero).

Application of Cramer's rule to the system of equations (A5) evaluated at $\gamma = 1$ and $\beta = 0$ yields equations (11) and (12) in the main text.

Repetition of the same exercise to evaluate the effects of changes in the spread of the distribution, requires that we sign the following expressions:

$$U_{L\gamma} = -cA(S)E\{U'(W)\varepsilon[1 - R_A(W)W_L L]\} \tag{A8}$$

$$U_{S\gamma} = (cA(S)L)E[R_A(W)U'(W)W_S\varepsilon]. \tag{A9}$$

Under CARA, the sign of (A8) depends on the sign of the covariance between $U'(W)$ and ε, which is positive. Hence $U_{L\gamma} < 0$. Similarly, since the covariance between the marginal utility of safety measures and the random variable ε is positive, $U_{S\gamma} > 0$ when $R_A(W)$ is constant. The same signs hold under DARA, since the marginal utility of labor is decreasing in ε, and the marginal utility of safety is increasing in ε.

c. *Other comparative static results*

The cross-derivatives with respect to the cost of injury compensation, c, and the output price, p, can be signed as done with the expressions considered above:

$$U_{Lc} = -E[U'(W)\tilde{A}] + E[R_A(W)U'(W)W_L\tilde{A}L] < 0 \tag{A10}$$

$$U_{Sc} = -E[U'(W)A_s(\varepsilon + 1)L] + E[R_A(W)U'(W)W_S\tilde{A}L] > 0. \tag{A11}$$

The signs of U_{Lc} and U_{Sc} are those above, provided $R_A(W)$ is nonincreasing.

The cross-derivative with respect to the mandated safety standards, $(\partial U/\partial L\partial S_R)$, is

$$U_{LSr} = -E[U'(W)](\omega' + c)A_s + E[U''(W)W_L W_S] > 0. \tag{A12}$$

Those with respect to output prices are

$$\begin{aligned} U_{Lp} &= Q_L E[U'(W)] + Q(L)E[U''(W)W_L] = \\ &= E\{U'(W)[Q_L - Q(L)R_A(W)W_L]\} \end{aligned} \tag{A13}$$

$$U_{Sp} = E[U''(W)W_S Q(L)] = Q(L)E[-R_A(W)U'(W)W_S]. \tag{A14}$$

If $R_A(W)$ is constant, then $U_{Lp} > 0$ and $U_{Sp} = 0$. If $R_A(W)$ is decreasing, the sign of (A13) stays positive, while (A14) is negative.

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Notes

1. This assumes, however, that the firm has no guess as of the possible value of workers' care. This treatment can be considered as alternative to one in which the firm can affect workers' behavior by providing incentives or penalties. Also, I assume that workers' care is not dependent on firm's safety measures.
2. The problem is, in some sense, analytically akin to that of nonpoint pollution control under uncertainty. In that case, monitoring of individual polluters is difficult due to the nonlocalized nature of the emissions (take acid rains as an example), even in a world of certainty. In addition, inference of pollution control measures undertaken based on ambient pollution levels can be misleading due to the random distribution of pollution levels.
3. An additive version of the risk function

$$\tilde{A} = A(S) + \varepsilon$$

was also used in a preliminary version of the model. This version will provide a useful comparison with the multiplicative risk function whenever the results from the latter are ambiguous.
4. From the assumption that any risky firm pays compensating wage differentials, it follows that $\omega_0 > w_{RF}$, where w_{RF} is the wage paid in the risk-free sector of the economy.
5. It is assumed that the firm operates in a system similar to workers' compensation in the United States. The firm pays a benefit equal to c for each injury that takes place and in exchange the worker foregoes the right to sue the firm in court. The benefit c can be mandated by law. Our model assumes that the firm does not insure the expected cost of injuries on the market.
6. Since the model is static, productive capital can be considered as given. A previous version of the model incorporating capital as well as labor in the production function Q, showed that the demand for capital behaves exactly as the demand for labor.
7. At the aggregate level, the fall in the demand for labor affects workers as a group and may represent an extra cost of the regulation.

References

BARTEL, A.P. and THOMAS, G. [1985]: "Direct and indirect effects of regulation: A new look at OSHA's impact," *Journal of Law and Economics,* 28(2), 1–25.

BRIYS, E., SCHLESINGER, H., and SCHULENBURG, J.M. [1991]: "Reliability of risk man-

agement: Market insurance, self-insurance and self-protection reconsidered," *Geneva Papers on Risk and Insurance Theory,* 16(1), 45–58.
BUTLER, R.J. and WORRALL, J.D. [1983]: "Workers' compensation: Benefit and injury claims rates in the seventies," *Review of Economics and Statistics,* 65(4), 580–589.
DIONNE, G. and ST-MICHEL, P. [1991]: "Workers' compensation and moral hazard," *Review of Economics and Statistics,* 73(2), 236–244.
ELDER, H.W. [1985]: "An economic analysis of factor usage and workplace regulation," *Southern Economic Journal,* 52(2), 315–331.
REA, S.A., Jr. [1981]: "Workmen's compensation and occupational safety under imperfect information," *American Economic Review,* 71(1), 80–93.
RUSER, J.W. [1985]: "Workers' compensation insurance, experience rating and occupational injuries," *Rand Journal of Economics,* 16(4), 487–503.
SANDMO, A. [1971]: "On the theory of the competitive firm under price uncertainty," *American Economic Review,* 61(1), 65–73.
SCHNEIDER, T. [1992]: "Economic behaviour of the firm and prevention of occupational injuries," *Geneva Papers on Risk and Insurance Theory,* 17(1), 77–85.
SIDER, H. [1983]: "Safety and productivity in underground coal mining," *Review of Economics and Statistics,* 65(May), 225–233.
VISCUSI, W.K. [1986]: "The impact of occupational safety and health organisation, 1973–1983," *Rand Journal of Economics,* 17(4), 567–580.