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Correction to

Finite Rings with a Specified Group of Units

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Ian Stewart

Dr. G.L.C. Bond of the University of Reading has pointed out a mistake in my above paper. In the proof of Lemma 3.2 I made the inadvertent assumption that all rings are commutative, and in fact the lemma fails for the ring of 2×2 upper triangular matrices over $\mathbb{Z}_p$, p prime.

Fortunately the error can be confined to Lemma 3.2, as was also pointed out by Dr. Bond. The only place where 3.2 is used is in the proof of 3.3, and here its use can be avoided as follows: we have $R/J \cong T_1 \oplus \cdots \oplus T_n$ where each $T_i \cong \mathbb{Z}_2$, and we pick elements $e_j \in R$ which map by the canonical homomorphism $R \to R/J$ to the identity of T_j. We find that $e_n = x + a$ where $x \in J$ and $a \in \operatorname{Ann}(J)$. At this point we wish to assert that $e_n^\lambda \in \operatorname{Ann}(J)$ for some integer $\lambda > 0$. In the paper we invoked 3.2; instead we show that $\lambda = c-1$ will do, where c is the index of nilpotency of J. For then

$$\begin{aligned} e_n^{c-1} &= (x+a)^{c-1} \\ &= x^{c-1} + z \end{aligned}$$

where $z \in \operatorname{Ann}(J)$ (which is an ideal of R by 3.1). Thus $e_n^{c-1} J = (x^{c-1} + z) J = 0$ since $J^c = 0$, and similarly $J e_n^{c-1} = 0$. Therefore $e_n^{c-1} \in \operatorname{Ann}(J)$. The proof of 3.3 now carries through with λ replaced by $c-1$ (or by 1 when $c=1$) with no further changes.

Thus all the results can be resuscitated, with the exception of Lemma 3.2.

Dr. Ian Stewart
Mathematics Institute
University of Warwick
Coventry CV 4 7 AL
England

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