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Excursions of a Markov Process Induced by Continuous Additive Functionals

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Summary. Let $\hat{L}$ be a continuous additive functional with support C of a Hunt process $X=\{X_t; t\geqq 0\}$. Let $S=\{S_t; t\geqq 0\}$ be the inverse of $\hat{L}$ and put $Y_t = X_{S_t}$. For each time of discontinuity u of S, let Z_u be the corresponding excursion of X outside of C. The conditional structure of the excursion process $\{Z_u; u\geqq 0\}$ given the paths of $Y=\{Y_t; t\geqq 0\}$ is studied. It is shown that conditionally, given Y, the excursion process is a Poisson random measure.

1. Introduction

Consider a Hunt process $X=(\Omega, \mathcal{M}, \hat{\mathcal{M}}_t, X_t, \hat{\theta}_t, P^x)$ with state space $(E, \mathscr{E})$ where E is a Borel subset of a compact space and Δ is a point not in E. Let $\hat{L} = \{\hat{L}_t; t\geqq 0\}$ be a continuous additive functional (CAF) of X so that $t\to\hat{L}_t(\omega)$ is continuous and nondecreasing for all $\omega\in\Omega$. Let C denote the support of $\hat{L}$ and $S = \{S_t; t\geqq 0\}$ denote the inverse of $\hat{L}$. If we put $Y_t = X_{S_t}$, then $Y=\{Y_t; t\geqq 0\}$ is a strong Markov process and is roughly speaking the restriction of X to the set C.

Let $J(\omega)$ be the closure of the set $\{t: X_t(\omega)\in C\}$. If $I(\omega)$ is a contiguous interval of $J(\omega)=(u, u+h)$, the mapping

$$Z(s,\omega)=\begin{cases} X_{s+u}(\omega) & \text{if } 0\leqq s<h,\\ \Delta & \text{if } s\geqq h\end{cases}$$

is called the excursion of X corresponding to $I(\omega)$. Each contiguous interval corresponds to a time of discontinuity of S. If t is a jump time of $S(\omega)$ we will write $Z_t(\omega)$ for the excursion of X corresponding to the interval $I(\omega) = (S_{t-}(\omega), S_t(\omega))$; that is, $Z_t(s,\omega)=X_{S_{t-}+s}(\omega)$ for $0\leqq s<S_t(\omega)-S_{t-}(\omega)$ and $Z_t(s,\omega)=\Delta$ for $s\geqq S_t(\omega)-S_{t-}(\omega)$. If t is not a jump time of S, then we put $Z_t(\omega) = [\Delta]$ where $[\Delta]$ is the constant mapping from $\mathbb{R}_+$ into $E\cup\{\Delta\}\setminus C$ having the value Δ; $((E\cup\{\Delta\})\setminus C=\{x\in E\cup\{\Delta\}: x\notin C\})$.

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The excursion process $Z=\{Z_t; t\geqq 0\}$ takes values in W, the collection of right continuous mappings from $[0,\infty)$ into $E\cup\{\Delta\}$ that are absorbed at Δ. We will denote the σ-algebra on W induced by the coordinate mappings by $\mathscr{W}$.

Let μ be the random counting measure on $[0,\infty)\times W$ induced by the times of discontinuity of S and the corresponding excursions. μ is called the excursion counting measure induced by $\hat{L}$. If $C=\{a\}$, then μ is related to the excursion point process defined by Itô [9]. He showed that μ is a Poisson random measure in this case. For arbitrary C, μ is no longer Poisson because the excursions are no longer independent.

The structure of the process of excursions from a general Borel set B not necessarily the support of a CAF has been studied recently by Getoor and Sharpe [6, 7], Gzyl [8], and Maisonneuve [11]. In [11] Maisonneuve also considers the excursion process as defined here and obtains Lévy system type results for it. Our results differ from these in that we will study the conditional structure of the excursion process given the paths of Y; more precisely, given $\mathscr{H}$ which is the completion of $\sigma(Y_t; t\geqq 0)$ with respect to the family of measures $\{P^\nu: \nu$ is a finite measure$\}$. ($\sigma(\cdot)$ denotes the σ-algebra generated by $(\cdot)$.) The results have applications to the boundary problem of Markov processes; in particular to the problem of determining the class of all possible Hunt processes whose stopped process at the hitting time of a fixed set is a given one. They also have applications to the study of a Hunt process in the neighborhood of a fixed set.

In the next section we state the problem more precisely and show that there exists a regular version P_ω of $P(\cdot\,|\,\mathscr{H})$ on $\sigma(Y_t, S_t, Z_t; t\geqq 0)$. Further, the excursions are conditionally independent given $\mathscr{H}$ and, if T is a time of discontinuity of S, then the conditional distribution of the excursion Z_T given $\mathscr{H}$ depends only on Y_{T-} and Y_T.

In Section 3 we study the conditional structure of the excursion counting measure μ given $\mathscr{H}$. We show that μ is an additive random measure with respect to P_ω; that is, $\mu(A_1),\ldots,\mu(A_n)$ are conditionally independent random variables given $\mathscr{H}$ whenever $A_1,\ldots,A_n$ are disjoint measurable subsets of $[0,\infty)\times W$. Further, $\mu=\mu^f+\mu^d$ where μ^f and μ^d are independent additive random measures with respect to P_ω. In addition, μ^d is a Poisson random measure.

We then obtain results concerning μ in the case in which, roughly, excursions start and end at only countably many points of C. As an example of the type of results we are interested in, suppose the support set of $\hat{L}$ consists of two points a and b. In this case, with respect to P_ω, μ^f is the additive random counting measure induced by those excursions $\{Z_{T_j}\}$ for which $Y_{T_j-}\neq Y_{T_j}$ and μ^d is the Poisson random measure induced by excursions $\{Z_{T_j}\}$ for which $Y_{T_j-}=Y_{T_j}$. Further, $\mu^d=\mu_a+\mu_b$ where μ_a and μ_b are independent Poisson random measures with respect to P_ω. For $i=a, b$, the mean measure m_i^ω of μ_i is such that $m_i^\omega([0,t]\times B)=L_t^i(\omega)\,N^i(B)$ for $B\in\mathscr{W}$ and $t\geqq 0$ where $L^i=\{L_t^i; t\geqq 0\}$ is the local time of Y at the point i and N^i is a possibly σ-finite measure on $(W,\mathscr{W})$.

