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Financing and the Demand for Corporate Insurance

MARTIN F. GRACE
College of Business Administration, Georgia State University, Atlanta, GA 30303

MICHAEL J. REBELLO
College of Business Administration, Georgia State University, Atlanta, GA 30303

Abstract

In this paper we examine the insurance decision of a firm with private information regarding its cash flows and insurable losses. We show that, even in the absence of bankruptcy costs and information production by insurers, the firm's attempts to hedge its information risk can induce it to demand insurance. If higher operating revenues are accompanied by a lower insurance risk, the firm will choose to self-insure. In contrast, if higher operating revenues are accompanied by a higher insurance risk, the firm will demand insurance. In fact, if its insurable losses are relatively small, the firm will fully insure its losses. Further, if there exists considerable uncertainty regarding the firm's insurance risk, the level of coverage demanded by the firm is dependent on its private information, with higher levels of coverage signaling favorable information regarding the firm's future operations.

Key words: Adverse Selection, Corporate Insurance, Signaling

1. Introduction

A recent survey by Tillinghast [1990] found that corporations expend significant resources on obtaining property and liability loss insurance: an average corporation spends almost $6 million, a median firm spends $2.3 million, and the largest purchaser spends $162 million on property and liability insurance.[1] For some time researchers have been trying to ascertain why corporations purchase such significant amounts of insurance. Earlier explanations of this phenomenon were based on the notion that because corporate insurance helped them diversify risk, risk-averse investors would prefer that corporations purchase insurance (see, e.g., Borch [1961]). More recently, though, researchers have argued that because of the separation of ownership and control institutionalized in the modern corporation, investors are able to effectively hedge against insurance risk through diversification (see, e.g., Mayers and Smith [1982]). Consequently, the corporate demand for insurance cannot be driven solely by diversification motives.

To explain the corporate demand for insurance, researchers now focus their attention on the role of insurance in reducing costs incurred by corporations. For example, Main [1983] argues that corporations can reduce their tax liability by purchasing insurance. MacMinn [1987] demonstrates that insurance purchases can increase firm value by helping firms avoid financial distress costs. Yet another explanation for the corporate demand for insurance is based on the ability of insurance to increase firm efficiency by mitigating agency conflicts between corporate stakeholders. For example, Mayers and Smith [1982] argue that insurance helps in the monitoring of contract compliance and bonds investment decisions (see also MacMinn and Han [1990]).

While the tax, transaction cost, and moral-hazard–based insurance literature is extensive, there has been little attention devoted to the impact of adverse selection on the demand for corporate insurance.[2] One paper that does analyze the corporate demand for insurance in an adverse-selection context is Thakor [1982]. Thakor demonstrates that when firms have private information regarding their cash flows and insurance companies can generate information about these cash flows, the purchase of bond insurance can act as a signal of firm quality, enabling firms to avoid adverse-selection costs associated with financing decisions.

In this paper we further examine the influence of asymmetric information on the corporate demand for insurance. We consider the case of a firm with private information regarding its quality—i.e. its expected operating revenues and insurance risk. The remaining agents in the economy, including insurers, know only the probability distribution over firm quality and are able to obtain information regarding the firm's quality only by observing the firm's insurance decision. We assume that the firm has access to a positive net present value (NPV) project and has to decide on its insurance coverage at the same time it is undertaking the project. Even if it purchases no insurance, the firm has insufficient funds to undertake the project. To raise the finances required to undertake the project and to purchase insurance, the firm issues bonds.[3]

Because the firm has private information regarding its insurance risk, there exists the potential for incorrectly assessing its insurance risk. Further, there also exists the potential for the market to incorrectly assess the default risk on the firm's bonds. In general, the lower the investors' assessment of the firm's operating revenues relative to its true operating revenues, the greater the adverse-selection cost from the overestimation of the firm's default risk. Similarly, the lower the insurers' assessment of the firm's insurance risk relative to its true insurance risk, the greater the insurance related benefits from adverse selection.

Thus, if higher operating revenues are associated with a higher insurance risk, adverse-selection costs from the underestimation of operating revenues are mitigated by gains from the underestimation of the firm's insurance risk. This occurs if, for example, the firm's private information relates to future demand for its products, and the probability that its plants will be destroyed increases with output. In this case, if the firm's private information is favorable—i.e., the firm can expect both higher operating revenues and higher insurable losses—the underestimation of operating revenues is accompanied by the underestimation of the firm's insurance risk. In contrast, if higher operating revenues are associated with a lower insurance risk, adverse-selection costs from the overestimation of the firm's default risk are compounded by adverse-selection costs from the overestimation of the firm's insurance risk. This occurs if, for example, the firm's private information pertains to the efficiency of its operations, and increased efficiency reduces the probability of incurring insurable losses. In this case, if the private information is favorable—i.e., the firm knows that it is efficient—the underestimation of operating revenues is accompanied by the overestimation of insurance risk.

In our framework, the firm's problem reduces to one of minimizing the adverse selection costs resulting from the incorrect assessment of both its default risk and its insurance risk. In the event that higher operating revenues are associated with a lower insurance risk, minimization of the adverse-selection costs induces the firm to not purchase insurance. In direct contrast, if higher operating revenues are associated with a higher insurance risk, the firm will, in general, purchase insurance. If insurance losses are relatively small or the firm's capital requirements are relatively large, the firm will always choose to fully insure its losses. However, despite fully insuring its losses, the firm is unable to fully offset the adverse selection costs it incurs because of the market's overestimation of its default risk. In contrast, if its capital requirements are moderate, the firm may be able to avoid adverse-selection costs by unambiguously signaling its private information through its coverage choice. Favorable information is signaled through the purchase of a high level of insurance coverage, while unfavorable information is signaled through the choice of a low level of insurance coverage.

These results generate a variety of interesting empirical implications. For example, they show that firms with favorable information will purchase higher levels of coverage. This implies that, on average, the purchase of high levels of insurance coverage should be accompanied by an upward revision in firms' security prices. Conversely, the purchase of low insurance coverage should be accompanied by a downward revision in firms' security prices. Further, on average, firms that purchase high levels of coverage should display higher operating revenues than firms that purchase low levels of coverage. Our results also show that firms that purchase higher levels of insurance coverage will tend to have a higher insurance risk. Thus, one would expect the marginal cost of insurance to be increasing in the level of coverage. Finally, our results establish that firms operating in industries where expected losses are relatively small will either purchase no insurance or will fully insure their losses, while the coverage choices of firms operating in industries with relatively large losses will tend to be less polarized. This implies that the variance in coverage levels across firms will be greater in industries with small insurable losses.

Our model differs from the traditional Rothschild and Stiglitz [1976]–type approach to addressing adverse-selection problems in insurance markets as it recognizes that insurance purchasers face resource constraints and that because of these resource constraints insurance purchases might be tied to decisions such as corporate financing decisions. By considering the interaction between the insurance decision and financing decisions, our approach provides additional insights into the corporate demand for insurance. Our model and results also differ substantially from those of Thakor [1982], who demonstrates that corporations may demand bond insurance in an adverse-selection context. Thakor's results are predicated on the assumption that insurers are able to expend resources to obtain information regarding firm quality. However, we show that, even when information production by insurers is not feasible, firms may demand insurance to reduce financing-related adverse-selection costs. The only similarity between our results and those of Thakor is that both sets of results predict that, on average, firms will demand more insurance if they have favorable information regarding their future prospects.

This analysis also has some commonalities with the literature on design of insurance contracts based on the endogenous categorization of risks. For example, Bond and Crocker [1991] develop a model in which consumers have private information regarding their insurance risks and these risks are correlated with the consumption of certain goods. They show that because the cost of changing consumption patterns to misrepresent insurance risk may be too high, adverse-selection problems in the insurance market can be solved by offering consumption-contingent insurance contracts. Our results, in the case where the firm requires a moderate amount of capital and higher operating revenues are associated with a higher insurance risk, are also based on a similar tradeoff: the cost of changing insurance coverage to misrepresent default risk may be too high. Thus, a firm can be *categorized* based on the demand for insurance. Our model, however, differs from that of Bond and Crocker in one important aspect. In our context, the firm has no inherent preference for insurance, the consumption of which is used to categorize it. The firm's demand for insurance is driven solely by its desire to minimize the mispricing of its bond contract. In contrast, the insured's inherent preference for goods correlated with insurance risk is the critical factor driving Bond and Crockers' results.

The remainder of the paper is organized as follows. Section 2 contains a description of the model. In section 3 we examine the pricing of insurance and financing contracts in an adverse selection context. Section 4 is devoted to an examination of the firm's insurance decision. The final section presents some additional comments and a summary of our results. Proofs of all claims are presented in the appendix.

