The Geneva Papers on Risk and Insurance Theory, 21: 191–210 (1996)

Consumer Risk Perceptions and Information in Insurance Markets with Adverse Selection

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Abstract

Standard models of adverse selection in insurance markets assume policyholders know their loss distributions. This study examines the nature of equilibrium and the equilibrium value of information in competitive insurance markets where consumers lack complete information regarding their loss probabilities. We show that additional private information is privately and socially valuable. When the equilibrium policies separate types, policyholders can deduce the underlying probabilities from the contracts, so it is information on risk type, rather than loss probability per se, that is valuable. We show that the equilibrium is "as if" policyholders were endowed with complete knowledge if, and only if, information is noiseless and costless. If information is noisy, the equilibrium depends on policyholders' prior beliefs and the amount of noise in the information they acquire.

Key words: adverse selection, hidden information, informational equilibrium, learning

1. Introduction

It is well recognized that the Pareto optimal allocations normally associated with competitive markets may not be reached whenever incomplete or asymmetric information regarding product quality or riskiness exists (e.g., Arrow [1963], Akerlof [1970]). In insurance markets, consumers are usually assumed to know their own loss distribution, while insurers generally do not possess information regarding the loss distributions of particular insureds. The problem resulting from this information asymmetry is known as adverse selection. The standard solution to adverse selection problems in insurance markets is to employ a self-selection mechanism. The insurer establishes a menu of contracts, each specifying a level of coverage at a particular price, and each designed to appeal to a particular risk type. Consumers reveal their risk characteristics by self-selecting a particular price-quantity policy. A complete characterization of competitive equilibrium depends on whether the contracts offered must break even individually or jointly (see Rothschild and Stiglitz [1976], Wilson [1977], Miyazaki [1977], and Spence [1978]).

An important assumption in adverse-selection models is that each individual is exogenously endowed with full knowledge of his or her loss distribution. This assumption may be reasonable for certain insurance markets, such as the smoker versus nonsmoker classification in life insurance. Even there, smokers may only know that their risk is increased but may not know the risk precisely. The assumption that individuals know their loss distributions may be less representative of other markets (such as automobile insurance, where all drivers may believe themselves to be above average). In such markets insurers may have better information regarding loss distributions than policyholders[1]. In such markets it is reasonable to ask how consumers' perceptions of the risks they face affect the market equilibrium.

Recent research has begun to explore the question of market equilibrium and the value of information when individuals are not endowed with complete knowledge of their risk characteristics. Kleindorfer and Kunreuther [1983] assume that each consumer knows his or her risk type but misestimates the true loss probability. Doherty and Schlesinger [1995] assume each individual knows his or her risk type and probability of loss but allow the loss amount to be randomly perturbed or noisy. Both of these papers are concerned with how consumers' risk perceptions affect the existence and form of equilibrium and take each consumer's information set as given and unalterable. Crocker and Snow [1992] and Doherty and Thistle [1996] analyze insurance markets where consumers initially know only the population loss probabilities and type distribution but can acquire additional information about their own risk characteristics. Crocker and Snow show that, if insurers can observe policyholders' informational (informed/uninformed) status and policyholders have no prior information, then the Pareto optimal policies involve no signaling. This implies that the private value of information to policyholders is negative. If policyholders have some prior information, then additional information has positive or negative value depending on whether or not the benefits of improved screening outweigh the costs of increased signaling. Doherty and Thistle assume that consumers' informational status is private information and show that, whether or not there is prior information, additional private information on risk type has nonnegative value to consumers.

This article extends prior research regarding the implications of consumers' information for insurance markets. As in Kleindorfer and Kunreuther, we assume a consumer may misestimate his or her objective loss probability. In addition, we assume a consumer does not necessarily know his or her individual risk type. As in Doherty and Thistle, we focus on the value of information to consumers and consumers' incentives to learn their risk types and/or loss probabilities, although our results also have implications for the social value of information. Doherty and Thistle analyze the value of information to policyholders in competitive insurance markets assuming Nash equilibrium. We show that Doherty and Thistle's main result—that if informational status is unobservable, then information has nonnegative value to consumers—is quite robust. We show that this result holds for both noiseless and noisy information and does not depend on either the market structure or equilibrium concept.

We then consider the relationship between consumers' information and insurance market equilibria. While we emphasize the Nash and Miyazaki-Spence (MS) equilibria, we also discuss Wilson's [1977] anticipatory equilibrium, Riley's [1979, 1985] reactive equilibrium, and monopoly equilibrium. This allows us to consider when the equilibria are "as if" consumers were endowed with complete knowledge of their loss probabilities, or, in Doherty and Thistles' terminology, when the equilibrium is "fully informationally productive". We show that consumers can extract the loss probabilities from a separating equilibrium. This implies that it is information on risk type, rather than loss probability per se, that is valuable. An equilibrium is fully informationally productive if, and only if, it is a separating equilibrium, the information structure noiselessly reveals risk type, and the information structure is costless. If the information available to consumers is noisy (that is, consumers may receive an incorrect signal of their risk type), we show that an informationally productive MS equilibrium always exists; the equilibrium contracts depend on individuals' prior beliefs and the amount of noise in the information they acquire.

In Section 2 we analyze the decision problem of an individual consumer. In Sections 3 and 4 we analyze markets with multiple objective risk classes. The model is the standard two-risk-class model of adverse selection in insurance markets. Section 3 examines the case of perfectly informative or noiseless signals. Section 4 considers the case where the signal that consumers observe does not perfectly reveal their risk class or is noisy. Section 5 concludes the paper. We show in an appendix that the results regarding signal extraction apply to an arbitrary n risk-class environment whether the insurance market is monopolistic or competitive.

2. Optimal insurance purchases with unknown loss probabilities

In this section we analyze the optimal insurance decision of a consumer who is not endowed with knowledge of his or her risk type. Throughout the article we assume consumers are expected utility maximizers. All consumers have initial nonstochastic wealth w and the increasing, concave von Neumann-Morgenstern utility function $u(\cdot)$. Each consumer faces a possible loss $x (0 < x < w)$, which occurs with probability T. We assume T and x are fixed, so that there is no normal hazard.

An insurance contract or policy c consists of a premium and a level of coverage: $c = (p, q)$. A consumer is assumed to choose the policy that maximizes expected utility, given his or her assessment of the probability of incurring a loss. Expected utility for a consumer with policy c and subjective loss probability ω is

$$v(c, \omega) = (1 - \omega)u(w - p) + \omega u(w - p - x + q). \tag{1}$$

We assume that each consumer knows the probability of loss belongs to a set $\Omega \subseteq [0, 1]$ but does not know his or her particular loss probability T. The consumer has a continuous prior density over possible loss probabilities $\pi(\omega)$. An information structure Φ consists of a space of signals Y and a function $\phi : \Omega \to Y$. If the signal y is observed, then the individual's loss probability is in $\Omega(y) = \phi^{-1}(y) \subseteq \Omega$. The prior probability of receiving the signal y is $\Pi(y) = \int_{\Omega(y)} \pi(\omega)\, d\omega$. The consumer's posterior beliefs, $\pi(\omega \mid y)$, regarding the loss probability are a Bayesian revision of the prior

$$\begin{aligned} \pi(\omega \mid y) &= \frac{\pi(\omega)}{\int_{\Omega(y)} \pi(\hat{\omega})\, d\hat{\omega}} & \omega \in \Omega(y) \\ \pi(\omega \mid y) &= 0 & \omega \notin \Omega(y). \end{aligned} \tag{2}$$

We assume that the priors are strictly positive on Ω. This implies that $\Pi(y) > 0, \forall y \in Y$, so that every signal is regarded as possible. This in turn implies that the posterior beliefs $\pi(\omega \mid y)$ are well defined for all loss probabilities and all signals. That is, we assume that the individual can always interpret the signal, and, if the information structure is informative, the signal changes beliefs.