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2. The Conditional Distribution of the Excursion Process

In this section we will show the existence of a conditional distribution for the excursion process given the paths of Y. We will first introduce some notation.

Let $\mathbb{R}_+=[0,\infty)$, $\overline{\mathbb{R}}_+=[0,+\infty]$ and $\mathscr{R}_+$ (respectively, $\overline{\mathscr{R}}_+$), be the Borel subsets of $\mathbb{R}_+$ (respectively $\overline{\mathbb{R}}_+$). Let $(F,\mathscr{F})$ be a measurable space. If f is a real valued measurable function on $(F,\mathscr{F})$ we will write $f\in\mathscr{F}$. If further, f is bounded (respectively positive), we will write $f\in b\mathscr{F}$ (respectively $f\in p\mathscr{F}$). For each $x\in F$, let ε_x denote the Dirac measure that puts its unit mass at x. By a transition kernel N from $(F,\mathscr{F})$ into a measurable space $(G,\mathscr{G})$ is meant a mapping $N: F\times\mathscr{G}\to\overline{\mathbb{R}}_+$ such that the mapping $A\to N(x,A)$ is a σ-finite measure on $\mathscr{G}$ for each fixed $x\in F$ and that $x\to N(x,A)$ is in $p\mathscr{F}$ for each fixed $A\in\mathscr{G}$.

Let E be a Borel subset of a compact space $\tilde{E}$ and $\mathscr{E}$ be the Borel subsets of E. Let $\Delta\in\tilde{E}$ be a point not in E. Put $E_\Delta=E\cup\{\Delta\}$ and write $\mathscr{E}_\Delta$ for the σ-algebra in E_Δ generated by $\mathscr{E}$. We will write $\mathscr{E}^*$ (respectively $\mathscr{E}_\Delta^*$), for the σ-algebra of universally measurable sets over $(E,\mathscr{E})$ (respectively $(E_\Delta,\mathscr{E}_\Delta)$).

Let $X=(\Omega,\mathscr{M},\hat{\mathscr{M}}_t,X_t,\hat{\theta}_t,P^x)$ be a Hunt process with state space $(E,\mathscr{E})$ and infinite lifetime. Let $\hat{L}=\{\hat{L}_t;t\geqq 0\}$ be a CAF of X so that $t\to\hat{L}_t(\omega)$ is continuous and nondecreasing for all $\omega\in\Omega$. Let $C\in\mathscr{E}^*$ denote the support of $\hat{L}$; that is, if $R=\inf\{t:\hat{L}_t>0\}$, then $C=\{x:P^x(R=0)=1\}$. Let $S_t=\inf\{u:\hat{L}_u>t\}$, $Y_t=X_{S_t}$, $\theta_t=\hat{\theta}_{S_t}$, and $M_t=\hat{M}_{S_t}$. Then $Y=(\Omega,\mathscr{M},\mathscr{M}_t,Y_t,\theta_t,P^x)$ is a strong Markov process taking values in the state space $(C,\mathscr{C})$ where $\mathscr{C}=C\cap\mathscr{E}^*$; (cf. [2], Chapter V, (2.11)). Y is roughly speaking the X process restricted to the set C. We will put $C_\Delta=C\cup\{\Delta\}$ and $\mathscr{C}_\Delta=\mathscr{C}\cap C_\Delta$. It is not hard to show that the following result holds.

(2.1) **Proposition.** *$(Y,S)=(\Omega,\mathscr{M},\mathscr{M}_t,(Y_t,S_t),\theta_t,P^y)$ is a Markov additive process* (MAP) *with state space $(C\times\mathbb{R}_+,\mathscr{C}\times\mathscr{R}_+)$ in the sense of Çinlar* [3]*; that is,*

a) *$(\Omega,\mathscr{M},\mathscr{M}_t,Y_t,\theta_t,P^y)$ is a Markov process on $(C,\mathscr{C})$;*

b) *the mapping $t\to S_t(\omega)$ is right continuous, has left-hand limits, and satisfies $S_0(\omega)=0$, $S_t(\omega)=S_\zeta(\omega)$ for all $t\geqq\zeta=\inf\{u: Y_u=\Delta\}$ P^y-almost surely for each $y\in C$;*

c) *for each $t\in\mathbb{R}_+$, S_t is $\mathscr{M}_t$-measurable;*

d) *for each $t\in\mathbb{R}_+$, $A\in\mathscr{C}$, $B\in\overline{\mathscr{R}}_+$, the mapping $y\to P^y\{Y_t\in A, S_t\in B\}$ of C into $[0,1]$ is in $\mathscr{C}$;*

e) *for each t, $s\in\mathbb{R}_+$, $S_{t+s}=S_t+S_s\circ\theta_t$ almost surely P^y, $y\in C$;*

f) *for all t, $s\in[0,\infty)$, $y\in C_\Delta$, $A\in\mathscr{C}_\Delta$, and $B\in\overline{\mathscr{R}}_+$*

$$P^y(Y_s\circ\theta_t\in A, S_s\circ\theta_t\in B\mid\mathscr{M}_t)=P^{Y_t}(Y_s\in A, S_s\in B).$$

We will assume throughout this paper that S is quasi-left-continuous: that is, $\lim_{n\to\infty} S_{T_n}=S_T$ almost surely P^y, $y\in C_\Delta$ for any increasing sequence of $\{\mathscr{M}_t\}$-stopping times $\{T_n\}$ so that $\lim_{n\to\infty} T_n=T$. This assumption implies that the Markov process Y is also quasi-left-continuous. It follows from Theorem (2.23) and Corollary (4.19) of Çinlar [3] that S has this property in the important special case in which Y is a regular pure jump process.