2. The model

Consider a single-period, two-date economy with dates indexed by $d = 0, 1$. All agents in the economy are risk neutral and the risk-free interest rate is 0. Insurance and securities markets are competitive, ensuring that insurance is sold at actuarially fair prices, and whenever possible, projects are financed at zero expected profits to security buyers.[4] At date 0 a firm has access to a positive NPV project requiring an investment of $\$I$. The project generates a cash flow at date 1. Prior to undertaking the project, the firm must decide on a level of insurance coverage and issue bonds to finance both the project and the insurance policy. The financing terms and insurance coverage together determine the existing equityholders' share of the date 1 cash flows. Both the insurance and the financing decisions are made by a manager who maximizes the date 0 expected value of these cash flows.[5] The firm's type, denoted by $t \in T = \{G, B\}$, can either be good (G) or bad (B) and is known only by the manager. All other agents have homogeneous beliefs regarding the firm's type.[6] The uninformed agents' common assessment that the firm is type G is represented by $\phi \in (0, 1)$, and this probability distribution over firm types is common knowledge.

At date 0, first the firm chooses a level of insurance coverage, K. This coverage choice causes insurers and investors to revise their beliefs regarding the firm's type. This revision only depends on the message selected by the firm. We denote the common revised belief that the firm is type G by $\mu(G \mid K)$. Insurers respond to the firm's message by posting a premium for the desired coverage. Investors respond to the message either by setting a face value, F, for the bonds and providing the firm with the funds required to finance the project and purchase insurance or by deciding not to finance the firm's plans—i.e., setting $F = N$.

If the firm is type t and succeeds in financing the project and purchasing insurance coverage K, it obtains a date 1 cash flow of $\tilde{X}_t(K)$. This cash flow represents the project's operating revenue less the firm's net loss, where the net loss represents the difference between the firm's insurable loss and the insurance compensation it receives. The operating revenues and insurable losses are assumed to be independently distributed and have two point supports $\{L, H\}$ and $\{0, M\}$ respectively, where $H > H - M \geq L$ and $M > 0$. To abstract from the impact of the limited liability feature of financial contracts, we assume that the date 1 cash flow is never negative—i.e., $L - M \geq 0$.

As the insurance coverage pays off only when the firm incurs a loss, given coverage K, the firm's net loss has the two-point support $\{0, M - K\}$. Thus, the firm's net cash flow has the four-point support: $\{L, H\} - \{0, M - K\}$. If the firm is type t, we assume that it has probability $P_{tH} \in (0, 1)$ of realizing operating revenue H and probability $1 - P_{tH}$ of realizing operating revenue L. Similarly, if the firm is type t, it has probability $P_{tM} \in (0, 1)$ of realizing insurable loss M and probability $1 - P_{tM}$ of realizing no insurable loss. It follows that, if the firm is type t, it has probabilities $P_{tH}\,(1 - P_{tM})$, $P_{tH}\,P_{tM}$, $(1 - P_{tH})(1 - P_{tM})$, and $(1 - P_{tH})\,P_{tM}$ of realizing cash flows H, $H - M + K$, L, and $L - M + K$ respectively. Further, if it is type t and purchases coverage K, the expected value of the firm's date 1 cash flow can be represented by $V_t(K)$, where

$$V_t(K) \equiv L + P_{tH}(H - L) - P_{tM}(M - K).$$

We formalize the cash-flow ordering across firm types by assuming that, for each level of insurance coverage, the value of the firm's cash flows if it is type G is greater than the value of its cash flows if it is type B—i.e., $V_G(K) > V_B(K)$ for all $K \in [0, M]$. Note that because this restriction also holds for $K = M$, it also implies that the firm has a higher probability of realizing an operating revenue H if it is type G—i.e., $P_{GH} > P_{BH}$. This assumption does not, however, preclude the firm from having higher insurable losses if it is type G—i.e., there exist values of the other parameters such that $P_{GM} > P_{BM}$ is consistent with this assumption.

The division of the firm's date 1 cash flows determines the payoffs to the firm's claimants. If the firm's date 1 realized cash flow is high enough, bondholders are paid off in full, and equityholders retain the residual cash flow. Otherwise, in the event of a low cash-flow realization, the bondholders take over the firm's assets. Thus, the bondholders' expected payoff contingent on the firm being type t, purchasing coverage K, and issuing bonds with a face value of F, can be represented by $w(t, K, F)$, where

$$w(t, K, F) \equiv \begin{cases} E\{Min\{\tilde{X}_t(K), F\}\} & \text{if } F \neq N \\ 0 & \text{otherwise.} \end{cases}$$

The equityholders receive the firm's cash flows net of the payments to bondholders. Thus, the expected payoff to the firm's equityholders, if the firm is type t, purchases coverage K, and issues bonds with a face value F, can be represented by $u(t, K, F)$, where

$$u(t, K, F) \equiv \begin{cases} E\{Max\{\tilde{X}_t(K) - F, 0\}\} & \text{if } F \neq N \\ 0 & \text{otherwise.} \end{cases}$$

As insurers are liable only if the firm suffers a loss, if the firm is type t, their expected date 1 insurance payment is $P_{tM} K$. Let $P_M(\delta) = \delta P_{GM} + (1 - \delta) P_{BM}$. It follows that, if insurers assess probability δ to the firm being type G, their expected date 1 insurance payments can be represented by $\Gamma(\delta, K)$, where $\Gamma(\delta, K) \equiv P_M(\delta)K$, for all $K \in [0, M]$. Similarly, if they assess probability δ to the firm being type G, the bondholders' expected payoffs can be represented by $W(\delta, K, F)$, where

$$W(\delta, K, F) \equiv \delta w(G, K, F) + (1 - \delta) w(B, K, F).$$

In equilibrium, agents' actions are determined by their conjectures regarding other agents' actions and their impact on the payoffs described above. To focus solely on equilibria in which the firm's insurance decision is relevant, we assume that the firm always chooses to invest and investors always finance the investment and desired level of insurance coverage. Further, to ensure that in the event that the firm issues debt, the adverse-selection costs are not solely a product of the incorrect assessment of its insurable losses, we restrict our attention to cases where the firm defaults if it realizes operating revenue L.[7] Finally, to simplify the analysis, we restrict our attention to cases where the firm does not default if it realizes operating revenue H.

More formally, we restrict our analysis to the subset of the parameter space satisfying the restrictions $I > L$, and

$$(H - M)P_{tH} + L(1 - P_{tH}) - M(1 - P_{tH})P_{tM} > I + \max_{t} P_{tM} M \qquad \text{(FI)}$$

for $t = B, G$. The restriction $I > L$ ensures that the firm will default on its debt if it realizes the cash flow L or $L - M + K$. The restriction (FI) ensures that the manager will always prefer to invest and will always be able to raise the desired financing. It also ensures that the firm will never default on its debt if it realizes the cash flow H or $H - M + K$. Thus, together, the two restrictions generate the desired simplification.

In the sequel we examine the equilibria of this model. We begin by examining the pricing decisions of investors and insurers and their implications for the adverse-selection costs incurred by the firm. Then we proceed with an examination of the firm's equilibrium decisions.

3. Pricing decisions and adverse-selection costs

In this section, we derive the pricing functions for the firm's bonds and insurance contract. Then we examine the implications these pricing functions have for the adverse-selection costs incurred by the firm.

The pricing decisions of the insurers and investors are a function of their beliefs regarding the firm's type. If insurers and investors assess probability δ to the firm being type G, contingent on it purchasing coverage K, their expectation regarding the firm's date 1 cash flow can be represented by $V(\delta, K)$, where

$$V(\delta, K) \equiv V_G(K) + (1 - \delta)V_B(K).$$

Similarly, when investors and insurers assess probability δ to the firm being type G, their assessment of the probabilities of the cash-flow realizations H, $H - M + K$, L, and $L - M + K$ can be represented by $P_{HO}(\delta)$, $P_{HM}(\delta)$, $P_{LO}(\delta)$, and $P_{LM}(\delta)$, respectively, where

$$P_{HO}(\delta) \equiv \delta P_{GH}(1 - P_{GM}) + (1 - \delta)P_{BH}(1 - P_{BM}),$$
$$P_{HM}(\delta) \equiv \delta P_{GH}P_{GM} + (1 - \delta)P_{BH}P_{BM},$$
$$P_{LO}(\delta) \equiv \delta(1 - P_{GH})(1 - P_{GM}) + (1 - \delta)(1 - P_{BH})(1 - P_{BM}),$$

and

$$P_{LM}(\delta) \equiv \delta(1 - P_{GH})P_{GM} + (1 - \delta)(1 - P_{BH})P_{BM}.$$

Table 1. Operating revenue.