An information structure induces a partition $\{\phi^{-1}(y), y \in Y\}$ of Ω and can in fact be identified with the partition. For the completely uninformative information structure $\phi^{-1}(y) = \Omega, \forall y \in Y$. One information structure Φ_1 is "more informative" than Φ_0 if it induces a finer partition of Ω. Following Doherty and Thistle [1996], we will say that an equilibrium is informationally productive if the equilibrium policies reflect more information than the completely uninformative information structure. An equilibrium is fully informationally productive if the equilibrium contracts reflect the most informative information structure in the economy. We assume that consumers have an all-or-nothing choice between remaining uninformed and acquiring the information structure and will choose to acquire an information structure if it has nonnegative net value—that is, if it increases ex ante expected utility.

Under the initial completely uninformative information structure, the consumer's maximization problem is defined by

$$\max_{c} \int_{\Omega} v(c, \omega)\pi(\omega)\, d\omega. \tag{3}$$

If a signal y is observed, then the policyholder's problem becomes

$$\max_{c} \int_{\Omega} v(c, \omega)\pi(\omega \mid y)\, d\omega. \tag{4}$$

Since expected utility is linear in the probability of loss, this is equivalent to maximizing expected utility with the probability of loss equal to $E(\omega \mid y)$. The ex ante expected utility of the information structure Φ is

$$V(\Phi) = \int_{Y} \left\{ \max_{c} v(c, E(\omega \mid y)) \right\} \Pi(y)\, dy, \tag{5}$$

where $\Pi(y)$ is the prior probability that $\omega \in \Omega(y)$—that is, the prior probability of receiving the signal y.

Insurers offer any level of coverage less than or equal to x at a premium that offers nonnegative expected profits:

$$p \geq Tq, \quad q \leq x. \tag{6}$$

Insurance is supplied in a competitive market by risk-neutral, expected-profit-maximizing firms who are assumed to know T. This implies that, in equilibrium, firms earn zero expected profits, so that the first inequality in (6) holds as an equality.

A consumer may overestimate, correctly estimate, or underestimate the loss probability T. Consider the preferred contract choices of the three types of consumers, taking the information structure as fixed. Given the estimate of the loss probability, the consumer's problem is essentially that of buying a policy with a premium loading. Individuals who underestimate the probability of loss view an objectively actuarially fair policy as having a positive loading, while individuals who overestimate the loss probability view the policy as having a negative loading. The comparative statics of the consumer's problem here are the same as for the problem of purchasing a policy with a premium loading as shown by Mossin [1968]. Consumers who do not underestimate the loss probability ($E(\omega) \geq T$) will buy full coverage, while consumers who do underestimate the loss probability ($E(\omega) < T$) will buy less than full coverage.

One would expect that information regarding the true loss probability would be valuable to the consumer. The value of information depends on how well consumers understand the insurance market—particularly, how well they understand the importance of competition.

Proposition 1: (a) *If the consumer does not know that the market is competitive, additional information on the probability of loss has nonnegative value.* (b) *If the consumer knows the market is competitive, the value of information on the probability of loss is zero.*

Proof. (a) Letting Φ_0 be the consumer's initial information set, then Φ_1 represents additional information if it induces a finer partition of Ω than Φ_0. Then, as shown in, for example, Laffont ([1989], p. 59), $V(\Phi_1) \geq V(\Phi_0)$. (b) The terms of the policy to be purchased are observable, so that if the consumer knows that policies are actuarially fairly priced, then $E\{\omega\} = p/q = T$ and $V(\Phi_1) = V(\Phi_0)$. □

The first result essentially follows from the fact that the consumer can better adapt his actions with additional information. The second part of the proposition points out that the loss probability can be computed from the terms of the policy. Since information has nonnegative value, consumers will choose to learn the probability of loss if they can do so costlessly. Both results can be strengthened somewhat. First, suppose that the consumer overestimates the loss probability $E\{\omega\} \geq T$ and is therefore constrained to the objectively actuarially fair policy $c^* = (Tx, x)$. At this policy, $v(c^*, \omega) = u(w - Tx)$, which is independent of the subjective loss probability; consequently, the value of information is zero. Second, suppose that the consumer does not know that the market is competitive but does know that policies will not be sold at an expected loss. Then, observing the terms of the policy, $E\{\omega\} \leq T$; a consumer will not overestimate the loss probability and, when the inequality is strict, will regard less than full coverage as optimal. In the case, the information on the loss probability leads the individual to increase coverage and has positive value.

3. Risk perceptions with multiple-risk classes

The presence of more than one risk class (that is, more than one true loss probability) complicates the analysis. In particular, it becomes important to distinguish between knowledge of the risk class and knowledge regarding the probability of loss. A consumer may know that he or she is, say, a high-risk individual without knowing the probability of loss for the class.

We assume the same market characteristics as in the preceding section with the exception that an individual may now have a true loss probability equal to either T_H or T_L, where $T_H > T_L$. We assume there are N_H and N_L individuals of type H and L, respectively. An individual may continue to have a subjective estimate of his or her loss probability that may differ from the true probability. We assume there are n_H individuals with prior $\pi_H(\omega)$, who estimate the loss probability as $s_H = E_H\{\omega\} = \int \omega\pi_H(\omega)\,d\omega$. There are also n_L individuals with prior $\pi_L(\omega)$, who estimate the loss probability as $s_L = E_L\{\omega\} = \int \omega\pi_L(\omega)\,d\omega$, where $s_L \leq s_H$. We assume that priors are strictly positive on Ω and that $T_L < s_H$ and $T_H > s_L$. We refer to individuals with the prior $\pi_H(\omega)(\pi_L(\omega))$ as subjective type *H*s (subjective type *L*s). We let n_{ij} be the number of subjective type i policyholders who are in fact of type j. We assume that consumers' priors tend to be correct in the sense that $n_{ii} > n_{ij}$; this assumption insures that the subjective loss probability of the high- (low-) risk group is greater (less) than the overall population loss probability[2].

We assume that the number of subjective and objective high- and low-risk individuals is common knowledge[3]. We also assume that the priors $\pi_H(\omega)$ and $\pi_L(\omega)$ are common knowledge. An individual's subjective risk type is private information. We assume that insurers know the objective loss probabilities T_H and T_L. In contrast to standard adverse-selection models, we assume that an individual does not know his or her objective risk type or objective loss probability[4].

An information structure reveals the individual's type if there are signals y_H, y_L such that $T_H \in \Omega_H = \varphi^{-1}(y_H)$ and $T_L \in \Omega_L = \varphi^{-1}(y_L)$ and $\omega \in \Omega_H, \omega' \in \Omega_L$ implies $\omega > \omega'$. That is, Ω is partitioned as $\{\Omega_H, \Omega_L, \Omega_0\}$ where Ω_0 may be empty or may be further partitioned; the existence of the set Ω_0 allows us to consider cases where $\{\Omega_H, \Omega_L\}$ do not cover Ω. If an information structure reveals the individual's risk type, the policyholder revises his or her prior $\pi_i(\omega)$ to the posterior $\pi_i(\omega \mid y)$ according to Bayes rule. An information structure can reveal the individual's type without revealing the individual's probability of loss. An information structure reveals the individual's probability of loss if there are signals y_H, y_L such that $\varphi^{-1}(y_H) = \{T_H\}$ and $\varphi^{-1}(y_L) = \{T_L\}$. If an information structure reveals the individual's probability of loss, then the posterior is $\pi_i(T_j \mid y_j) = 1, i, j = H, L$. Obviously, if an information structure reveals the individual's loss probability, then it reveals his or her type[5].