We will now turn our attention to the excursions of X outside of C. If t is a jump time of $S(\omega)$, let

$$Z_t(s,\omega)=\begin{cases}X_{S_{t-}+s}(\omega) & \text{for } 0\leqq s<S_t(\omega)-S_{t-}(\omega),\\ \Delta & \text{for } s\geqq S_t(\omega)-S_{t-}(\omega);\end{cases}$$

that is, $Z_t(\omega)=\{Z_t(s,\omega); s\geqq 0\}$ is the excursion of X corresponding to the interval $(S_{t-}(\omega), S_t(\omega))$. If t is not a jump time of $S(\omega)$ put $Z_t(\omega)=[\Delta]$.

Fix $\Gamma\in\mathscr{W}$ and put $S_t^\Gamma=\int_0^t 1_\Gamma\circ Z_u\,dS_u$; (where $1_\Gamma(x)=1$ if $x\in\Gamma$ and 0 otherwise).

$S^\Gamma=\{S_t^\Gamma; t\geqq 0\}$ is a right continuous nondecreasing process and jumps at those times of discontinuity of S for which the corresponding excursion is in Γ; the height of the jump is the same as that of S. We have the following result:

(2.2) **Proposition.** *$(\Omega, \mathscr{M}, \mathscr{M}_t, Y_t, (S_t^\Gamma, S_t), \theta_t, P^y)$ is a MAP with state space $(C\times\overline{\mathbb{R}}_+^2, \mathscr{C}\times\overline{\mathscr{R}}_+^2)$.*

Proof. Note that $Z_s\circ\theta_t=Z_{s+t}$. Hence, by (2.1e) $S_{t+s}^\Gamma-S_t^\Gamma=S_s^\Gamma\circ\theta_t$ almost surely P^y, $y\in C$. Since S_t is a $\{\hat{\mathscr{M}}_t\}$-stopping time, the result now follows from Proposition (2.1) and the strong Markov property for X.

We now give some more notations. Let $\mathscr{K}_t^0=\sigma(Y_s; s\leqq t)$, $\mathscr{L}_t^0=\sigma(Y_s, S_s; s\leqq t)$, $\mathscr{L}_t^0(\Gamma)=\sigma(Y_s, S_s, S_s^\Gamma; s\leqq t)$, $\mathscr{F}_t^0=\sigma(X_s; s\leqq S_t)$, $\mathscr{G}_t^0=\sigma(Y_s, S_s, Z_s; s\leqq t)$, $\mathscr{K}^0=\bigvee_t \mathscr{K}_t^0$, $\mathscr{L}^0=\bigvee_t \mathscr{L}_t^0$, $\mathscr{L}^0(\Gamma)=\bigvee_t \mathscr{L}_t^0(\Gamma)$, $\mathscr{F}^0=\bigvee_t \mathscr{F}^0$ and $\mathscr{G}^0=\bigvee_t \mathscr{G}_t^0$. Let $\mathscr{K}$ (respectively $\mathscr{L}$, $\mathscr{L}^\Gamma$, $\mathscr{G}$, $\mathscr{F}$), denote the completion of $\mathscr{K}^0$ (respectively $\mathscr{L}^0$, $\mathscr{L}^0(\Gamma)$, $\mathscr{G}^0$, $\mathscr{F}^0$), with respect to the family of measures $\mathscr{P}=\{P^\nu: \nu$ is a finite measure on $(E_\Delta, \mathscr{E}_\Delta)\}$. Let $\mathscr{K}_t$ (respectively $\mathscr{L}_t$, $\mathscr{L}_t^\Gamma$, $\mathscr{G}_t$, $\mathscr{F}_t$), denote the completion of $\mathscr{K}_t^0$ (respectively $\mathscr{L}_t^0$, $\mathscr{L}_t^0(\Gamma)$, $\mathscr{G}_t^0$, $\mathscr{F}_t^0$), in $\mathscr{K}$ (respectively $\mathscr{L}$, $\mathscr{L}^\Gamma$, $\mathscr{G}$, $\mathscr{F}$), with respect to $\mathscr{P}$.

By Proposition (2.20) of Çinlar [3] there exists a regular version P_ω^Γ of $P^y(\cdot\,|\,\mathscr{K})$ on $\mathscr{L}^\Gamma$ which is further independent of $y\in C_\Delta$. By (2.22) and (2.23) of Çinlar [3], $(\Omega, \mathscr{L}^\Gamma, \mathscr{L}_t^\Gamma, S_t, P_\omega^\Gamma)$ is a process with independent increments so that

(2.3) $\quad S_t=A_t+S_t^f+S_t^d$

where $\sigma(S_t^f; t\geqq 0)$ and $\sigma(S_t^d; t\geqq 0)$ are conditionally independent given $\mathscr{K}$ with respect to P^y, $y\in C_\Delta$. Further, the following hold:

a) $A=\{A_t; t\geqq 0\}$ is an additive functional of Y;

b) $S^f=\{S_t^f; t\geqq 0)$ is a pure jump process;

(Y, S^f) is a quasi-left-continuous MAP; there exists a sequence of $(\mathscr{K}_t)$-stopping times which exhausts the jumps of S^f;

c) $S^d=\{S_t^d; t\geqq 0\}$ is a pure jump process;

(Y, S^d) is a MAP; S^d is a stochastically continuous process with independent increments over $(\Omega, \mathscr{L}^\Gamma, P_\omega^\Gamma)$.

(2.4) **Lemma.** *There exists a decomposition* (2.3) *such that A is a CAF.*

Proof. Consider any decomposition (2.3) and put

$$A_t'=A_t-\sum_{s\leqq t}(A_s-A_{s-}), \qquad S_t'^f=S_t^f+A_t-A_t'.$$

Then $S=A'+S'^f+S^d$; A is a CAF of Y; and S'^f has properties (2.3)b. □

By the last result, we need only to consider times of discontinuity of S due to S^f and S^d. We will now establish the existence of conditional probability laws for the individual excursions. To this end let $V_t^\Gamma = \int_0^t 1_\Gamma \circ Z_u \, dS_u^f$ and $U_t^\Gamma = \int_0^t 1_\Gamma \circ Z_u \, dS_u^d$. We will first consider the process $V^\Gamma = \{V_t^\Gamma;\, t \geqq 0\}$.