Loss	L	H	
0	$P_{LO}(\delta)$	$P_{HO}(\delta)$	$1 - P_M(\delta)$
M	$P_{LM}(\delta)$	$P_{HM}(\delta)$	$P_M(\delta)$
	$1 - P_H(\delta)$	$P_H(\delta)$	

Let the term $P_H(\delta)$ represent the probability that investors and insurers will assess to the realization of *operating revenue H* if they assess probability δ to the firm being type G—i.e., $P_H(\delta) = P_{HO}(\delta) + P_{HM}(\delta) = \delta P_{GH} + (1 - \delta)P_{BH}$. Table 1 provides a systematic representation of these probabilities, given that investors and insurers assess probability δ to the firm being type G.

Insurance pricing implications

Competition in the insurance market ensures that every insurance contract generates zero expected profits for the insurer. Thus, given the insurers' belief that the firm is type G with probability δ, the premium charged as a function of the market's belief and the firm's coverage can be represented by $\Pi^*(\delta, K)$, where

$$\Pi^*(\delta, K) = \Gamma(\delta, K) = KP_M(\delta).$$

If the firm was known to be type $G(B)$, its insurance premium for coverage K would be $\Pi^*(1, K)$ ($\Pi^*(0, K)$). It follows that if insurers assess probability δ to the firm being type G and $\Pi^*(\delta, K) > (<) \Pi^*(1, K)$, insurance coverage is overpriced (underpriced) if the firm is type G. Because $\Pi^*(\delta, K) - \Pi^*(1, K) = (1 - \delta)(P_{BM} - P_{GM})K$, if the firm is type G and insurers assess probability $\delta \neq 1$ to the firm being type G, it loses as a result of adverse selection if $P_{BM} > P_{GM}$ and benefits from adverse selection if $P_{BM} < P_{GM}$. Thus, if it has favorable information regarding its operating revenues, by purchasing insurance, the firm incurs an adverse-selection cost if higher operating revenues are associated with a lower insurance risk and obtains a benefit from adverse selection if higher operating revenues are associated with a higher insurance risk. Further, because

$$\frac{\partial(\Pi^*(\delta, K) - \Pi^*(1, K))}{\partial K} = (1 - \delta)(P_{BM} - P_{GM}),$$

the magnitude of the cost increases with the level of coverage if higher operating revenues associated with a lower insurance risk.

Similarly, if insurers assess probability $\delta \neq 0$ to the firm being type G and $\Pi^*(\delta, K) > (<) \Pi^*(0, K)$, insurance coverage is overpriced (underpriced) if the firm is type B. It follows that if the firm is type B and insurers assess probability δ to the firm being type G, it incurs an adverse-selection cost if $P_{BM} < P_{GM}$ and obtains a benefit from adverse selection if $P_{BM} > P_{GM}$. Further, because

$$\frac{\partial(\Pi^*(\delta, K) - \Pi^*(0, K)}{\partial K} = (1 - \delta)(P_{GM} - P_{BM}),$$

the magnitude of the cost increases with the level of coverage if higher operating revenues associated with higher insurance risk.

Bond pricing implications

Competition in the bond market ensures that investors will finance the firm's investment and insurance policy as long as they receive bonds whose expected payoff equals the firm's financing needs. Thus, if investors assess probability δ to the firm being type G, they will finance the firm as long as the face value of the bonds, $F = F^*(\delta, K)$, where

$$W(\delta, K, F^*(\delta, K)) = I + \Pi^*(\delta, K).$$

The following result characterizes the functional form of $F^*(\delta, K)$ implied by the above equality.

Lemma 1: *If investors and insurers assess probability δ to the firm being type G, then in return for financing the firm's bond issue, investors will demand bonds with a face value of $F^*(\delta, K)$, where*

$$F^*(\delta, K) = H - \frac{V(\delta, K) - P_{HM}(\delta)(M - K)}{P_H(\delta)}$$

and $H - M > F^*(\delta, K) > L$ for all $\delta \in [0, 1]$ and $K \in [0, M]$.

As the firm's cash-flow distribution is dependent on its type, the default risk of its bonds is also dependent on its type. If investors assess too high a default risk to the bonds, the firm incurs adverse-selection costs. In contrast, if investors assess too low a default risk to the bonds, the firm benefits from adverse selection. Because both the firm's cash-flow distribution and the amount of financing it requires are determined by its insurance coverage decision, as the following analysis demonstrates, the adverse-selection costs incurred by the firm vary with the amount of insurance purchased.

Let $U(t, \delta, K)$ represent the date 0 expected payoff to the equityholders if the firm is type t, it chooses coverage K, and its insurance and bond contracts are priced as if it is type G with probability δ, i.e.,

$$U(t, \delta, K) \equiv u(t, K, F^*(\delta, K)) = E\{Max\{\tilde{X}_t(K) - F^*(\delta, K), 0\}\}.$$

From lemma 1 it follows that $H - M > F^*(\delta, K) > L$ for all $\delta \in [0, 1]$ and $K \in [0, M]$. Thus, the equityholders do not receive any payoff if cash flow L or $L - M + K$ is realized. However, if cash flow H is realized, the equityholders receive $H - F^*(\delta, K)$, and if cash flow $H - M + K$ is realized, the equityholders receive $H - F^*(\delta, K) - M + K$. Table 2 illustrates these event-contingent payoffs to equityholders.

It follows that

$$E\{Max\{\tilde{X}_t(K) - F^*(\delta, K), 0\}\} = P_{tH}(H - F^*(\delta, K)) - P_{tH}P_{tM}(M - K).$$

This, in turn, implies that

$$U(t, \delta, K) = P_{tH}\left(\frac{V(\delta, K) - I + P_{HM}(\delta)(M - K)}{P_H(\delta)}\right) - P_{tH}P_{tM}(M - K).$$

If the firm is type G and investors and insurers assess probability δ to the firm being type G, the firm incurs an adverse-selection cost if $U(G, \delta, K) < U(G, 1, K)$ and obtains a benefit if $U(G, \delta, K) > U(G, 1, K)$. Similarly, if the firm is type B and investors and insurers assess probability δ to the firm being type G, the firm incurs an adverse-selection cost if $U(B, \delta, K) < U(B, 0, K)$ and obtains a benefit if $U(B, \delta, K) > U(B, 0, K)$. When $P_{GM} < P_{BM}$, the firm has a lower default probability if it is type G—i.e., it has lower probabilities of realizing cash flows L and $L - M + K$. It also has a lower insurance risk. Thus, if both the insurance contract and the bond contract are priced at pooled terms, the firm incurs an adverse selection cost if it is type G. In contrast, if the firm is type B, it benefits from adverse selection because the market underestimates both its insurance and default risks. This argument is formalized in the following lemma.

Lemma 2: *If* $P_{GM} < P_{BM}$, *then (i)* $U(G, \delta, K) < U(G, 1, K)$ *for all* $\delta \in [0, 1)$ *and* $K \in [0, M]$, *and (ii)* $U(B, \delta, K) > U(B, 0, K)$ *for all* $\delta \in (0, 1]$ *and* $K \in [0, M]$.

Table 2. Operating revenue.

Loss	L	H
0	0	$H - F^*(\delta, K)$
M	0	$H - F^*(\delta, K) - M + K$

The impact of the insurance decision on the magnitude of adverse-selection costs is subtle. If $P_{GM} < P_{BM}$, increased coverage has two effects. It magnifies adverse-selection costs incurred because of the purchase of overpriced insurance, and because it reduces the loss to bondholders when cash flow $L - M + K$ is realized, it reduces the type sensitivity of default risk on the bonds. The first effect increases adverse-selection costs, and the second reduces them. However, the first effect dominates, and increased insurance coverage results in increased adverse-selection costs if the firm is type G. Conversely, increased coverage increases the benefit from adverse selection if the firm is type B. When $P_{GM} > P_{BM}$, though, increased coverage has the opposite effect. If the firm is type G, it benefits from the purchase of increased coverage, and if the firm is type B, it incurs a cost if it increases its insurance coverage. This argument is formalized in the following lemma.