We need to consider the incentive of a consumer to learn his or her risk type and/or loss probability. We assume that consumers' informational (informed/uninformed) status is not observed by insurers. This implies that the insurer must offer the same set of contracts to all individuals; the contracts cannot depend on individual's informational status. If a consumer becomes informed, then his or her risk type and/or loss probability are private information.

At any market outcome that solves the adverse-selection problem, the self-selection constraints must hold

$$v(c_H, s_H) \geq v(c_L, s_H) \tag{7}$$

$$v(c_L, s_L) \geq v(c_H, s_L). \tag{8}$$

The self-selection constraints can create incentives for a policyholder to learn his or her risk type.

Proposition 2: *For any fixed set of contracts $\{c_H, c_L\}$ such that the self-selection constraints hold, the value of information regarding risk type is nonnegative.*

Proof. Let Φ_0 be the initial, uninformative information structure and Φ_1 an information structure that reveals the individual's risk type. Let s_H^* be the loss probability for which (7) is binding—that is, $v(c_H, s_H^*) = v(c_L, s_H^*)$. For $i = H, L$, define Y_{iH} by $y \in Y_{iH}$ iff $E_i\{\omega \mid y\} \geq s_H^*$, and define Y_{iL} by $y \in Y_{iL}$ iff $E_i\{\omega \mid y\} < s_H^*$. The self-selection constraints imply that if $y \in Y_{iH}$ the policyholder will choose c_H, while if $y \in Y_{iL}$ the policyholder will choose c_L. Let $\Pi_{iH} = \int_{\Omega(Y_{iH})} \pi_i(\omega) d\omega$ be the prior probability that $y \in Y_{iH}$ and Π_{iL} the prior probability that $y \in Y_{iL}$. Let $s_{iH} = E_i\{\omega \mid y \in Y_{iH}\}$ and $s_{iL} = E_i\{\omega \mid y \in Y_{iL}\}$ be the respective mean posterior estimates of the loss probability. Observe that

$$E_i\{\omega\} = E_y\{E_i\{\omega \mid y\}\} = \int_Y \int_\Omega \omega \pi_i(\omega \mid y) \Pi_i(y)\, d\omega\, dy$$

so that

$$s_i = \Pi_{iH} s_{iH} + \Pi_{iL} s_{iL}. \tag{9}$$

For a policyholder with prior $\pi_i(\omega)$, the ex ante change in expected utility is

$$\begin{aligned} I_i &= V_i(\Phi_1) - V_i(\Phi_0) \\ &= \int_Y \max\{v(c_H, E_i\{\omega \mid y\}), v(c_L, E_i\{\omega \mid y\})\} \Pi_i(y)\, dy - v(c_i, s_i) \\ &= \int_{Y_{iH}} v(c_H, E_i\{\omega \mid y\}) \Pi(y)\, dy + \int_{Y_{iL}} v(c_L, E_i\{\omega \mid y\}) \Pi(y)\, dy - v(c_i, s_i) \\ &= \Pi_{iH}\, v(c_H, s_{iH}) + \Pi_{iL}\, v(c_L, s_{iL}) - v(c_i, s_i), \quad i = H, L. \end{aligned} \tag{10}$$

Upon adding and subtracting $\Pi_{ij} v(c_i, s_{ij})$, rearranging, and substituting for H and L, one obtains

$$I_i = \{\Pi_{ii} v(c_i, s_{ii}) + \Pi_{ij} v(c_i, s_{ij}) - v(c_i, s_i)\} + \Pi_{ij}[v(c_j, s_{ij}) - v(c_i, s_{ij})]. \tag{11}$$

The term in braces vanishes, using (9) and the fact that expected utility is linear in probabilities. Then

$$I_i = \Pi_{ij}[v(c_j, s_{ij}) - v(c_i, s_{ij})]. \tag{12}$$

Since priors are strictly positive on Ω, Y_{ij} is nonempty, so that $\Pi_{ij} > 0$. For any $y \in Y_{ij}$, the posterior estimate of the loss probability is such that the contract c_j is preferred—that is, $v(c_j, E_i\{\omega \mid y \in Y_{ij}\}) - v(c_i, E_i\{\omega \mid y \in Y_{ij}\}) \geq 0$. Since $s_{ij} = E_i\{\omega \mid y \in Y_{ij}\}$, it follows that the term in brackets in nonnegative and therefore $I_i \geq 0$. □

Proposition 2 states that the value of information is nonnegative. From (12), the value of information to both subjective high- and low-risk individuals depends on the set of contracts $\{c_H, c_L\}$. By further specifying the contracts, the result can be made somewhat sharper:

Corollary 1: (a) *If* $c_H = c_L$, *then* $I_H = I_L = 0$. *Suppose* $c_H \neq c_L$. (b) *If* $s_H \geq s_H^*$ *and* $s_{HL} < s_H^*$, *then* $I_H > 0$, *while if* $s_{HL} = s_H^*$, *then* $I_H = 0$. (c) *If* $s_L < s_H^*$ *and* $s_{LH} > s_H^*$, *then* $I_L > 0$, *while if* $s_{LH} = s_H^*$, *then* $I_L = 0$.

Proof. (a) Immediate from (12). (b) If $s_H \geq s_H^*$ and $s_{HL} < s_H^*$, then the individual's contract choice given the posterior is different than the contract choice given the prior, (7) implies that $v(c_L, s_{HL}) > v(c_H, s_{HL})$, and from (12), $I_H > 0$. If $s_{HL} = s_H^*$, then (7) implies that $v(c_L, s_{HL}) = v(c_H, s_{HL})$, and from (12), $I_H = 0$. (c) If $s_L < s_H^*$ and $s_{LH} > s_H^*$, then the individual's contract choice given the posterior is different than the contract choice given the prior, (7) implies that $v(c_H, s_{LH}) > v(c_L, s_{LH})$, and from (12), $I_L > 0$. If $s_{LH} = s_H^*$, then (7) implies that $v(c_L, s_{LH}) = v(c_H, s_{LH})$, and from (12), $I_L = 0$. □

We should point out that Proposition 2 does not assume any particular market structure or equilibrium concept. The key assumption in Proposition 2 is that the set of policies is fixed. The set of contracts may be fixed, for example, due to regulation or due to insurers' inability to observe whether policyholders are informed or uninformed. The self-selection constraints, (7) and (8), determine which policy is preferred by an individual. The self-selection constraints imply that the policy c_H is preferred by those with high estimated loss probabilities (no less than s_H^*), while the policy c_L is preferred by those with low estimated loss probabilities (strictly less than s_H^*). This implies that an informed individual can choose the policy better suited to his or her true objective loss probability.