(2.5) **Proposition.** *There exist transition probability kernels F and K from $(C_\Delta^2, \mathscr{C}_\Delta^2)$ into $(\overline{\mathbb{R}}_+, \overline{\mathscr{R}}_+)$ and from $(C_\Delta^2 \times \overline{\mathbb{R}}_+, \mathscr{C}_\Delta^2 \times \overline{\mathscr{R}}_+)$ into $(W, \mathscr{W})$ respectively, such that $F(y, y, \cdot) = \varepsilon_0$ for $y \in C_\Delta$ and for an arbitrary fixed jump time T of V^Γ*

$$P^y(V_T^\Gamma - V_{T-}^\Gamma \in A \mid \mathscr{K}) = \int_A K(Y_{T-}, Y_T, s; \Gamma)\, F(Y_{T-}, Y_T; ds)$$

for $A \in \overline{\mathscr{R}}_+$ and $y \in C_\Delta$.

Proof. It follows from Theorem (4.8) of Çinlar [3] that there exists a transition probability kernel F from $(C_\Delta^2, \mathscr{C}_\Delta^2)$ into $(\overline{\mathbb{R}}_+, \overline{\mathscr{R}}_+)$ with $F(y, y, \cdot) = \varepsilon_0$ such that, if T is a jump time of S^f, then

$$P^y(S_T - S_{T-} \in A \mid \mathscr{K}) = F(Y_{T-}, Y_T; A), \qquad A \in \overline{\mathscr{R}}_+,\ y \in C_\Delta.$$

By the definition of V^Γ and Proposition (2.2), (Y, V^Γ) is a MAP with the same properties as (Y, S^f). Thus, there exists a transition probability kernel F_Γ such that for any jump time T of V^Γ

$$\begin{aligned} F_\Gamma(Y_{T-}, Y_T; A) &= P^y(V_T^\Gamma - V_{T-}^\Gamma \in A \mid \mathscr{K}) \\ &= P^y(Z_T \in \Gamma,\, S_T^f - S_{T-}^f \in A \mid \mathscr{K}) \\ &\leqq F(Y_{T-}, Y_T; A), \qquad A \in \overline{\mathscr{R}}_+. \end{aligned}$$

Hence, by the Radon-Nikodym theorem and the special natures of $(C, \mathscr{C})$ and $(\overline{\mathbb{R}}_+, \overline{\mathscr{R}}_+)$ there exists a nonnegative measurable function $(x, y, s) \to K(x, y, s; \Gamma)$ on $(C_\Delta^2 \times \overline{\mathbb{R}}_+, \mathscr{C}_\Delta^2 \times \overline{\mathscr{R}}_+)$ such that

$$F_\Gamma(Y_{T-}, Y_T; A) = \int_A K(Y_{T-}, Y_T, s; \Gamma)\, F(Y_{T-}, Y_T; ds).$$

W is the complement of an analytic set in a compact metric space; (cf. Maisonneuve [11]). Since $\Gamma \in \mathscr{W}$ is arbitrary we can further choose K so that the mapping $\Gamma \to K(x, y, s; \Gamma)$ is a probability on $(W, \mathscr{W})$ for fixed $x, y \in C_\Delta$ and $s \in \overline{\mathbb{R}}_+$. □

We will now consider the process $U^\Gamma = \{U_t^\Gamma;\, t \geqq 0\}$.

(2.6) **Proposition.** *There exists a continuous additive functional B of Y and a transition probability kernel H from $(C_\Delta, \mathscr{C}_\Delta)$ into $(\overline{\mathbb{R}}_+ \times W, \overline{\mathscr{R}}_+ \times \mathscr{W})$ such that*

$$E^y[\exp\{-\lambda U_t^\Gamma\} \mid \mathscr{K}] = \exp\left\{-\int_0^t \int_{(0,\infty]} (1 - e^{-\lambda z})\, H(Y_s; dz \times \Gamma)(z \wedge 1)^{-1}\, dB_s\right\}$$

for $y \in C_\Delta$, $t \geqq 0$ and $\lambda \geqq 0$.

Proof. Since S^d is an increasing process, Corollary (2.25) of Çinlar [3] yields

$$E^y[\exp\{-\lambda S_t^d\} \mid \mathscr{K}] = \exp\{-\int (1 - e^{-\lambda z})\, D_t(dz)\}, \qquad y \in C_\Delta,$$

for any $\lambda \geqq 0$ where, for $\omega \in \Omega$ fixed, the measure

$$B_t(\omega, A) = \int_A (z \wedge 1)\, D_t(dz, \omega)$$

is finite; $B_t(\omega, \{0\}) = 0$; and where, if

$$B_t(\omega) = B_t(\omega, \overline{\mathbb{R}}_+),$$

then $B = (B_t)_{t \geqq 0}$ is an increasing CAF of Y. Since (Y, U^Γ) is a MAP of the same type as (Y, S^d), the same result holds with $U_t^\Gamma, D_t^\Gamma, B_t^\Gamma(\omega, A)$ and B_t^Γ replacing $S_t^d, D_t, B_t(\omega, A)$ and B_t respectively. Further, for fixed $A \in \overline{\mathscr{R}}_+$, $\{B_t^\Gamma(\cdot, A); t \geqq 0\}$ is a CAF of Y.

By the definition of U^Γ, $D_t^\Gamma(A, \omega)$ is conditionally, given $\mathscr{K}$, the expected number of jumps of S that take place before time t whose heights are in set A and whose corresponding excursions are in set Γ. Thus $D_t^\Gamma(A, \omega) \leqq D_t(A, \omega)$ for all $A \in \mathscr{R}_+$. Hence, $B_t^\Gamma(\omega, A) \leqq B_t(\omega)$ for $t \geqq 0$. By the "Radon-Nikodym" theorem for CAF's (Benveniste and Jacod [1]) there exists a nonnegative measurable function $y \to H(y; A \times \Gamma)$ on $(C_\Delta, \mathscr{C}_\Delta)$ such that

$$B_t^\Gamma(\omega, A) = \int_0^t H(Y_s; A \times \Gamma)\, dB_s(\omega).$$

By the special natures of $(\overline{\mathbb{R}}_+, \overline{\mathscr{R}}_+)$ and $(W, \mathscr{W})$, H can further be chosen so that for fixed $y \in C_\Delta$, $D \to H(y; D)$ is a probability on $(\overline{\mathbb{R}}_+ \times W, \overline{\mathscr{R}}_+ \times \mathscr{W})$.