Lemma 3: *(i) $U(G, \delta, K)$ is strictly decreasing (increasing) in K for all $\delta \in [0, 1)$ if and only if $P_{GM} < (>) P_{BM}$. (ii) $U(B, \delta, K)$ is strictly decreasing (increasing) in K for all $\delta, \in (0, 1]$ if and only if $P_{BM} < (>) P_{GM}$.*

As the above result suggests, if $P_{GM} > P_{BM}$, the firm is type G, and both its insurance and bond contracts are priced at pooled terms, the firm can offset the adverse-selections costs it incurs because of the overestimation of the default risk on its bonds by increasing its insurance coverage. In fact, if the firm purchases sufficient coverage, it is able to completely offset the adverse-selection costs arising from the overestimation of its default risk. Let the coverage level K^D be defined as follows:

$$K^D \equiv M - \frac{P_{GH}[V_B(0) - I] - P_{BH}[V_G(0) - I]}{P_{GH}P_{BH}(P_{GM} - P_{BM})}. \tag{EK^D}$$

The following result demonstrates that if the firm is type G and it purchases coverage greater than K^D, it gains from adverse selection. However, if it purchases coverage less than K^D, it incurs an adverse-selection cost. In contrast, if the firm is type B, it incurs an adverse-selection cost if it purchases coverage greater than K^D, and gains from adverse selection if it purchases coverage less than K^D.

Lemma 4: *If $P_{GM} > P_{BM}$ and $K^D \in [0, M]$, then (i) $U(t, \delta, K)$ is constant in δ if $K = K^D$, $U(t, \delta, K)$ is strictly increasing in δ if $K \in [0, K^D)$, and $U(t, \delta, K)$ is strictly decreasing in δ if $K \in (K^D, M)$.*

Together, the above results demonstrate that, when higher operating revenues are associated with a higher insurance risk—i.e., $P_{GM} > P_{BM}$—the firm may be able to trade adverse-selection costs on one contract against gains from adverse selection on the other contract. In contrast, when higher operating revenues are associated with lower insurable losses—i.e., $P_{GM} < P_{BM}$—this tradeoff is not available. In the following section, we demonstrate that, in the former case, the firm will, in general, choose to purchase insurance. In the latter case, however, the firm will never choose to purchase insurance.

4. The insurance decision

In this section we examine the firm's optimal insurance decision. First we examine the insurance decision when higher operating revenues are associated with a lower insurance risk—i.e., $P_{GM} < P_{BM}$. Then we characterize the optimal insurance decision when higher operating revenues are associated with a higher insurance risk. In the former case, we demonstrate that there exist only pooling equilibria in which the firm chooses not to purchase insurance. In the latter case, we show that there exist pooling equilibria characterized by varying levels of coverage. In this case, there also exist separating equilibria in which the firm purchases higher coverage if it has favorable information regarding operating its operating revenues—i.e., if it is type G.

To characterize the firm's optimal insurance decision, we use the Perfect Bayesian Equilibrium (PBE) concept (see, e.g., Fudenberg and Tirole [1991], ch. 8.)[8] A PBE is characterized by the insurance coverage for the firm and a response strategy and beliefs for investors and insurers, such that each player's action is optimal given the others' decisions. Whenever possible, in a PBE, the market's beliefs are derived using the Bayes' rule.[9] Otherwise, they are unrestricted.

More formally, in our setting, a PBE is characterized by a firm strategy K^{t*}, a bond pricing function, an insurance premium function, and posterior probability that the firm is type G, $\mu^*(G \mid K)$, which satisfy the following conditions:

Condition 1: K^{t*} maximizes the payoff to type t, conditioned on investor and insurer responses—i.e., $K^{t*} \in \arg\max \{U(t, K, F^*(\mu^*(G \mid K), K)) \mid K \in [0, M]\}$ for $t \in T$.

Condition 2: Insurance contracts generate zero expected profits for insurers—i.e., the premium posted is $\Pi^*(\mu^*(G \mid K), K)$ for all $K \in [0, M]$.

Condition 3: Bond contracts generate zero expected profits for investors—i.e., the face value of the bonds is $F^*(\mu^*(G \mid K), K)$ for all $K \in [0, M]$.

Condition 4: If K is an equilibrium path action—i.e., whenever there exists at least one type such that $K^{t*} = K$— the posterior probability $\mu^*(G \mid K)$ is determined by Bayes's rule. In the case of an off-equilibrium action—i.e., $K \neq K^{t*}$ for $t = B, G$—the market's belief is unrestricted.

In a pooling equilibrium, if the firm selects the equilibrium coverage level, no information regarding firm type is conveyed to the market. Thus, Bayes's rule requires that if the firm chooses coverage $K = K^*$, where K^* represents the equilibrium coverage level, the market's posterior belief $\mu^*(G \mid K^*) = \phi$. Similarly, in a separating equilibrium, if the firm selects K^{G*}, the level of coverage conjectured to be chosen by it if it is type G, Bayes's rule requires that the market's posterior belief $\mu^*(G \mid K^{G*}) = 1$. If the firm selects K^{B*}, the level of coverage conjectured to be chosen by it if it is type B, the market's posterior belief is $\mu^*(G \mid K^{B*}) = 0$.

Because the market's beliefs in response to an off-equilibrium action are unrestricted by the PBE concept, there exist a multitude of PBE of our model. To highlight the tradeoffs available to the firm and robustness of our results, we focus our attention on those PBE that survive the Universal Divinity refinement of Banks and Sobel [1987].[10] Universal Divinity restricts the market's beliefs on the observation of an off-equilibrium action by requiring it to assess a probability of 1 to the type that is most likely to defect from the equilibrium to that particular action. However, when both types are equally likely to defect to an off-equilibrium action, Universal Divinity does not restrict the market's beliefs if the action is observed. Let $OEB_t(K)$ denote the set of market beliefs for which the equity-holders receive a higher payoff if the firm is type t and it defects from the equilibrium coverage level K^{t*} to the coverage level K—i.e.,

$$OEB_t(K) \equiv \{\delta \mid \delta \in [0, 1] \text{ and } U(t, \delta, K) \geq U(t, \mu^*(G \mid K^{t*}), K^{t*})\}.$$

In our setting Universal Divinity requires that the market assess probability 1 (0) to the firm being type G if the firm chooses coverage K and $OEB_B(K) \subset (\supset)$ $OEB_G(K)$. Further, if $OEB_B(K) = OEB_G(K)$, Universal Divinity places no restriction on the market's beliefs if the firm chooses coverage K.

The equilibria surviving Universal Divinity are characterized by insurance decisions that are very intuitive. The firm's insurance decision is driven by the desire to minimize the adverse-selection costs borne by it. Whenever possible, the firm chooses its coverage so as to unambiguously signal its type to the market. By doing so it is able to avoid bearing any adverse selection costs. However, when this is not possible—i.e., it is forced to "pool"—the firm chooses its insurance coverage so as to offset the adverse-selection costs arising from the overestimation of its default risk to the greatest possible extent.

Regardless of whether the firm is forced to pool or is able to signal its type to the market, its insurance decision is determined primarily by the relationship between operating revenues and insurable losses across types. When higher operating revenues are associated with a lower insurance risk, regardless of the level of coverage, if the firm is type G, it has both a lower default risk and a lower insurance risk. Thus, if the firm is type B and both contracts are priced at pooled terms, it is able to benefit from both the assessment of too low an insurance risk and the assessment of too low a default risk (see lemma 2). It follows that the firm will be forced to pool and incur adverse selection costs if it is type G. Because insurance is "overpriced" if the firm is type G, these adverse-selection costs are minimized if it does not purchase any insurance. Thus, when higher operating revenues are associated with lower insurable losses, there exist only pooling equilibria in which, regardless of its type, the firm chooses to purchase no insurance. This argument is formalized in the following proposition.

Proposition 1: *If $P_{GM} < P_{BM}$, there exist only pooling equilibria in which the firm purchases no insurance.*

A change in the relationship between operating revenues and insurable losses across firm types can lead to a complete reversal of this result. Consider the case where higher operating revenues are associated with a higher insurance risk. If the firm is type *G*, because it has a lower default risk, it incurs an adverse-selection cost if it purchases no insurance and all contracts are priced at pooled terms. From lemma 3, it follows that the purchase of insurance will reduce this adverse-selection cost. In contrast, if the firm is type *B*, the purchase of increased coverage will decrease its benefit from adverse selection. In fact, for sufficiently high coverage levels, the benefit it obtains from the assessment of too low a default risk to its bonds may be more than offset by the adverse selection costs it incurs from the purchase of overpriced insurance. When these coverage levels are not feasible—i.e., they entail the purchase of more than 100 percent coverage—if it is type *B*, the firm will always prefer its insurance and bond contract to be priced at pooled terms. Because it is forced to pool, if it is type *G*, the firm will demand full coverage to minimize its adverse-selection costs. This argument is formalized in the following proposition.