The self-selection constraints hold at the Nash, Miyazaki-Spence (MS), anticipatory, reactive, and monopoly equilibria[6]. The Nash equilibrium contracts can be characterized as maximizing the expected utility of the lowest (subjective) risk class, subject to the self-selection constraints and the requirement that each policy earn zero expected profit. The MS equilibrium contracts can be characterized as the solution to the same problem, except that the set of contracts is required to earn zero expected profit jointly rather than individually. Thus, the MS contracts allow cross-subsidization, while the Nash contracts do not. If the MS contracts have zero cross-subsidy, then they are identical to the Nash contracts. An analytical advantage of the MS contracts is that they always exist. Existence of the Nash contracts requires the Rothschild-Stiglitz condition.

Rothschild-Stiglitz Condition: *Let* T_A *be the average objective loss probability for the two risk classes. Let c be any contract that has nonnegative expected profit on the pool—that is,* $p \geq T_A q$. *Then* $v(c_L, s_L) \geq v(c, s_L)$.

Graphically, the Rothschild-Stiglitz condition implies that the pooled fair odds line does not intersect the low-risk group's indifference curve through the low-risk contract. If the Rothschild-Stiglitz condition holds, then the Nash contracts exist. The anticipatory equilibrium contracts are the Nash contracts if they exist; otherwise the equilibrium contract is the contract on the pooled fair odds line most preferred by the low risks. The reactive equilibrium contracts are the efficient separating contracts, each earning zero expected profit[7]. Proposition 2 applies to all of these equilibria and implies that all policyholders will choose to become informed about their risk types if the information is costless.

Since information is endogenous, the equilibrium conditions must be extended to include the informational decisions. We refer to these as *informational equilibria.* Informational equilibrium requires that policyholders make the expected utility-maximizing decision regarding whether to become informed, given the contracts available. Equilibrium also requires that insurers' expectations regarding consumers' informational decisions are fulfilled. An important question is whether these informational equilibria are informationally productive. Proposition 2 implies that, if insurers expect consumers to remain uninformed and offer contracts based on policyholders' priors, then policyholders will become informed, and insurers' expectations will not be fulfilled. Then either the contracts will not earn zero expected profit, or at least some policyholders' expected utility can be increased. Thus, the contract set based on policyholders' priors is not an equilibrium contract set.

Proposition 2 further implies that, if insurers expect policyholders to become informed and offer contracts based on policyholders' posteriors, then their expectations will be fulfilled. The equilibrium contract set will be based on policyholders' posterior estimates of their loss probabilities. Since the information structure is noiseless, there are two risk classes if consumers learn their risk types. This has important implications for the pricing of insurance policies. If the contracts separate types and satisfy the self-selection constraints, then c_H will be bought only by objective type *H*s, and c_L will be bought only by objective type *L*s. If policyholders understand that policies earn zero expected profit, they can then deduce their loss probabilities. Under these conditions an information structure that reveals the actual probability of loss has no marginal value beyond an information structure that reveals type.

To see this let $C^* = \{c_H^*, c_L^*\}$ be the equilibrium policies for fully informed policyholders; these may be Nash, anticipatory, reactive, or MS equilibrium policies. $E_i\{\omega \mid y, C\}$ is the subjective estimate of the loss probability given the signal and the observed set of policies C. Suppose that Φ_1, an information structure that reveals loss type, but not the probability of loss, is costlessly available. Let Φ^* be a costless information structure that reveals the probability of loss.

Proposition 3: *If C^* is a separating equilibrium ($c_H^* \neq c_L^*$), the* (a) *the informational equilibrium contract set is C^*,* (b) $E_i\{\omega \mid y \in Y_{iH}, C^*\} = T_H$ *and* $E_i\{\omega \mid y \in Y_{iL}, C^*\} = T_L, i = H, L,$ *and* (c) $V_i(\Phi^*) - V_i(\Phi_1) = 0, i = H, L.$

Proof. As previously discussed, Proposition 2 implies the equilibrium contract set cannot be based on policyholders' priors. Now suppose insurers offer C^*. Proposition 2 also implies that policyholders will learn their risk types, consistent with insurers' expectations. Then c_H^* is bought only by objective type Hs, and c_L^* is bought only by objective type Ls.

At the Nash, reactive, and separating anticipatory equilibrium policies, each risk type yields zero expected profit, so $T_i = p_i/q_i$. For the MS equilibrium, the zero expected profit constraint is

$$N_H(p_H - T_H q_H) + N_L(p_L - T_L q_L) = 0. \tag{13}$$

The self-selection constraint (7) is binding at the MS equilibrium. Since type Hs receive full insurance, this self-selection constraint can be written as

$$(1-T_H)u(w-p_L)+T_H u(w-p_L-x-q_L) = u(w-p_H), \quad \text{or} \quad T_H\alpha = \beta, \quad (14)$$

where $\alpha = u(w-p_L)-u(w-p_L-x-q_L)$ and $\beta = u(w-p_L)-u(w-p_H)$. Solving (13) and (14) yields $T_H = \beta/\alpha$ and $T_L = [N_H(p_H-(\beta/\alpha)q_H)+N_L p_L]/N_L q_L$. Since the loss probabilities can be deduced from the terms of the policies, $E_i\{\omega \mid y_j, C^*\} = T_j$, for $i, j = H, L$. Since the loss probabilities are known, $V_i(\Phi^*) - V_i(\Phi_1) = 0$. □

Equilibrium in the case of perfectly informative signals involves rational expectations on the part of both insurers and consumers. The fixed set of policies and self-selection constraints create an incentive for policyholders to become informed. At the separating equilibrium C^*, policyholders with prior π_H have a strictly positive value of information. Since the self-selection constraint is binding, the value of information is zero for policyholders with prior π_L[8]. Insurers deduce the policyholders will become informed and offer the corresponding set of contracts. The constraints and the structure of the equilibrium contracts is such that policyholders can deduce the objective loss probabilities. (We show in the appendix that this is true for an arbitrary number of risk classes.) The resulting equilibrium is "as if" policyholders were initially endowed with knowledge of their loss probabilities. Hence, if a perfectly informative information structure is available, one can endogenously derive the usual assumption that policyholders know their true loss probabilities. Put differently, the equilibrium is fully informationally productive.

However, this is not true if the full information anticipatory or MS equilibrium C^* is a pooling equilibrium. In either case, $c_H^* = c_L^* = (\alpha T_A x, \alpha x)$, where $0 < \alpha < 1$ at the anticipatory equilibrium and $\alpha = 1$ at the MS equilibrium.

Corollary 2: *If C^* is a pooling equilibrium ($c_H^* = c_L^*$) and the information structure, Φ_1, noiselessly reveals type but not the loss probability, then, in general,* (a) *C^* is not an informational equilibrium contract set and* (b) $E_i\{\omega \mid y \in Y_{ij}, C^*\} \neq T_j, i, j = H, L$.

Proof. Again, Proposition 2 implies the equilibrium contract set cannot be based on policyholders' priors, and that, consistent with insurers' expectations, policyholders will learn their risk type at C^*. However, (7), hence (14), holds identically for all values of T_H. Given the signal and the observed contracts, the posterior subjective loss estimates must satisfy

$$N_H E_i\{\omega \mid y \in Y_{iH}, C^*\} + N_L E_i\{\omega \mid y \in Y_{iL}, C^*\} = p/q. \quad (15)$$

While these estimates are consistent with the pooling equilibrium at C^*, they need not equal (T_H, T_L).