Let $\{U_j\}$ be the finite or countable collection of jump times of S. The proof of following result shows that the conditional distribution of the excursion Z_{U_j} *given the process* Y depends only on Y_{U_j-} and Y_{U_j}.

(2.7) **Proposition.** *The random variables* $\{Z_{U_j}\}$ *are conditionally independent given* $\mathscr{K}$.

Proof. Propositions (2.5), (2.6) and the proof of Theorem (2.2) of Çinlar [4] imply that there exists a continuous additive functional $\bar{A}$ of Y and a transition kernel $\bar{Q}$ from $(C_\Delta, \mathscr{C}_\Delta)$ into $(C_\Delta \times \overline{\mathbb{R}}_+ \times W, \mathscr{C}_\Delta \times \overline{\mathscr{R}}_+ \times \mathscr{W})$ such that for any $f \in p(\mathscr{C}_\Delta \times \mathscr{C}_\Delta \times \overline{\mathscr{R}}_+ \times \mathscr{W})$ and $y \in C_\Delta$

$$E^y\Big[\sum_{s \leqq t} f(Y_{s-}, Y_s, S_s - S_{s-}, Z_s)\, I_{\{Y_{s-} \neq Y_s\} \cup \{S_{s-} \neq S_s\}}\Big]$$
$$= E^y\left[\int_0^t d\bar{A}_s \int_{C \times \overline{\mathbb{R}}_+ \times W} \bar{Q}(Y_s; dx, ds, dz)\, f(Y_s, x, s, z)\right].$$

Fix $\varepsilon > 0$ and let $\tau = \inf\{u : S_u - S_{u-} > \varepsilon\}$. Since S is quasi-left-continuous by assumption, τ is a totally inaccessible $\{\mathscr{M}_t\}$-stopping time. Let $M \in p\mathscr{M}_{\tau-}$, $f \in p\mathscr{W}$, and $G \in p\mathscr{M}$. By the results of Weil [12] on conditioning on the strict past, for $y \in C_\Delta$

$$\begin{aligned} E^y&[M(G \circ \theta_\tau)\, f \circ Z_\tau] \\ &= E^y[M(f \circ Z_\tau)\, E^{Y_\tau}[G]] \\ &= E^y[M E^y[f \circ Z_\tau E^{Y_\tau}[G] \mid \mathscr{M}_{\tau-}]] \\ &= E^y\Big[M \int_{C_\Delta \times W} Q_\varepsilon(Y_{\tau-}; dx, dz)\, f(z)\, E^x[G]\Big] \end{aligned}$$

where

$$Q_\varepsilon(x; D\times\Gamma)=\frac{\bar{Q}(x; D\times(\varepsilon,\infty]\times\Gamma)}{\bar{Q}(x; C_\Delta\times(\varepsilon,\infty]\times W)}, \quad D\in\mathscr{C}_\Delta \text{ and } \Gamma\in\mathscr{W}.$$

Let $K_\varepsilon(x; B)=Q_\varepsilon(x; B\times W)$. Note that $P\{Y_\tau\in B\,|\,\mathscr{M}_{\tau-}\}=K_\varepsilon(Y_{\tau-}; B)$. By the Radon-Nikodym theorem and the special nature of $(C,\mathscr{C})$, for fixed $\Gamma\in\mathscr{W}$ there exists a nonnegative measurable function $(x,y)\to q_\varepsilon(x,y;\Gamma)$ on $(C_\Delta^2,\mathscr{C}_\Delta^2)$ such that

$$Q_\varepsilon(x; B\times\Gamma)=\int_B q_\varepsilon(x,y;\Gamma)\,K_\varepsilon(x; dy).$$

By the special nature of $(W,\mathscr{W})$ we can further choose q_ε so that the mapping $\Gamma\to q_\varepsilon(x,y;\Gamma)$ is a probability on $(W,\mathscr{W})$ for fixed $x, y\in C_\Delta$. Now

$$\begin{aligned} &E^y[M(G\circ\theta_\tau)f\circ Z_\tau]\\ &\quad=E^y[M\int_C\int_W q_\varepsilon(Y_{\tau-},y;dz)\,f(z)\,K(Y_{\tau-};dy)\,E^y[G]]\\ &\quad=E^y[M\int_W q_\varepsilon(Y_{\tau-},Y_\tau;dz)\,f(z)\,E^{Y_\tau}[G]]\\ &\quad=E^y[M(G\circ\theta_\tau)\int_W q_\varepsilon(Y_{\tau-},Y_\tau;dz)\,f(z)]. \end{aligned}$$

Since $\mathscr{K}$ is generated by sets of the form $M(G\circ\theta_\tau)$, it follows that the conditional distribution of Z_τ given $\mathscr{K}$ depends only on $Y_{\tau-}$, Y_τ. Since for each jump time of S there exists $\varepsilon>0$ such that the jump time is of the form of τ or one of its iterates, the result follows.

We now come to the result concerning the existence of a conditional probability law for the excursion process given $\mathscr{K}$.

(2.8) **Theorem.** *There exists a regular version of $P^y(\cdot\,|\,\mathscr{K})$ on $\mathscr{G}$ which is further independent of $y\in C_\Delta$.*

Proof. Let $\{T(j)\}$ (respectively $\{T^f(j)\}$), be the finite or denumerable collection of jump times of S^d (respectively S^f). Put $\Delta S(j)=S_{T(j)}-S_{T(j)-}$ and $Z(j)=Z_{T(j)}$ (respectively $\Delta S^f(j)=S_{T^f(j)}-S_{T^f(j)-}$ and $Z^f(j)=Z_{T^f(j)}$).