Proposition 2: *If $P_{GM} > P_{BM}$, for relatively small M or relatively large I, there exist only pooling equilibria in which the firm purchases full insurance coverage.*

While the firm is never able to avoid incurring adverse-selection costs if its insurable losses are small or its capital requirements are large, this is not the case when higher operating revenues are associated with a higher insurance risk, insurance risk differs greatly across types, and the level of investment required for the project is moderate—i.e., $P_{GM}/P_{BM} > P_{GH}/P_{BH}$ and *I* is not large. In this case, because *I* is not large, the amount of financing required is not very large. Thus, the adverse-selection consequences from financing are also not "large." Further, because of the sensitivity of insurance risk to firm type, the adverse selection consequences from the purchase of insurance *are* "large." Consequently, if the firm is type *B* and all contracts are priced at pooled terms, for sufficiently high coverage—i.e., for $K > K^D$—the firm incurs an adverse-selection cost because of the assessment of too high a risk to its insurance contract. Thus, if it is type *B*, the firm will prefer not to have its contracts priced at pooled terms for $K > K^D$. Consequently, if the firm is type *G*, it will not be forced to pool if it chooses a coverage level greater than K^D. Further, if the firm is type *G* and all contracts are priced at pooled terms, for sufficiently low coverage—i.e., $K < K^D$—the firm incurs an adverse-selection cost because of the assessment of too high a default risk to its bonds. Thus, if it is type *G*, the firm will prefer not to have its contracts priced at pooled terms for $K < K^D$. It follows then that the firm will unambiguously signal its private information by choosing a coverage level greater (lower) than K^D if it is type G (B). This argument is formalized in the following proposition.

Proposition 3: *If $P_{GM}/P_{BM} > P_{GH}/P_{BH}$, for intermediate values of I, $K^D \in (0, M)$, the firm finances with debt and chooses coverage $K^{G*} \in (K^D, M]$ if it is type G, and finances with debt and chooses coverage $K^{B*} \in [0, K^D)$ if it is type B.*

When $P_{GM}/P_{BM} > P_{GH}/P_{BH}$ and I is not large, in addition to the separating equilibria described above, there also exist pooling equilibria in which the firm does not incur any adverse-selection costs. In these pooling equilibria, regardless of its type, the firm chooses coverage K^D. Lemma 4 demonstrates that, at this level of coverage, the adverse-selection costs resulting from the incorrect assessment of risk on one contract are exactly offset by the gains from the incorrect assessment of the other contract's risk. Thus, regardless of its type, the firm's adverse-selection cost is minimized by the choice of coverage K^D. This argument is formalized in the following proposition.

Proposition 4: *If $P_{GM}/P_{BM} > P_{GH}/P_{BH}$, for intermediate values of I, $K^D \in (0, M)$, and there exist pooling equilibria in which, regardless of its type, the firm chooses coverage K^D.*

Taken together, the last two results demonstrate that, when I is not large and $P_{GM}/P_{BM} > P_{GH}/P_{BH}$, there exist equilibria that support the choice of coverage levels ranging from [0, M]. Further, regardless of its type, the firm incurs no adverse selection costs. This contrasts directly with our earlier results, which show that if $P_{GM} < P_{BM}$, M is relatively small, or I is relatively large, the firm always chooses extreme coverage levels—i.e., it chooses $K \in \{0, M\}$. Further, it always incurs adverse-selection costs if it is type G. Collectively, propositions 1–4 show that insurance is an effective tool in mitigating adverse-selection problems and is most effective in doing so when capital requirements are not large and higher operating revenues are associated with higher insurable losses.

5. Conclusion

In this paper we examined the insurance decision of a firm with private information regarding its future cash flows and insurable losses. We show that the firm will choose not to purchase insurance if higher operating revenues are accompanied with a *lower* insurance risk. In contrast, if higher operating revenues are accompanied by a *higher* insurance risk, in general, the firm will purchase insurance. In fact, if insurable losses are relatively small, the firm will always choose to purchase full coverage. We also establish that if there exists considerable uncertainty regarding the firm's insurance risk, the insurance decision may act as a signal of firm quality, with higher coverage levels signaling higher expected cash flows. In addition to providing conditions under which firms will demand insurance, because the decision to *not* purchase insurance can be interpreted as a decision to self-insure, our analysis also provides insights into the reasons why firms may choose to self-insure.

These results were derived in the context of a stylized model, in which the distributions of the firm's operating revenues and insurable losses have two point distributions and investors are risk-neutral. However, the tradeoffs on which they are based will be present even in more general models. For example, consider the case where the firm's operating revenues and insurable losses have arbitrary distributions. Even in this context, as long as the incorrect assessment of the firm cash-flow distributions results in the firm incurring adverse-selection costs on its financing contract and benefiting from adverse selection on its insurance contract, it will choose to purchase insurance to minimize its losses. In contrast, if it incurs adverse-selection costs on both its bond contract and its insurance contract, the firm would choose not to purchase insurance.

Similar results can also be obtained in a model in which agents are risk averse and the risk-free rate is positive. Given an appropriate state space, the manager's private information can be interpreted as information regarding the likelihood of the occurrence of a particular set of states. The effect of risk aversion on valuation can be incorporated into the model by interpreting the probability distributions as being induced by a martingale measure over the state-space rather than a subjective probability distribution. Finally, discount rates can be incorporated into the analysis by interpreting the cash flows as discounted cash flows.

Appendix

Proof of lemma 1: First we show $H - M > F^*(\delta, K) > L$ for all $\delta \in [0, 1]$ and $K \in [0, M]$. Then we obtain the functional form of $F^*(\delta, K)$.

Note that

$$W(\delta, K, F^*(\delta, K)) = \delta E\{Min\{F^*(\delta, K), \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{F^*(\delta, K), \tilde{X}_B(K)\}\}.$$

Next note that, if $F^*(\delta, K) \leq L$, then $\text{Min}\{F^*(\delta, K), x\} \leq \text{Min}\{L, x\}$ for all x. Further, because $L \geq \text{Min}\{L, x\}$ for all x,

$$\delta E\{Min\{F^*(\delta, K), \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{F^*(\delta, K), \tilde{X}_B(K)\}\} \leq \delta E\{Min\{L, \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{L, \tilde{X}_B(K)\}\} \leq L.$$

Because $I > L$ and $\Pi^*(\delta, K) \geq 0$ for all $\delta \in [0, 1]$ and $K \in [0, M]$, this implies that $W(\delta, K, F^*(\delta, K)) \leq L < I \leq I + \Pi^*(\delta, K)$. Thus, in equilibrium, it cannot be the case that $F^*(\delta, K) \leq L$.

Now note that, if $F^*(\delta, K) \geq H - M$, then $\text{Min}\{F^*(\delta, K), x\} \geq \text{Min}\{H - M, x\}$ for all x. It follows that

$$\delta E\{Min\{F^*(\delta, K, \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{F^*(\delta, K), \tilde{X}_B(K)\}\} \geq \delta E\{Min\{H - M, \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{H - M, \tilde{X}_B(K)\}\}.$$

Further, because $H - M > L$ and $K \in \{0, M]$,

$$E\{Min\{H - M, \tilde{X}_t(K)\}\} = (H - M)P_{tH} + L(1 - P_{tH})(1 - P_{tM}) + (L - M - K)(1 - P_{tH})P_{tM} = (H - M)P_{tH} + L(1 - P_{tH}) - M(1 - P_{tH})P_{tM} + K(1 - P_{tH})P_{tM} \geq (H - M)P_{tH} + L(1 - P_{tH}) - M(1 - P_{tH})P_{tM}, \quad \text{(A.1)}$$

for t $\in$ {B, G}. But (FI) implies that

$$(H - M)P_{tH} + L(1 - P_{tH}) - M(1 - P_{tH})P_{tM} > I + \max_t P_{tM} M.$$

Because $\Pi^*(\delta, K) \leq \max_t P_{tM} M$ for all $K \in [0, M]$ and $\delta \in [0, 1]$, (A1) and (FI) together imply that $E\{\text{Min}\{H - M, \tilde{X}_t(K)\}\} > I + \Pi^*(\delta, K)$ for $t \in \{B, G\}$, $\delta \in [0, 1]$ and $K \in [0, M]$. But this implies that, if $F^*(\delta, K) \geq H - M$,

$$W(\delta, K, F^*(\delta, K)) = \delta E\{Min\{F^*(\delta, K), \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{F^*(\delta, K), \tilde{X}_B(K)\}\} \geq \delta E\{Min\{H - M, \tilde{X}_G(K)\}\} + (1 - \delta)E\{Min\{H - M, \tilde{X}_B(K)\}\} > I + \Pi^*(\delta, K),$$

for all $\delta \in [0, 1]$ and $K \in [0, M]$. It follows that, in equilibrium, it cannot be the case that, $F^*(\delta, K) \geq H - M$.