The policy c_L^* provides coverage αx at the premium $\alpha T_A x$. Consider the policy obtained by reducing the premium for c_L^* by δ and reducing coverage by $\gamma\alpha x$—that is, $c_L^{**} = (\alpha T_A x - \delta, (1-\gamma)\alpha x)$. Then, for the set of contracts $\{c_H^*, c_L^{**}\}$, expected profit is

$$N_H(\alpha T_A x - T_H \alpha x) + N_L(\alpha T_A x - \delta - T_L(1-\gamma)\alpha x) = N_L\alpha[-\delta + T_L\gamma x], \quad (16)$$

which is nonnegative if $0 < \delta \leq T_L \gamma x$. Suppose that low risk policyholders underestimate their loss probabilities. Then (in (p, q)-space) the line through c_L^* with slope T_L cuts the low risk-indifference curve through c_L^* to the left of c_L^* [9]. Then we can find values of δ and γ such that low risks strictly prefer c_L^{**} to c_L^*. Since the high risk-indifference curve through $c_L^* (= c_H^*)$ is steeper than T_L, high risks strictly prefer c_H^* to c_L^{**}. The set of contracts $\{c_H^*, c_L^{**}\}$ satisfies the self-selection constraints and earns strictly positive expected profit. Thus, C^* cannot be an informational equilibrium contract set. □

The full information contracts C^* are subject to the self-selection and breakeven constraints, based on the objective loss probabilities. If C^* is a separating set of contracts, then policyholders can deduce the objective loss probabilities, and subjective and objective loss probabilities are the same. If C^* is a pooling set of contracts, then policyholders can no longer deduce the loss probabilities from the contracts, and, given an information structure that reveals type only, the subjective estimates of the loss probabilities will generally differ from objective probabilities. If low risks underestimate T_L, then they perceive that their expected utility is maximized at a lower premium and coverage than c_L^*. Then the difference between the subjective and objective loss probabilities can profitably be exploited, breaking the pooling equilibrium. This is not true if high risks underestimate their loss probability; in this case C^* is an informational anticipatory or MS equilibrium. We should point out that, if it exists, individuals choose to become informed at a pooling equilibrium, so that pooling equilibria are informationally productive, but not fully informationally productive.

We have assumed that individuals have partial prior information. We should point out that Propositions 2 and 3 do not depend on this assumption. If individuals have no prior information, then prior densities are the same, $\pi_H = \pi_L$. Then the value of information is strictly positive if all individuals underestimate T_H, $(E\{\omega\} \geq T_H)$; otherwise the value of information is zero.

4. Imperfectly informative signals

The assumptions that policyholders have access to perfect information or access to no information on their own risk characteristics seem equally unrealistic. A reasonable alternative is that policyholders may have access to "noisy" information; that is, there is a positive probability that they will receive the "wrong" signal. While it is still true that noisy information is valuable to policyholders, the market equilibrium is markedly different from the noiseless case.

An information structure with noise can be characterized by the conditional density of the signals, $f(y \mid \omega)$. This gives the probability of the signal, conditional on the true probability of loss[10]. Then, if the prior is $\pi(\omega)$, the posterior density of the loss probability, given the signal y is received, is

$$\pi(\omega \mid y) = \frac{f(y \mid \omega)\pi(\omega)}{\int_\Omega f(y \mid \hat{\omega})\pi(\hat{\omega})\, d\hat{\omega}}. \tag{17}$$

As before, a completely uninformed policyholder faces the decision problem (3), while observing the signal y leads to the decision problem (4); the ex ante expected utility of a noisy information structure is again given by (5).

In a market where all consumers have the same probability of loss, the analog of Proposition 1 holds for the noisy information case for essentially the same reasons. If consumers know that the market is competitive, the loss probability can be computed from the terms of the policy, and additional information has no value. If consumers do not know that the market is competitive, then additional noisy information is valuable since the consumer can make a better choice regarding his or her policy purchase.

When there are multiple risk classes and information structures are noisy, the concept of an information structure that reveals the individual's risk type or probability of loss is not well defined. In general, noisy information structures lead to a posterior density that may put positive probability on all possible values of the loss probability. Still, given reasonable assumptions regarding the state and signal space and unobservable informational status, a noisy information structure has positive value to policyholders and an informational equilibrium exists.

In order to demonstrate the value of information and the existence of equilibrium, we make some simplifying assumptions on the economic environment. We assume that the "state space" is $\Omega = \{T_H, T_L\}$ and the signal space is $Y = \{y_H, y_L\}$. An information structure is then characterized by ρ, the probability that the signal correctly indicates the consumer's loss probability; we assume $1/2 \leq \rho \leq 1$. The case $\rho = 1/2$ corresponds to an uninformative structure; correct and incorrect signals are equally likely. The case $\rho = 1$ corresponds to an noiseless or perfectly informative structure; the signal is always correct. Information structures with $1/2 < \rho < 1$ correspond to noisy information, with larger values of ρ representing better information. We let Φ_ρ denote the information structure with parameter ρ. We assume that Ω, Y, and ρ are common knowledge.

We retain the assumption that policyholders have prior densities $\pi_H(\omega)$ or $\pi_L(\omega)$. For individuals with prior density $\pi_i(\omega)$ who receive signal y_j, the posterior density is $\pi_i(\omega \mid y_j)$, and the posterior estimate of the loss probability is $s_{ij} = E_i\{\omega \mid y_j\}$. The prior probability of observing the signal y_j is Π_{ij}.

Provided consumers view insurers' policy offerings as fixed, a set of contracts $\{c_H, c_L\}$ based on policyholders' priors cannot be a rational expectations equilibrium in the above described economic environment.

Proposition 4: *For any fixed set of contracts $\{c_H, c_L\}$ such that the self-selection constraints hold, the noisy information structure has nonnegative value.*

Proof. Policyholders with prior loss densities $\pi_i(\omega)$ observing signal y_j form the expected loss probability $E_i\{\omega \mid y_j\}$ and will choose c_H or c_L as $E_i\{\omega \mid y_j\}$ is greater than or less than s_H^*. The former occurs with probability Π_{iH} and the later with probability Π_{iL}. Then the value of information is computed as in (10) to (12). □

Corollary 3: (a) *If $c_H = c_L$, then $I_H = I_L = 0$. Suppose $c_H \neq c_L$.* (b) *If $s_H \geq s_H^*$ and $s_{HL} < s_H^*$, then $I_H > 0$, while if $s_{HL} = s_H^*$, then $I_H = 0$.* (c) *If $s_L < s_H^*$ and $s_{LH} > s_H^*$, then $I_L > 0$, while if $s_{LH} = s_H^*$, then $I_L = 0$.*

Proof. By the same argument as Corollary 1. □

Hence, for essentially the same reasons as when information is noiseless, insurers should expect their policyholders to become informed and make their contract offerings accordingly. The nature of equilibrium in the noisy information case is substantially different from the equilibrium in the noiseless case.

We first consider the case where consumers have no partial prior information, so the prior densities are the same, $\pi_H = \pi_L$. The loss probability of individuals who observe the signal y_j is $\zeta_j = \rho T_j + (1-\rho)T_i$. We let C_ρ denote the MS equilibrium contract set when the information structure is Φ_ρ. We assume henceforth that C_ρ is a separating set of contracts. It should be clear from the discussions of Propositions 2 and 3 that the critical factor is that C_ρ is a separating set of contracts, rather than the use of the MS equilibrium concept. As in the case of noiseless information structures, equilibria may fail to exist if C_ρ is a set of pooling contracts.

Proposition 5: *If C_ρ is a separating set of contracts, then* (a) *an informational MS equilibrium exists, and* (b) $E\{\omega \mid y_j, C_\rho\} = \zeta_j$, $j = H, L$.