Let N be the collection of all counting measures on $(\mathbb{R}_+^2\times W, \bar{\mathscr{R}}_+^2\times\mathscr{W})$ that are finite on compact sets. N is metrizable and we write $\mathscr{N}$ for the Borel subsets of N. Let ν^f and ν^d be mappings from $(\Omega,\mathscr{G})$ into $(N,\mathscr{N})$ such that for $g\in p(\bar{\mathscr{R}}_+^2\times\mathscr{W})$

$$\int g\,d\nu^f=\sum_j g(T^f(j),\Delta S^f(j),Z^f(j))$$

and

$$\int g\,d\nu^d=\sum_j g(T(j),\Delta S(j),Z(j));$$

that is, ν^f (respectively ν^d), is the random counting measure induced by the times of discontinuity of S^f (respectively S^d), and the corresponding magnitudes of the jumps and excursions.

Fix $\omega \in \Omega$ and let K, F, H, and B be as in Propositions (2.5) and (2.6). By results concerning the construction of probabilities on $(N, \mathcal{N})$ (cf. Jagers [10]), there exist probabilities $\tilde{P}^1_\omega$ and $\tilde{P}^2_\omega$ on $(N, \mathcal{N})$ such that for any $g \in p(\bar{\mathscr{R}}^2_+ \times \mathscr{W})$

$$\begin{aligned}\tilde{E}^1_\omega[\exp\{-\textstyle\int g\,dm\}] &= \prod_{T^f(j,\omega)} \int \exp\{-g(T^f(j,\omega), s, z)\} K(Y_{T^f(j)-}(\omega), Y_{T^f(j)}(\omega), s; dz) \\ &\quad \cdot F(Y_{T^f(j)-}(\omega), Y_{T^f(j)}(\omega); ds)\end{aligned}$$

and

$$\begin{aligned}\tilde{E}^2_\omega[\exp\{-\textstyle\int g\,dm\}] &= \exp\{-\textstyle\int (1-e^{-g(t,s,z)}) H(Y_t(\omega); ds, dz)(s \wedge 1)^{-1}\, dB_t(\omega))\}.\end{aligned}$$

Thus, there exists a probability on $(\Omega, \mathscr{G})$ such that for Γ and Λ in $\mathcal{N}$

$$P_\omega(v^f \in \Gamma, v^d \in \Lambda) = \tilde{P}^1_\omega(\Gamma)\,\tilde{P}^2_\omega(\Lambda).$$

That P_ω is a version of $P^y(\cdot \mid \mathscr{K})$ on $\mathscr{G}$ for all $y \in C_\Delta$ follows from (2.3) and Propositions (2.5), (2.6), and (2.7). □

When deleting the ω we will simply write P for the version of $P^y(\cdot \mid \mathscr{K})$ in Theorem (2.8).

3. The Excursion Counting Measure

In this section we will study the structure of the excursion counting measure μ. We will first give some definitions.

Let $(\bar{G}, \bar{\mathscr{G}})$ be a measurable space and $(\bar{\Omega}, \bar{\mathscr{M}}, \bar{P})$ be a probability space. The mapping $v: \bar{\mathscr{G}} \times \bar{\Omega} \to \mathbb{R}_+$ is said to be a random measure on $(\bar{G}, \bar{\mathscr{G}})$ provided: a) $A \to v(A, \omega)$ is a measure on $(\bar{G}, \bar{\mathscr{G}})$ for fixed $\omega \in \bar{\Omega}$; and b) $\omega \to v(A, \omega)$ is in $\bar{\mathscr{M}}$ for fixed $A \in \bar{\mathscr{G}}$. v is said to be an additive random measure over $(\bar{\Omega}, \bar{\mathscr{M}}, \bar{P})$ if $v(A_1), \ldots, v(A_n)$ are independent random variables whenever $A_1, \ldots, A_n$ are disjoint sets in $\bar{\mathscr{G}}$. An additive random measure v is said to be a Poisson random measure (PM) with mean measure n if for each $A \in \bar{\mathscr{G}}$ with $n(A) < \infty$, $v(A)$ has a Poisson distribution with parameter $n(A)$; (if $n(A) = \infty$, then $v(A) = \infty$ a.s.). An easy characterization of a PM is given by the following result whose proof will be omitted.

(3.1) **Lemma.** *A random measure v is a PM with σ-finite mean measure n if and only if*

$$\bar{E}[\exp\{-\textstyle\int f\,dv\}] = \exp\{-\textstyle\int (1-e^{-f})\,dn\}$$

for all $f \in p\bar{\mathscr{G}}$.

Recall that $\{T(j)\}$ (respectively $\{T^f(j)\}$), are the times of discontinuity of S^d (respectively S^f) and $Z(j)$ (respectively $Z^f(j)$), is the excursion corresponding to $T(j)$ (respectively $T^f(j)$). Let μ^f and μ^d be the random counting measures on $(\mathbb{R}_+ \times W, \bar{\mathscr{R}}_+ \times \mathscr{W})$ such that for any $g \in p(\bar{\mathscr{R}}_+ \times \mathscr{W})$

$$\int g\,d\mu^f = \sum_j g(T^f(j), Z^f(j)) \quad \text{and} \quad \int g\,d\mu^d = \sum_j g(T(j), Z(j)).$$

Then $\mu=\mu^f+\mu^d$ is the random counting measure induced by the times of discontinuity of S and the corresponding excursions. The next result follows from Theorem (2.8) and Lemma (3.1).

(3.2) **Theorem.** *The random measures μ^f and μ^d are independent additive random measures over $(\Omega, \mathscr{G}, P_\omega)$. Further, μ^d is a PM with mean measure M_ω such that for $t \geqq 0$ and $\Gamma \in \mathscr{W}$*

$$M_\omega([0,t]\times\Gamma)=\int_0^t \int_0^\infty H(Y_s(\omega); dz\times\Gamma)(z\wedge 1)^{-1}\, dB_s(\omega).$$

Assume for the moment that Y is a regular pure jump process. By Corollary (4.8) of Çinlar [3]

$$S_t^f=\sum_{s\leqq t}(S_s^f-S_{s-}^f)\,1_{\{Y_{s-}\neq Y_s\}}.$$

Since $t\to S_t^d$ is stochastically continuous over $(\Omega, \mathscr{G}, P_\omega)$, the jump times of S^d and Y do not coincide P^y-almost surely for $y\in C$. Hence, in this case, almost surely, μ^f is the counting measure induced by the times of discontinuity of S and the corresponding excursions $\{Z_{U_j}\}$ for which $Y_{U_j-}\neq Y_{U_j}$; μ^d is the counting measure induced by the times of discontinuity of S and the corresponding excursions $\{Z_{U_j}\}$ for which $Y_{U_j-}=Y_{U_j}$.