Because $L < F^*(\delta, K) < H - M$ for all $K \in [0, M]$ and $\delta \in [0, 1]$, it follows that $F^*(\delta, K) > L - M + K$, and $F^*(\delta, K) < H - M + K$. Thus,

$$W(\delta, K, F^*(\delta, K)) = P_{HO}(\delta)F^*(\delta, K) + P_{HM}(\delta)F^*(\delta, K) + P_{LO}(\delta)L + P_{LM}(\delta)(L - M + K).$$

Setting $W(\delta, K, F^*(\delta, K))$ equal to $I + \Pi^*(\delta, K)$ and solving for $F^*(\delta, K)$, we obtain

$$F^*(\delta, K) = \frac{I + \Pi^*(\delta, K) - P_{LO}(\delta)L - P_{LM}(\delta)(L - M + K)}{P_{HO}(\delta) + P_{HM}(\delta)}$$

$$= \frac{I + P_M(\delta)K - (1 - P_H(\delta))L + P_{LM}(\delta)(M - K)}{P_H(\delta)}$$

$$= -\frac{-I - P_M(\delta)K + (1 - P_H(\delta))L + P_H(\delta)H - P_H(\delta H - (P_M(\delta) - P_{HM}(\delta))(M - K)}{P_H(\delta)}$$

$$= -\frac{-I + V(\delta, 0) - P_H(\delta)H + P_{HM}(\delta)(M - K)}{P_H(\delta)}$$

$$= H - \frac{V(\delta, 0) - I + P_{HM}(\delta)(M - K)}{P_H(\delta)}. \blacksquare$$

Proof of lemma 2: To establish this claim, we only need to show that $U(t, \delta, K)$ is strictly increasing in δ. By differentiating $U(t, \delta, K)$ with respect to δ, it can be seen that

$$\frac{\partial U(t, \delta, K)}{\partial \delta} = P_{tH}P_H(\delta)\left[\frac{(V_G(0) - V_B(0) + (P_{GH}P_{GM} - P_{BH}P_{BM})(M - K))}{[P_H(\delta)]^2}\right] - P_{tH}\left[\frac{(V(\delta, 0) - I + P_{HM}(\delta)(M - K))(P_{GH} - P_{BH})}{[P_H(\delta)]^2}\right]. \quad (A.2)$$

It follows that $\partial U(t, \delta, K)/\partial\delta \geq (<)\ 0$ if and only if

$$P_H(\delta)[V_G(0) - V_B(0) + (P_{GH}P_{GM} - P_{BH}P_{BM})(M - K)] - (V(\delta, 0) - I + P_{HM}(\delta)(M - K))(P_{GH} - P_{BH}) \geq (<)\ 0. \quad (A.3)$$

The left side of (A.3) reduces to

$$(I - L)(P_{GH} - P_{BH}) + M(P_{GH}P_{BM} - P_{BH}P_{GM}) - (M - K)P_{GH}(P_{BH}P_{BM} - P_{BH}P_{GM}).$$

Because $I > L$, $M > (M - K)P_{GH}$, $P_{BM} > P_{GM}$, and $P_{GH} > P_{BH}$, it follows that $P_{GH}P_{BM} - P_{BH}P_{GM} > P_{BH}P_{BM} - P_{BH}P_{GM}$ and the above expression is strictly positive. Thus, (A.2) is also strictly positive, implying that $\partial U(t, \delta, K)/\delta > 0$ for all $K \in [0, M]$ and $\delta \in [0, 1]$.■

Proof of lemma 3: (i) By differentiating $U(t, \delta, K)$ with respect to K, it can be seen that

$$\frac{\partial U(t, \delta, K)}{\partial K} = P_{tH}\left[P_{tM} - \frac{P_{HM}(\delta)}{P_H(\delta)}\right]. \quad (A.4)$$

It follows that $\partial U(G, \delta, K)/\partial K > (<)\ 0$ if and only if $P_{GM}P_H(\delta) - P_{HM}(\delta) > (<)\ 0$. The left side of this expression reduces to $(1 - \delta)P_{BH}(P_{GM} - P_{BM})$. Thus, for

$\delta \in [0, 1)$, it must be the case that $\partial U(G, \delta, K)/\partial K > (<) 0$ if and only if $P_{GM} > (<) P_{BM}$.

(ii) Note that from (A.4) it follows that $\partial U(B, \delta, K)/\partial K > (<) 0$ if and only if $P_{BM}P_H(\delta) - P_{HM}(\delta) > (<) 0$. The left hand side of this expression reduces to $\delta P_{GH}(P_{BM} - P_{GM})$. Thus, for $\delta \in (0, 1]$, it must be the case that $\partial U(B, \delta, K)/\partial K > (<) 0$ if and only if $P_{BM} > (<) P_{GM}$.■

Proof of lemma 4: Note that from (A.3) it follows that $\partial U(t, \delta, K)/\partial\delta \geq (<) 0$ if and only if

$$P_H(\delta)[V_G(0) - V_B(0) + (P_{GH}P_{GM} - P_{BH}P_{BM})(M - K)] \\ - [(V(\delta, 0) - I + P_{HM}(\delta)(M - K))(P_{GH} - P_{BH})] \geq (<) 0. \tag{A.5}$$

On simplification, the left side of (A.5) reduces to

$$P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I] + MP_{BH}P_{GH}(P_{GM} - P_{BM}) \\ - KP_{BH}P_{GH}(P_{GM} - P_{BM}). \tag{A.6}$$

On setting (A.6) equal to 0 and solving for K, one obtains K^D. Thus, $\partial U(t, \delta, K^D)/\partial\delta = 0$. Now note that because $\partial U(t, \delta, K^D)/\partial\delta = 0$ and $P_{GM} > P_{BM}$, (A.6) is negative (positive) for $K > (<) K^D$. It follows that $\partial U(t, \delta, K)/\partial\delta < (>) 0$ for $K > (<) K^D$ and $\delta \in [0, 1]$.■

Lemma A1: *If $U(t, \delta, K)$ is strictly increasing in δ for $t \in \{B, G\}$ and all $K \in [0, M]$, then (i) the pooling outcome $K^* \in [0, M]$ is supported by an Universally Divine Perfect Bayesian equilibrium if and only if $U(G, \phi, K^*) \geq U(G, \phi, K)$ for all $K \in [0, M]$, and (ii) under the same restrictions there exist no separating equilibria.*

Proof of lemma A1: First we prove (i). Then we prove (ii). To prove (i) we first establish sufficiency. Then we prove necessity.

Sufficiency. To prove sufficiency, we first establish the off-equilibrium path beliefs consistent with Universal Divinity. Then we demonstrate that, given these beliefs, the firm will have no incentive to defect.

Suppose that $U(G, \phi, K^*) \geq U(G, \phi, K)$ for all $K \in [0, M]$. Because $U(G, \delta, K)$ is increasing in δ for all $K \in [0, M]$, it must be the case that $U(G, \phi, K^*) \geq U(G, \phi, K) > U(G, \delta, K)$ for $K \neq K^*$ and $\delta \in [0, \phi)$. Thus, $OEB_G(K) \subseteq [\phi, 1]$ for all K. Now note that

$$\phi\, U(G, \phi, K) + (1 - \phi)\, U(B, \phi, K) = V(\phi, 0) - I. \tag{A.7}$$

It follows that, if $U(G, \phi, K^*) \geq U(G, \phi, K)$, then $U(B, \phi, K^*) \leq U(B, \phi, K)$. Further, because $U(B, \delta, K)$ is increasing in δ, it must be the case that $U(B, \delta,$

$K) \geq U(B, \phi, K) \geq U(B, \phi, K^*)$ for $K \neq K^*$ and $\delta \in [\phi, 1]$. Thus, $OEB_B(K) \supseteq [\phi, 1]$. Because $OEB_B(K) \supseteq [\phi, 1]) \supseteq OEB_G(K)$, it is consistent with Universal Divinity that the market's response to $K \neq K^*$, be conditioned on the belief that the firm is type G with probability $\delta = 0$.

If the market responds with the belief $\delta = 0$ to all off-equilibrium path actions, because $U(G, \delta, K)$ is strictly increasing in δ, it must be the case that $U(G, \phi, K^*) \geq U(G, \phi, K) > U(G, 0, K)$ for all $K \neq K^*$. Further, because $U(B, \delta, K)$ is strictly increasing in δ and $U(B, 0, K^*) = V_B(0) - I = U(B, 0, K)$, it must be the case that $U(B, \phi, K^*) > U(B, 0, K^*) = U(B, 0, K)$ for all $K \neq K^*$. This establishes sufficiency.