Proof. Proposition 4 implies that a contract set based on policyholders' priors cannot be an equilibrium contract set. Consider the contract set based on policyholders' posteriors. The MS contract set maximizes $v(c_L, \zeta_L)$, subject to the self-selection constraints (7) and (8) and the zero expected profit constraint $\sum_{j=H,L} n_j(p_j - \zeta_j q_j) = 0$. The existence of a solution is proved in Spence [1978]. Proposition 4 also implies that policyholders have an incentive to become informed at this contract set, consistent with insurers' expectations. The individual can solve the binding self-selection constraint and the zero profit constraint for the ζ_j. Since the group loss probabilities can be deduced from the terms of the policies, $E\{\omega \mid y_j, C_\rho\} = \zeta_j$ for $j = H, L$. □

In the case of no partial prior information, the value of information is given by (12), evaluated at the MS contract set C_ρ. Since the self-selection constraint (7) is binding, the value of information is zero in equilibrium. Since individuals choose to become informed, the equilibrium is informationally productive. The population loss probabilities, T_H and T_L, can be derived from the ζ_j. However, the equilibrium cannot be based on the T_j, since, due to the noise in the information structure, some individuals are misclassified. Thus, the equilibrium is not fully informationally productive unless ρ equals one. Crocker and Snow [1992] show that, when there is no prior partial information, the social value of additional private information is zero and the efficient contract involves no signaling (all individuals are pooled at $(T_A x, x)$). We also find that the social value of additional private information is zero. However, Propositions 3 and 5 imply that, when informational status is unobservable, the efficient contract typically will involve signaling.

We now turn to the case where individuals do have partial prior hidden information ($\pi_H \neq \pi_L$). In this market with imperfect information, there may be up to four distinct subjective risk types[11]. Unless priors are perfectly correlated with actual risk type or the signal is noiseless, each group will contain some true high risks and true low risks. Define $\eta_i = n_{ii}/n_i$ as the proportion of policyholders with prior i who are in fact type i. The signal y_i assigns an individual to the correct risk class with probability ρ. The proportion of high risks in each subjective probability class s_{ij} is then

$$\begin{aligned} H_{HH} &= \frac{\rho\eta_H}{\rho\eta_H + (1-\rho)(1-\eta_H)} \\ H_{HL} &= \frac{(1-\rho)\eta_H}{(1-\rho)\eta_H + \rho(1-\eta_H)} \\ H_{LH} &= \frac{\rho(1-\eta_L)}{(1-\rho)\eta_L + \rho(1-\eta_L)} \\ H_{LL} &= \frac{(1-\rho)(1-\eta_L)}{\rho\eta_L + (1-\rho)(1-\eta_L)}. \end{aligned} \tag{18}$$

Note that if the signal is informative ($\rho > .5$) and priors tend to be correct ($\eta_i > .5$), then $H_{HH} > H_{HL}$, $H_{LH} > H_{LL}$ and $H_{HH} > H_{LL}$. If the signal is informative ($\rho > .5$) and the high- and low-risk priors are not identical (that is, $s_{ii} \neq s_{ij}$), then $s_{HH} > s_{HL}$, $s_{LH} > s_{LL}$, and $s_{HH} > s_{LL}$. The remaining relations between proportions of high risks in the subjective risk classes and between the posterior estimates are ambiguous and depend on the structure and accuracy of priors and the informativeness of the signal.

If posterior means and proportions of objective high risks differ across subjective risk classes, there are still two possible cases:

Case	Subjective loss probability	Proportion of high risks
1	$s_{HH} > s_{LH} > s_{HL} > s_{LL}$	$H_{HH} > H_{LH} > H_{HL} > H_{LL}$
2	$s_{HH} > s_{HL} > s_{LH} > s_{LL}$	$H_{HH} > H_{HL} > H_{LH} > H_{LL}$

In either case, the contracts must satisfy the self-selection constraints:

$$v(c_{ij}, s_{ij}) \geq v(c_{hk}, s_{ij}), h, i, j, k = H, L, (i, j) \neq (h, k). \tag{19}$$

This allows us to state the following proposition:

Proposition 6: *For any set of contracts* $\{c_{HH}, c_{HL}, c_{LH}, c_{LL}\}$ *such that the self-selection constraints* (19) *hold, an information structure with noise has nonnegative value.*

Proof. Consider a policyholder with prior estimate of loss probability s_i, and suppose that, if uninformed, the policyholder would choose the policy c_{hk}. The value of a noisy information structure is

$$\begin{aligned} I_i &= \sum_{j=H,L} \Pi_{ij} v(c_{ij}, s_{ij}) - v(c_{hk}, s_i) \\ &= \left\{ \sum_{j=H,L} \Pi_{ij} v(c_{hk}, s_{ij}) - v(c_{hk}, s_i) \right\} + \sum_{j=H,L} \Pi_{ij}[v(c_{ij}, s_{ij}) - v(c_{hk}, s_{ij})]. \end{aligned} \tag{20}$$

The term in braces vanishes, using (19) and the fact that expected utility is linear in the probabilities. The self-selection constraints imply that each of the terms $v(c_{ij}, s_{ij}) - v(c_{hk}, s_{ij})$ in brackets is nonnegative. Therefore, the value of information is $I_i \geq 0$. □

Again, the fact that the set of policies is fixed and the self-selection constraints create an incentive for policyholders to become informed. As with Proposition 2, Propositions 4 and 6 do not assume any specific market environment or equilibrium concept. Also as with Proposition 2, the value of information depends on the specific set of contracts. Consider the MS equilibrium contracts, and suppose that the contracts are distinct. Then, for Case 1, the value of information is strictly positive, $I_H, I_L > 0$[12].

We now turn to the existence of equilibrium in an insurance market when policyholders have imperfect information. The nature of the equilibrium is different when information is noisy since the type i contract is not purchased exclusively by objective type i policyholders. The zero expected profit constraints then implies that the policies will not be priced at the objective loss probabilities. It follows that Proposition 3 cannot be extended to the case of noisy information; individuals' objective loss probabilities cannot be "backed out" of the policies. This implies that the equilibrium cannot be fully informationally productive. Let C_ρ denote the MS equilibrium contract set when the information structure is Φ_ρ, and let

$$\zeta_{ij} = H_{ij}T_H + (1 - H_{ij})T_L \tag{21}$$

be the objective loss probability for the group of individuals with prior π_i who receive signal y_j.

Proposition 7: *If C_ρ is a separating set of contracts, then* (a) *an informational MS equilibrium exists, and* (b) $E_i\{\omega \mid y_j, C_\rho\} = \zeta_{ij}, i, j = H, L$.

Proof. Proposition 4 implies that a contract set based on policyholders' priors cannot be an equilibrium contract set. Consider the contract set based on policyholder's posteriors. Let H_{ij} be the proportion of objective high risks with posterior s_{ij}. The MS contract set maximizes $v(c_{LL}, \zeta_{LL})$, subject to the self-selection constraints (19) and the zero expected profit constraint $\sum_{\forall ij} n_{ij}(p_{ij} - \zeta_{ij}q_{ij}) = 0$. The existence of a solution is proved in Spence [1978]. Proposition 6 implies that policyholders have an incentive to become informed at this contract set, consistent with insurers' expectations. The individual can solve the three binding self-selection constraints and the zero profit constraint for the ζ_{ij}. Since only the group loss probabilities can be deduced from the terms of the policies, $E_i\{\omega \mid y_j, C_\rho\} = \zeta_{ij}$, for $i, j = H, L$. □

From (18) and (21), each value of ρ leads to a unique value of the ζ_{ij}. Given the ζ_{ij}, the MS contract set is unique. Thus, each value of ρ yields a unique contract set C_ρ.