We will now study μ^d more closely. Unless otherwise stated we will put $S=S^d$ throughout the remainder of the paper. Thus $\mu=\mu^d$. Let D be the support of the additive functional B. We will not assume that Y is a regular pure jump process. However, we will make the assumption that D is discrete.

(3.3) **Lemma.** *For each $x\in D$ there exists a possibly σ-finite measure $A\to N^x(A)$ on $(W, \mathscr{W})$ such that*

$$M_\omega([0,t]\times\Gamma)=\sum_{x\in D} L_t^x(\omega)\,N^x(\Gamma), \qquad t\geqq 0,\ \Gamma\in\mathscr{W},\ \omega\in\Omega,$$

where $L^x=\{L_t^x; t\geqq 0\}$ is the local time at x for Y; $\Big($that is, L^x is a CAF of Y with support $\{x\}$ such that, if $R=\inf\{t: L_t^x>0\}$, then $E^y[e^{-R}]=E^y\left[\int_0^\infty e^{-t}\,dL_t^x\right]$ for $y\in C\Big)$.

Proof. Let $M_t(\Gamma,\omega)=M_\omega([0,t]\times\Gamma)$ for $\Gamma\in\mathscr{W}$ and $\omega\in\Omega$. Then $\Gamma\to M_t(\Gamma,\omega)$ is a possibly σ-finite measure on $\mathscr{W}$. Let $\Lambda_1=\{w\in W: w(1)\neq\Delta\}$ and $\Lambda_n=\left\{w\in W: w\left(\frac{1}{n}\right)\neq\Delta,\ w\left(\frac{1}{n-1}\right)=\Delta\right\}$ for $n=2,3,\ldots$. Then $\{\Lambda_n\}$ is countable collection of disjoint sets in $\mathscr{W}$ so that $W-[\Delta]=\bigcup_n \Lambda_n$ and $M_t(\Lambda_n)<\infty$ for $t\geqq 0$. Put $M_t^n(\Gamma)=M_t(\Gamma\cap\Lambda_n)$ for $\Gamma\in\mathscr{W}$ and let $M_t^n=M_t^n(W)$. Note that $\{M_t^n(\Gamma); t\geqq 0\}$ is a CAF of Y for each fixed $\Gamma\in\mathscr{W}$. Since $M_t^n=M_t^n(\Gamma)+M_t^n(\Gamma^c)$, by the "Radon-Nikodym" theorem for CAF's there exists a transition probability G_n from $(D,\mathscr{D})$ into $(W,\mathscr{W})$ so that

$$M_t^n(\Gamma)=\int_0^t G_n(Y_s,\Gamma)\,dM_s^n=\int_0^t G_n(Y_s,\Gamma)\,1_D(Y_s)\,dM_s^n$$

where $\mathscr{D}=D\cap\mathscr{C}$. For each $x\in D$, $\int_0^t 1_{\{x\}}(Y_s)\,dM_s^n$ is a CAF of Y with support $\{x\}$; hence it is a multiple of L^x. Therefore, there exist nonnegative constants c_x, $x\in D$, so that

$$M_t^n(\Gamma,\omega)=\sum_{x\in D} c_x\, G_n(x,\Gamma)\, L_t^x(\omega), \qquad \Gamma\in\mathscr{W},\ \omega\in\Omega.$$

Let $\tilde{N}(x,\Gamma\cap\Lambda_n)=c_x\, G_n(x,\Gamma)$. Put

$$N^x(\Gamma)=\sum_n \tilde{N}(x,\Gamma\cap\Lambda_n).$$

Since $M_t(\{[\Delta]\},\omega)=0$, $\Gamma\to N^x(\Gamma)$ is a possibly σ-finite measure on $(W,\mathscr{W})$ satisfying the conditions of the Lemma. □

For each $x\in D$, let $S_t^x=\sum_{s\leqq t}(S_s-S_{s-})\,1_{\{x\}}(Y_s)$, $\{T_j^x\}$ be the collection of jump times of $\{S_t^x; t\geqq 0\}$, $Z_j^x=Z_{T_j^x}$, and μ_x be the random counting measure on $(\bar{\mathbb{R}}_+\times W, \bar{\mathscr{R}}_+\times\mathscr{W})$ induced by $\{(T_j^x, Z_j^x)\}$; that is, for $g\in p(\bar{\mathscr{R}}_+\times\mathscr{W})$, $\int g\,d\mu_x=\sum_j g(T_j^x, Z_j^x)$. Since $S_t=\sum_{x\in D} S_t^x$, P-almost surely, $\mu=\sum_{x\in D}\mu_x$ P-almost surely.

(3.4) **Proposition.** a) *μ_x, $x\in D$, are independent PM's over $(\Omega,\mathscr{G},P_\omega)$; μ_x has mean measure m_x^ω where $m_x^\omega([0,t]\times\Gamma)=L_t^x(\omega)\,N^x(\Gamma)$ for $\omega\in\Omega$, $\Gamma\in\mathscr{W}$;*

b) *Let $U_j^x=L_{T_j^x}^x$ and v_x be the random counting measure on $(\bar{\mathbb{R}}_+\times W, \bar{\mathscr{R}}_+\times\mathscr{W})$ induced by $\{(U_j^x, Z_j^x)\}$. Then v_x is a PM over $(\Omega,\mathscr{G},P_\omega)$ with mean measure n_x^ω where*

$$n_x^\omega([0,t]\times\Gamma)=(t\wedge L_\infty^x(\omega))\,N^x(\Gamma);$$

c) *Given L_∞^x, v_x is independent of $\mathscr{K}$ for all P^y, $y\in C$.*
d) *Given $\sigma(L_\infty^x; x\in D)$, $\{v^x; x\in D\}$ are conditionally independent.*

Proof. Let $f\in p(\bar{\mathscr{R}}_+\times\mathscr{W})$. For $x\in D$

$$\begin{aligned}E[\exp\{-\textstyle\int f\,d\mu_x\}]&=E[\exp\{-\textstyle\sum_j f(T(j),Z(j))\,1_{\{x\}}(Y_{T(j)})\}]\\&=\exp\{-\textstyle\int(1-e^{-f(s,z)})\,dL_s^x\,N^x(dz)\}\end{aligned}$$

by Lemma (3.1), Theorem (3.2), and Lemma (3.3). a) now follows from Lemma (3.1). b) is proved in a similar fashion and c) and d) are immediate) from b). □

Note that if $L_\infty^x=\infty$ for all $x\in D$, then v_x, $x\in D$, are independent PM's.