Necessity. We establish necessity by means of a contradiction. Suppose that an equilibrium supports the purchase of coverage $K^* \in [0, M]$, and there exists K_O such that

$$U(G, \phi, K^*) < U(G, \phi, K_O). \tag{A.8}$$

Because $U(G, \delta, K)$ is increasing in δ for all $K \in [0, M]$, it must be the case that $U(G, \delta, K_O) \geq U(G, \phi, K_O) > U(G, \phi, K^*)$ for all $\delta \geq \phi$. Thus, $[\phi, 1] \subset OEB_G(K_O)$. From (A.7) it follows that because $U(G, \phi, K^*) < U(G, \phi, K_O)$, it must be the case that $U(B, \phi, K^*) > U(B, \phi, K_O)$. Because $U(B, \delta, K)$ is increasing in δ, it must be the case that $U(B, \delta, K_O) \leq U(B, \phi, K_O) < U(B, \phi, K^*)$ for $\delta \leq \phi$. Thus, $OEB_B(K_O) \subset [\phi, 1]$. Because $OEB_B(K_O) \subset [\phi, 1] \subset OEB_G(K_O)$, if the market observes the firm choosing K_O, Universal Divinity restricts its beliefs to $\delta = 1$. Now note that because $U(G, \delta, K)$ is increasing in δ for all $K \in [0, M]$, it must be the case that $U(G, \phi, K^*) < U(G, 1, K^*) = V_G(0) - I = U(G, 1, K_O)$. This implies that the pooling outcome K^* cannot be supported by an Universally Divine PBE, establishing necessity.

We use a proof by contradiction to demonstrate that there exist no separating equilibria. First suppose that there exists a separating equilibrium that in which the firm chooses K^{t*} if it is type t. Because, $U(B, 0, K^{B*}) = V_B(0) - I = U(B, 0, K^{G*})$, and $U(B, \delta, K)$ is increasing in δ for all $K \in [0, M]$, it must be the case that $U(B, 0, K^{B*}) = U(B, 0, K^{G*}) < U(B, 1, K^{G*})$. Thus, the firm would always defect to K^{G*} if it is type B. This contradiction establishes that cannot exist any separating equilibria when $P_{GM} < P_{BM}$ and completes the proof.■

Proof of proposition 1: From lemma 1 it follows that $U(G, \delta, K)$ is increasing in δ for all $K \in [0, M]$. Thus, it follows, from lemma A1, that there exist no separating equilibria. Further, to establish the existence of pooling equilibria that support the pooling outcome $K^* = 0$, we only need to establish that $U(G, \phi, 0) \geq U(G, \phi, K)$ for all $K \neq 0$. Because $P_{GM} < P_{BM}$, from lemma 2 it follows that $U(G, \phi, K)$ is strictly decreasing in K. Thus, $U(G, \phi, 0) > U(G, \phi, K)$ for all $K \in (0, M]$. This establishes that there exist pooling equilibria in which the firm chooses coverage $K^* = 0$. Further, because $U(G, \phi, K) < U(G, \phi, 0)$ for all $K \in [0, M]$, there cannot exist pooling equilibria in which the firm chooses $K \in (0, M]$.■

Proof of proposition 2: First we show that for sufficiently small M or sufficiently large I, $U(t, \delta, K)$ is increasing in δ for all $K \in [0, M]$. Then, using lemma A1, we establish the existence and uniqueness of pooling equilibria in which the firm chooses $K^* = M$ and establish that there cannot exist any separating equilibria or any pooling equilibria characterized by $K \in [0, M)$.

Note that, from (A.6) it follows that $U(t, \delta, K)$ is strictly increasing in δ if and only if

$$P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I] + (M - K)P_{BH}P_{GH}(P_{GM} - P_{BM}) > 0.$$

Because $K \in [0, M]$ and $P_{GM} > P_{BM}$,

$$P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I] + (M - K)P_{BH}P_{GH}(P_{GM} - P_{BM}) \geq P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I].$$

Thus, $P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I] > 0$ is sufficient for $U(t, \delta, K)$ to be strictly increasing in δ for all $K \in [0, M]$. Now note that $P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I]$ reduces to $(P_{GH} - P_{BH})[I - L] - M(P_{GH}P_{BM} - P_{BH}P_{GM})$. Because $P_{GH} > P_{BH}$ and $I > L$, the above expression is positive for $M = 0$. Further, because it is continuous in M, there exists $M^D > 0$ such that $(P_{GH} - P_{BH})[I - L] - M(P_{GH}P_{BM} - P_{BH}P_{GM})$ is strictly positive for all $M \in [0, M^D)$. Thus, $U(t, \delta, K)$ is increasing in δ for all $K \in [0, M]$ if $M \in [0, M^D)$. Next note that, if I is relatively large, $(P_{GH} - P_{BH})[I - L] - M(P_{GH}P_{BM} - P_{BH}P_{GM}) > 0$, which implies that $P_{BH}[V_G(0) - I] - P_{GH}[V_B(0) - I] > 0$. Thus, for sufficently large I, $U(t, \delta, K)$ is increasing in δ for all $K \in [0, M]$.

Let $0 < M < M^D$ or I be sufficiently large. Because $U(t, \delta, K)$ is increasing in δ for all $K \in [0, M]$, from lemma A1, it follows that there exist no separating equilibria. Further, there exist pooling equilibria characterized by $K^* = M$ if and only if $U(G, \phi, M) \geq U(G, \phi, K)$ for all $K \in [0, M]$. Because $P_{GM} > P_{BM}$, from lemma 2, it follows that $U(G, \phi, K)$ is strictly increasing in K. Thus, $U(G, \phi, M) > U(G, \phi, K)$ for all $K \in [0, M)$, implying that there exist pooling equilibria in which $K^* = M$. Further, because $U(G, \phi, K) < U(G, \phi, M)$ for all $K \in [0, M)$ there cannot exist any pooling equilibria in which $K^* \in [0, M)$.■

Lemma A2: *In a separating equilibrium, Universal Divinity restricts the market's response to an off-equilibrium path action K, to* $F^*(1, K)(F^*(0, K))$ *if* $U(t, \delta, K_O)$ *is strictly decreasing (increasing) in* δ*, and restricts the market's response to an off-equilibrium path action to* $F^*(\delta, K)$ *where* $\delta \in [0, 1]$ *if* $U(t, \delta, K)$ *is constant in* δ.

Proof of lemma A2: To establish the desired result, we only have to establish that the market's beliefs on which the responses specified in the lemma are conditioned satisfy the restrictions imposed by Universal Divinity.

Let K^{t*}, where $t = B, G$, be the equilibrium coverage level chosen by the firm if it is type t, and let K_O be an off-equilibrium path action—i.e., $K_O \neq K^{t*}$ for $t = B$ or G. Note that $U(G, 1, K_O) = V_G(0) - I = U(G, 1, K^{G*})$. Further, if $U(G, \delta, K_O)$ is strictly increasing (decreasing) in δ, then $U(G, 1, K^{G*}) = U(G, 1, K_O) > (<) U(G, \delta, K_O)$ for all $\delta \in [0, 1)$. This implies that $OEB_G(K_O) = \{1\}$ ($[0, 1]$) if $U(G, \delta, K_O)$ is strictly increasing (decreasing) in δ. Finally, if $U(G, \delta, K_O)$ is constant in δ, then $U(G, 1, K^{G*}) = U(G, 1, K_O) = U(G, \delta, K_O)$ for all $\delta \in [0, 1]$. Thus, in this case, $OEB_G(K_O) = [0, 1]$. Now note that $U(B, 0, K_O) = V_B(0) - I = U(B, 0, K^{B*})$. Further, if $U(B, \delta, K_O)$ is strictly decreasing (increasing) in δ, then $U(B, 0, K^{B*}) = U(B, 0, K_O) > (<) U(B, \delta, K_O)$ for all $\delta \in [0, 1]$. This implies that $OEB_B(K_O) = \{0\}$ ($[0, 1]$) if $U(B, \delta, K_O)$ is strictly decreasing (increasing) in δ. Finally, if $U(B, \delta, K_O)$ is constant in δ, then $U(B, 0, K^{B*}) = U(B, 0, K_O) = U(B, \delta, K_O)$ for all $\delta \in [0, 1]$. Thus, in this case, $OEB_B(K_O) = [0, 1]$. It follows that if $U(t, \delta, K_O)$ is strictly increasing (decreasing) in δ, then $OEB_G(K_O) \subset (\supset) OEB_B(K_O)$, and Universal Divinity restricts the market to assess probability $\delta = 0$ (1) to the firm being type G if it defects to K_O. Further, if $U(t, \delta, K_O)$ is constant in δ, then $OEB_B(K_O) = OEB_G(K_O)$, and the market's belief is unrestricted by Universal Divinity if the firm defects to K_O. This establishes the desired result.■

Proof of proposition 3: First we show that, if $P_{GM}/P_{BM} > P_{GH}/P_{BH}$ and I is not large, $K^D \in (0, M)$. Then we prove the remainder of the claim.