As in the case of noiseless information, consumers can extract the loss probabilities underlying the contracts, so we can take $s_{ij} = \zeta_{ij}$. Substituting (18) into (21), we see that, as ρ increases, ζ_{LH} increases from $(1 - \eta_L)T_H + \eta_L T_L$ to T_H, and ζ_{HL} decreases from $\eta_H T_H + (1 - \eta_H)T_L$ to T_L. Then $\zeta_{LH} < \zeta_{HL}$ for ρ sufficiently close to 1/2, so Case 2

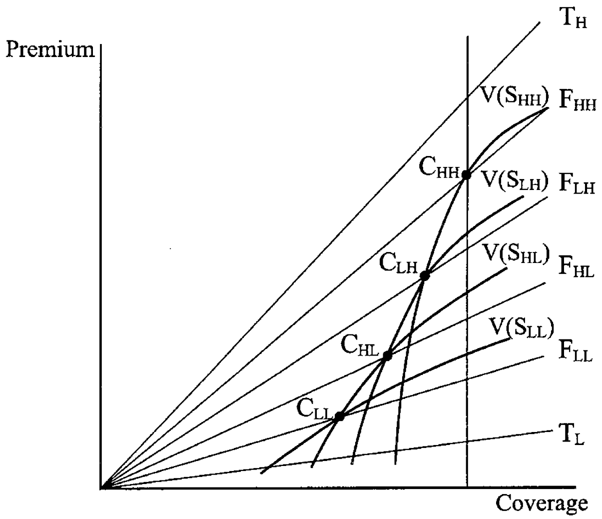

Figure 1. Separating equilibrium with noisy information.

holds, while $\zeta_{LH} > \zeta_{HL}$ for ρ sufficiently close to 1, so Case 1 holds. Then there is a value, say ρ', such that Case 2 holds for $\rho < \rho'$ and Case 1 holds for $\rho > \rho'$ in equilibrium.

Figure 1 shows a four-contract equilibrium with zero cross-subsidies. Individuals with high-risk priors who receive a high-risk signal purchase policy c_{HH} along fair odds line F_{HH}. Since $s_{HH} = \zeta_{HH}$, type HH individuals would purchase full coverage for a policy based on ζ_{HH}. Individuals with low-risk priors who receive high-risk signals purchase policy c_{LH} along fair odds line F_{LH}. Individuals with high-risk priors who receive low-risk signals purchase policy c_{HL} along fair odds line F_{HL}. Individuals with low-risk priors who receive low-risk signals purchase policy c_{LL} along fair odds line F_{LL}. Figure 1 is drawn under the assumption that Case 1 holds. For values of ρ such that Case 2 holds, the fair odds lines F_{HL} and F_{LH} would be interchanged.

The set of policies that comprise the MS equilibrium, such as the one illustrated in figure 1, depends on the informativeness of the information structure Φ_ρ to which consumers have access. For any $1/2 \leq \rho < 1$, the underlying group loss probabilities ζ_{ij} can be derived from the contracts. To obtain the underlying population loss probabilities T_H and T_L requires further solving (21). But, in general, this system of equations does not have a consistent solution. This leads to the following result:

Proposition 8: *If C^* is a separating set of contracts, then C^* is a fully informationally productive informational MS equilibrium if, and only if, $\rho = 1$.*

Proof. Equation (21) yields $\zeta_{iH} = T_H$ and $\zeta_{iL} = T_L$ if, and only if, $\rho = 1$. □

That is, the MS equilibrium is "as if" consumers were endowed with knowledge of their loss probabilities if, and only if, consumers have costless access to an information structure that noiselessly reveals their risk type. For any other value of ρ the equilibrium is informationally productive but not fully informationally productive.

Now consider how the value of information changes with ρ. Let $I_i(\rho) = V_i(\Phi_\rho) - V_i(\Phi_{1/2})$ denote the equilibrium value of the information structure Φ_ρ; this is given by (20) evaluated at C_ρ. We have $I_i(1/2) = 0$ for both subjective high and low risks. For any $1/2 < \rho \leq 1$, we have $I_H(\rho) > 0$, but no further restrictions can be placed on $I_H(\rho)$. From the discussions following Propositions 2, 6, and 8, we have that $I_L(\rho) = 0$ for $\rho < \rho'$, $I_L(\rho) > 0$ for $\rho > \rho'$, but $I_L(1) = 0$. This implies that $I_L(\rho)$ can be neither monotonically increasing nor concave in ρ.

The fact that $I_L(1) = 0$ implies that, if information is costly, then the MS equilibrium cannot be fully informationally productive and may not exist. At the equilibrium where contracts are based on the assumption that consumers are fully informed, subjective low risks will choose not to acquire costly information, and the equilibrium will fail to exist[13]. However, since $I_i(\rho) > 0$ for $\rho' < \rho < 1$, subjective low risks may be willing to acquire noisy information. That is, for at least some configurations of information costs, informationally productive equilibria may exist when fully informationally productive equilibria do not. More generally, proving existence of and characterizing equilibria when information is costly is likely to be difficult.

5. Conclusion

In this article we consider the possibility that policyholders may not be endowed with knowledge of their risk characteristics. We focus on the incentives of individuals to become informed and the implications of policyholders' information for insurance market equilibrium. We show that policyholders' information can have an important effect on the insurance market.

We show that, if consumers' informational status is unobservable, then information has nonnegative value to consumers. Further, this is true regardless of the market structure or the equilibrium concept. This extends the earlier results on the value of information of Crocker and Snow [1992], for the case of observable information status, and Doherty and Thistle [1996] for Nash equilibrium. Crocker and Snow show that the value of information may be positive or negative depending whether or not the benefits of improved screening outweigh the costs due to increased signaling. But if informational status is unobservable, the policies offered cannot depend on informational status. Then, when consumers become informed, signaling costs cannot be increased and screening is improved. This both creates a private incentive for individuals to become informed and implies that information is socially valuable.

We also show that with noiseless information, once consumers learn their objective type, they can deduce the objective loss probabilities from a separating equilibrium. This implies that information that reveals the individual's loss probabilities has no marginal value beyond information that reveals the individual's type. Put differently, given the distribution of risk types, there is a one-to-one relationship between the loss probabilities and the equilibrium contract set.

If information is noisy, then the MS equilibrium will be informationally productive but not fully so. The equilibrium set of policies will then depend on individuals' prior beliefs and the amount of noise in the information that they acquire. If information is costly, then equilibrium may not exist, and if it exists, it will not be fully informationally productive. The MS equilibrium is fully informationally productive, or "as if" consumers were endowed with knowledge of their risk characteristics, if, and only if, consumers have costless access to an information structure that perfectly reveals their risk type.

Appendix

We show that, if consumers know the risk class to which they belong, they can derive the underlying objective loss probabilities in the Nash, anticipatory, reactive, MS, and monopoly equilibrium for an arbitrary number of risk classes.

We assume there are n risk classes with objective loss probabilities $\theta_1 > \theta_2 > \cdots > \theta_n$. If the information structure is noiseless, the θ_i are the population loss probabilities T_i, and if the information structure is noisy, then θ_i are the objective loss probabilities ζ_{ij}, reindexed from highest to lowest. Let $c_i = (p_i, q_i)$ be the policy selected by risk class i. We assume the upward adjacent self-selection constraints are binding, so that $v(c_i, \theta_i) = v(c_{i+1}, \theta_i)$, $i = 1, 2, \ldots, n-1$; this is true at the all of the equilibria we are considering.