We will now turn our attention to the measure N^x, $x\in D$. For each $x\in D$, let $h_x(z)=P^z(Y_0=x)$ for $z\in E$. Put

$$K_t^x(z,du)=\begin{cases}\dfrac{1}{h_x(z)}P^z(S_0>t, X_t\in du)\,h_x(u) & \text{if } h_x(z)>0,\\ 0 & \text{otherwise.}\end{cases}$$

Then $\{K_t^x; t \geqq 0\}$ is a semigroup. For $v \in E \setminus C$, let $K^y(v, \cdot)$ be a regular version of the conditional distribution of the excursion Z_0 starting at time 0, given $Y_0 = X_{S_0} = y$ for the measure P^v; that is, $\Gamma \to K^y(v, \Gamma)$ is a probability on $(W, \mathscr{W})$ for fixed $v \in E \setminus C$ and $y \in D$; for fixed $\Gamma \in \mathscr{W}$, $(x, v) \to K^x(v, \Gamma)$ is measurable with respect to $\mathscr{D} \times \mathscr{E}^*$; and

(3.5) $\quad P^v(Z_0 \in \Gamma | Y_0) = K^{Y_0}(v, \Gamma) \qquad \Gamma \in \mathscr{W}$.

(3.6) **Proposition.** *Let $T = \inf\{t: S_t - S_{t-} > c\}$ for fixed $c > 0$. T is an $\{\mathscr{L}_t\}$-stopping time and $U = S_{T-} + c$ is a $\{\hat{\mathscr{M}}_t\}$-stopping time. For $\Gamma \in \mathscr{W}$ and $v \in E$*

$$P^v(Z_0 \circ \hat{\theta}_U \in \Gamma | \hat{\mathscr{M}}_U \vee \mathscr{K}) = K^{Y_T}(X_U, \Gamma).$$

Proof. Let $\mathscr{B}^1$, (respectively $\mathscr{B}^2$) be the completion of $\sigma(T, Y_u; u < T)$ (respectively $\sigma(Y_{T+u}; u \geqq 0)$), in $\mathscr{M}$ with respect to $\mathscr{P}$ Let $\tilde{\mathscr{K}}$ be the completion of $\sigma(\mathscr{B}^1, \mathscr{B}^2)$ in $\mathscr{M}$ with respect to $\mathscr{P}$. The proof of (2, Chap. III, (4.20)) shows that $\mathscr{K}^0 \subset \tilde{\mathscr{K}}$ and hence $\mathscr{K} \subset \tilde{\mathscr{K}}$. The inclusion is strict since $T \in \tilde{\mathscr{K}}$. Let $0 \leqq u_1 < \cdots < u_n$ and $A_1, \ldots, A_n \in \mathscr{C}$. There is $G \in b\mathscr{K}$ so that $G \circ \hat{\theta}_U$ is the indicator function of $\{Y_{T+u_i} \in A_i; i = 1, \ldots, n\}$. Let $R \in b\hat{\mathscr{M}}_U$ and $f \in b\mathscr{W}$. By the strong Markov property for X and (3.5)

$$\begin{aligned} E^x[R\{G(f \circ Z_0)\} \circ \hat{\theta}_U] &= E^x[R E^{X_U}[G K^{Y_0}(X_U; f)]] \\ &= E^x[R(G \circ \hat{\theta}_U) K^{Y_T}(X_U; f)] \end{aligned}$$

for $x \in E$. Since $\mathscr{B}^1 \subset \hat{\mathscr{M}}_U$ functions of the form $R(G \circ \hat{\theta}_U)$ generate $\hat{\mathscr{M}}_U \vee \tilde{\mathscr{K}}$,

$$E^x[f \circ Z_0 \circ \hat{\theta}_U | \hat{\mathscr{M}}_U \vee \tilde{\mathscr{K}}] = K^{Y_T}(X_U; B).$$

The result now follows. □

For $w \in W$, $w = \{w_s; s \geqq 0\}$, let $\delta(w) = \inf\{t > 0: w_t = \Delta\}$, and $\hat{\theta}_t w_s = w_{s+t}$ for $s, t \geqq 0$. Put $V_t(w) = w_t$.

(3.7) **Theorem.** *$\{V_t; t > 0\}$ is a Markov process over $(W, \mathscr{W}, N^x)$ with transition function $\{K_t^x; t \geqq 0\}$ for $x \in D$.*

Proof. For fixed $c > 0$ and $x \in D$ let $T = \inf\{t: S_t - S_{t-} > c, Y_t = x\}$. Let $Z = \{Z(s); s \geqq 0\}$ denote the corresponding excursion Z_T. By Proposition (3.4), for $B \in \mathscr{W}$ and a measurable set $A \subset E \setminus C$.

$$P(Z(c) \in A, Z \circ \hat{\theta}_c \in B) = \frac{N^x(V_c \in A, V \circ \hat{\theta}_c \in B)}{N^x(\delta > c)}.$$

Letting $B = W$ in the above expression we obtain

$$N^x(V_c \in A) = N^x(\delta > c) P(Z(c) \in A).$$

Hence, by Proposition (3.6)

$$N^x(V_c \in A, V \circ \hat{\theta}_c \in B) = \int_A N^x(V_c \in dy) K^x(y, B).$$

The result now follows by an induction argument. □

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