First note that, from (EK^D) it follows that, $K^D > 0$ if and only if $M P_{GH}P_{BH}(P_{GM} - P_{BM}) > P_{GH}(V_B(0) - I) - P_{BH}(V_G(0) - I)$. The right side of this expression reduces to $M (P_{BH}P_{GM} - P_{GH}P_{BM}) - (I - L)(P_{GH} - P_{BH})$. Thus, it follows that $K^D > 0$ if and only if $I - L > M[P_{BH}P_{GM}(1 - P_{GH}) - P_{GH}P_{BM}(1 - P_{BH})]/(P_{GH} - P_{BH})$. Next note that from ($EK^D$) it follows that because $P_{GM} > P_{BM}$, $K^D < M$ if and only if $P_{GH}(V_B(0) - I) - P_{BH}(V_G(0) - I) > 0$. This inequality reduces to $M(P_{BH}P_{GM} - P_{GH}P_{BM}) - (I - L)(P_{GH} - P_{BH}) > 0$, or equivalently, $I - L < M(P_{BH}P_{GM} - P_{GH}P_{BM})/(P_{GH} - P_{BH})$. Let

$$\Delta \equiv Max\{P_{BH}P_{GM}(1 - P_{GH}) - P_{GH}P_{BM}(1 - P_{BH}), 0\}M/(P_H - P_{BH}).$$

It follows that $K^D \in (0, M)$ if and only if $I - L \in (\Delta, (P_{BH}P_{GM} - P_{GH}P_{BM})M/(P_{GH} - P_{BH}))$. Because $[P_{BH}P_{GM}(1 - P_{GH}) - P_{GH}P_{BM}(1 - P_{BH})] = P_{BH}P_{GM} - P_{GH}P_{BM} - P_{GH}P_{BH}(P_{GM} - P_{BM}) < P_{BH}P_{GM} - P_{GH}P_{BM}$, and $P_{BH}P_{GM} - P_{GH}P_{BM} = P_{BH}P_{BM}(P_{GM}/P_{BM} - P_{GH}/P_{BH}) > 0$, the interval $(\Delta, P_{BH}P_{GM} - P_{GH}P_{BM})M/(P_{GH} - P_{BH}))$ is nonempty and all $z \in (\Delta, (P_{BH}P_{GM} - P_{GH}P_{BM})M/(P_{GH} - P_{BH}))$ are strictly positive. This shows that if I is not large, $K^D \in (0, M)$.

Let the market assess a probability of 1 (0) to the defector being type G if it defects to $K \in (K^D, M]$ ($[0, K^D]$), where $K \neq K^{t*}$, $t = B, G$. Because, (from lemma 4) $U(t, \delta, K)$ is constant in δ if $K = K^D$, $U(t, \delta, K)$ is strictly increasing in δ if $K \in [0, K^D)$, and $U(t, \delta, K)$ is strictly decreasing in δ if $K \in (K^D, M]$, it follows (from lemma A2) that these beliefs are consistent with Universal Divinity. Now note that $U(G, 1, K^{G*}) = V_G(0) - I = U(G, 1, K)$ for all $K \in [K^D, M]$. Further, because if $U(t, \delta, K)$ is increasing in δ for all $K \in [0, K^D]$, $U(G, 1, K^{G*}) = U(G, 1, K) \geq U(G, 0, K)$ for all $K \in [0, K^D]$. Also note that $U(B, 0, K^{B*}) = V_B(0) - I = U(B, 0, K)$ for $K \in [0, K^D]$. Further, because $U(t, \delta, K)$ is decreasing in δ for $K \in (K^D, M]$, $U(B, 0, K^{B*}) = U(B, 0, K) \geq U(B, 1, K)$ for $K \in (K^D, M]$. This completes the proof.■

Proof of proposition 4: First note that from the proof of proposition 3 it follows that if $P_{GM}/P_{BM} > P_{GH}/P_{BH}$ and I is not large, $K^D \in (0, M)$. Let $K^* = K^D$ be the equilibrium coverage level chosen by the firm, and let K_O be an off-equilibrium path action—i.e., $K_O \neq K^D$. To establish this claim we first show that Universal Divinity restricts the market to assess probability 1 (0) to the firm being type G if it defects to K_O where $K_O \in (K^D, M]([0, K^D))$. Then we show that, given these restrictions on the market's beliefs, the firm will choose coverage K^D regardless of its type.

Let $K_O \neq K^* = K^D$ be an off-equilibrium path action. Note that $U(G, 1, K_O) = V_G(0) - I = U(G, 1, K^{G*})$. Further, if $U(G, \delta, K_O)$ is strictly increasing (decreasing) in δ, then $U(G, 1, K^{G*}) = U(G, 1, K_O) > (<) U(G, \delta, K_O)$ for all $\delta \in [0, 1)$. This implies that $OEB_G(K_O) = \{1\}$ ([0, 1]) if $U(G, \delta, K_O)$ is strictly increasing (decreasing) in δ. Now note that $U(B, 0, K_O) = V_B(0) - I = U(B, 0, K^{B*})$. Further, if $U(B, \delta, K_O)$ is strictly decreasing (increasing) in δ, then $U(B, 0, K^{B*}) = U(B, 0, K_O) > (<) U(B, \delta, K_O)$ for all $\delta \in (0, 1]$. This implies that $OEB_B(K_O) = \{0\}$ ([0, 1]) if $U(B, \delta, K_O)$ is strictly decreasing (increasing) in δ. From the above arguments it follows that if $U(t, \delta, K_O)$ is strictly increasing (decreasing) in δ, then $OEB_G(K_O) \subset (\supset) OEB_B(K_O)$, and Universal Divinity restricts the market to assess probability $\delta = 0$ (1) to the firm being type G if it defects to K_O. Because (from lemma 4) $U(t, \delta, K)$ is strictly increasing (decreasing) in δ for $K \in [0, K^D)((K^D, M])$, it follows that Universal Divinity restricts the market to assess probability 0 (1) to the firm being type G if it chooses $K \in [0, K^D)((K^D, M])$. Now note that $U(G, 1, K^*) = V_G(0) - I = U(G, 1, K)$ for all $K \in (K^D, M]$. Further, because if $U(t, \delta, K)$ is strictly increasing in δ for all $K \in [0, K^D)$, $U(G, 1, K^*) = U(G, 1, K) > U(G, 0, K)$ for all $K \in [0, K^D)$. Also note that $U(B, 0, K^*) = V_B(0) - I = U(B, 0, K)$ for $K \in [0, K^D)$. Further, because $U(t, \delta, K)$ is strictly decreasing in δ for $K \in (K^D, M]$, $U(B, 0, K^{B*}) = U(B, 0, K) > U(B, 1, K)$ for $K \in (K^D, M]$. This completes our proof.■

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Notes

1. These figures, however, do not include the resources spent on self-insurance and risk-management programs. Thus, the amount spent on insurance activities is even greater than that reported above.
2. There has, however, been attention devoted to the analysis of adverse selection on the design of insurance contracts as well as the demand for insurance by agents other than corporations. For a recent survey of this literature see Dionne and Doherty [1992].
3. We restrict ourselves to examining the case where the firm issues debt to finance its project and insurance purchase since it can be shown that despite the increased complexity of the analysis similar results obtain regardless of the nature of the financial contract the firm uses to finance its investment and insurance coverage.
4. This assumption is consistent with Bertrand competition. See Kreps [1984] and Noe [1988] for similar assumptions.
5. See Myers and Majluf [1984] and Noe [1988] for a similar specification of the manager's objective function.
6. This assumption is made to simplify the analysis. In the absence of this restriction, the equilibria we characterize in the following sections continue to obtain. However, there may also exist equilibria other than the ones characterized in the paper.
7. The case where mispricing arises solely because of uncertainty regarding the firm's insurable losses can be incorporated in our analysis. However, doing so complicates the analysis and obfuscates some of the insights into the firm's insurance decision.
8. There is no loss of generality in focusing on outcomes supported by PBEs as for signaling games such as ours, every PBE is also a Nash sequential equilibrium (see, e.g., Fudenberg and Tirole [1991], ch. 8).
9. Henceforth, the term *market's beliefs* will be used to denote the common beliefs of insurers and investors.
10. We focus our attention on Universally Divine PBE as these highlight the intuitive tradeoff involved in the investment and financing decisions of our game. Other PBE can be easily constructed once these tradeoffs are explained.

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