At the Nash, separating anticipatory, and reactive equilibria, each risk type yields zero expected profit, $p_i - \theta_i q_i = 0$, so that $\theta_i = p_i/q_i$, $i = 1, \ldots, n$.

Now consider pooling anticipatory equilibrium and MS equilibrium. As in (14) in the text, the self-selection constraints can be written as

$$\theta_i \alpha_i = \beta_i, i = 1, \ldots, n-1, \tag{22}$$

where $\beta_i = u(w - p_{i+1}) - u(w - p_i)$ and $\alpha_i = \beta_i + u(w - p_i - x + q_i) - u(w - p_{i+1} - x + q_{i+1})$. Then we have $\theta_i = \beta_i/\alpha_i$, $i = 1, \ldots, n-1$, so long as $c_i \neq c_{i+1}$. At an anticipatory equilibrium with two or more types pooled, each policy offered yields zero profit. Letting P be the set of types that are pooled ($i, j \in P$ iff $c_i = c_j$), the zero profit constraint for each policy is $\sum_{i \in P} N_i(p_i - \theta_i q_i) = 0$, where the N_i can be interpreted as either the number or proportion in risk class i. If two types, $i-1$ and i $(i < n)$ are pooled, then from the upward adjacent self-selection constraint we have $\theta_i = \beta_i/\alpha_i$, and θ_{i-1} can be found from the profit constraint. If three or more risk types are pooled, with type i the least risky in the pool ($P = \{i - j, \ldots, i\}$, $j \geq 2$, and $i < n$), then from the upward adjacent self-selection constraint we again have $\theta_i = \beta_i/\alpha_i$, but the profit constraint cannot be solved for the remaining θ's. If the pool contains the lowest risk type, type n, then, since there is no upward adjacent selection constraint, we cannot determine θ_n. In particular, if there are only two risk types, then we cannot solve for θ_1 and θ_2.

At the MS equilibrium, the set of policies offered must earn zero expected profit, but no individual policy need break even. The zero profit constraint is $\sum_{\forall i} N_i(p_i - \theta_i q_i) = 0$, where the summation runs over all types. We again have $\theta_i = \beta_i/\alpha_i$, $i = 1, \ldots, n-1$, so long as $c_i \neq c_{i+1}$. Then $\theta_n = [\sum^{n-1} N_i(p_i - (\beta_i/\alpha_i)q_i) + N_n p_n]/N_n q_n$. If the contracts are distinct, then all of the objective loss probabilities can be derived from the MS equilibrium. If risk classes $i - j$ through i are pooled, then we can solve for θ_i (unless $i = n$), but the other θ's are indeterminate. If there are only two risk types, then we cannot solve for θ_1 and θ_2 at a pooling MS equilibrium.

The signal extraction problem can also be solved by consumers if insurance is provided by a monopolist. We assume all n risk classes are served in equilibrium. The upward adjacent self-selection constraints are binding, so that (21) holds. The participation constraint for group n (the least risky) is also binding, $v(c_n, \theta_n) = v(0, \theta_n)$. This can also be written as (22) upon taking $p_{n+1} = 0$ and $q_{n+1} = 0$. Then we have $\theta_i = \beta_i/\alpha_i, i = 1, \ldots, n$. If the contracts are distinct, then all of the objective loss probabilities can be derived from the monopoly equilibrium. If risk classes $i-1$ and i are pooled, then we can solve for all of the probabilities except θ_{i-1}. This, combined with the arguments given in the proofs, implies that the analogues of Propositions 3, 5, 7, and 8 are valid at a monopoly equilibrium.

Acknowledgments

Dr. Ligon wishes to thank the College of Commerce and Business Administration and the Department of Economics, Finance, and Legal Studies of the University of Alabama for their financial support. The comments of participants at the 1992 meeting of the American Risk and Insurance Association and the 1993 meeting of the Risk Theory Seminar are appreciated. We would also like to thank Harris Schlesinger (the editor) and two anonymous referees for their helpful suggestions.

Notes

1. Schlesinger and Venezian ([1986], p. 229) make this point in a different context: "Insurers often have better information regarding loss exposures, however, than do those who are insured. The arguments that are put forth to justify the collection of industry statistics for ratemaking suggest that an enormous volume of data is required to provide accurate estimates of the probability and severity of loss".
2. The π_i could be derived, for example, if consumers initially had a common prior and then (as in the next section) observed a private binary signal that was imperfectly correlated with their type. (The common prior and the informativeness of the signal are common knowledge). Then the π_i could be computed from the common prior and the signal by Bayes rule. The proportion of subjective Hs (Ls) would be the probability of a "correct" signal. If the objective type distribution is common knowledge, the n_{ij} could also be computed. We simply assert the π_i exist (however derived) and, along with the type distributions, are common knowledge.
3. The number of subjective type i policyholders in $n_i = n_{ii} + n_{ij}$, while the number of objective type i policyholders is $N_i = n_{ii} + n_{ji}$.
4. Adverse-selection problems are modeled as games of incomplete information in which nature assigns individuals' risk types and loss probabilities. In standard adverse-selection models, the population loss probabilities are assumed to be common knowledge, and individuals observe nature's move but insurers do not. Also, the analysis can be generalized to multiple-risk classes (see, for example, Spence [1978]); however, the two-risk-class case highlights the analytical issues that are of interest.
5. If the objective loss probabilities T_H and T_L are common knowledge, then an information structure that reveals type also reveals the loss probability.
6. See Rothschild and Stiglitz [1976] and Spence [1978] on the Nash and MS equilibria. See Stiglitz [1977] for a discussion on monopoly insurance markets.
7. The anticipatory and reactive equilibria differ when the Rothschild-Stiglitz condition does not hold. Under the anticipatory equilibrium firms considering introducing a new policy assume policies rendered unprofitable by the new policy will be withdrawn. Under the reactive equilibrium, firms considering introducing a new policy assume rivals will react by introducing additional policies if doing so is strictly profitable. If the withdrawal or introduction, respectively, of policies would render the contemplated new policy unprofitable, it is not introduced. Wilson [1977] proves an anticipatory equilibrium always exists. Riley [1979, 1985] and Engers and Fernandez [1987] prove a reactive equilibrium always exists and is unique.

8. The existence of the informational equilibrium at C^* clearly depends on our assumptions that information is costless and that policyholders choose to become informed if the value of information is zero. If, in the alternative, we assume that policyholders do not to become informed when the value of information is zero, or if we assume that information has a strictly positive cost (however small), then an informational equilibrium may fail to exist, or, if equilibrium exists, then some policyholders remain uninformed.
9. That is, at c_L^*, the slope of the low risk's indifference curve is $E_i\{\omega \mid y \in Y_{iL}, C^*\}$, which is less than T_L by assumption.
10. An information structure without noise, such as those analyzed in the previous section, is the special case where the conditional density of the signal, $f(y \mid \omega)$, is degenerate.
11. There will be fewer than four classes if subjective risk groups have equal posterior means or equal proportions of objective high-risk individuals.
12. To see this substitute the appropriate values in (20). If two or more groups are pooled then the value of information may be zero.
13. This problem is related to Grossman-Stiglitz's [1980] conclusion on the impossibility of informationally efficient markets. A similar existence problem is also discussed in Doherty and Thistle [1996].

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