TROJAN-TYPE ORBITS IN THE SYSTEM OF TWO GRAVITATIONAL CENTERS WITH VARIABLE MASSES AND SEPARATION

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Abstract. Trojan type orbits in the system of two gravitational centers with variable separation are studied within the framework of the restricted problem of three bodies. The backward numerical integration of the equations of motion of the bodies starting in the triangular libration points L_4 and L_5 (reverse problem) finds the breakdown of librations as the separation decreases because of the mass gain of the smaller component and an approach of the body of negligible mass to the latter up to the distance below its sphere of action with a relative velocity approximately equal to the escape one on this sphere. The breakdown of librations about L_5 occurs under the mass gain of the smaller component considerably larger than in the case of L_4 and implications are made for the asymmetry of the number of librators about L_4 and L_5 in the solar system (Greeks and Trojans). Other parameters of the libration motion near 1/1 commensurability are obtained, namely, the asymmetry of the libration amplitudes about the triangular points as well as the values of periods and amplitudes within the limits of those for real Trojan asteroids. Trojans could be supposedly formed inside the Proto-jupiter and escape during its intensive mass loss.

1. Introduction

In the present work we treat the possibility of formation of Trojan orbits in a system of two gravitational centers with variable separation. We have already studied the motion of a large number of small bodies in this system by a computer simulation within the framework of the planar restricted three-body problem in order to study the process of formation of the inner planets of the solar system and of the asteroidal belt structure (Djakov and Reznikov, 1980, 1981; hereafter referred to as Papers 1 and 2, respectively).

It was found that for the mass ratio of centers $C1$ and $C2$ of the order of several thousands ($M_{C1}/(M_{C1}+M_{C2}) = \mu \sim 10^{-3}$) the bodies of small mass $m \ll M_{C1}$ formed according to the model (Paper 1) inside the Roche lobe of $C1$ leave the latter during its mass loss and further revolve about $C2$ giving rise to a regular structure of their mean motion distribution in a form of the alternating gaps and clusters in radial direction. In more details this process was studied in a region corresponding to the asteroid belt of the solar system (Paper 2) where it was obtained both the main gaps and the cluster of the bodies near the commensurability 3/2 of their mean motion and that of $C1$. The initial conditions of their motion considered in Papers 1 and 2, i.e. those of an escape from the Roche lobe of $C1$ through the first Lagrangian point, gave rise to the orbits completely inside $C1$'s orbit. Other conditions leading to the trajectories partially or completely outside were not considered.

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It was supposed in Paper 2 that among those could be orbits with $C1$'s mean motion, i.e. those of Trojan type of the Sun-Jupiter system. The examination of this assumption by the computer simulation is a goal of the present study.

This work is connected in part with Kuiper's idea (Kuiper, 1951) of Trojans as former satellites of Jupiter under condition of considerable mass loss from the latter. This problem was studied analytically by Rabe (1954) and numerically by Horedt (1974). We ought to emphasize, however, the main difference between that idea and our model including both the mass loss of $C1$ and the related variable separation which has not yet been adapted to the problem of Trojan origin.

In order to maintain, in principle, the possibility of formation of the Trojan orbits in such a system and to obviate the enormous difficulties arising due to uncertainties in the starting conditions inside the Roche lobe of $C1$ we seek for solution of an inverse problem – i.e., we integrate backward the equations of motion of the bodies originally in the orbits at 1/1 commensurability with the mean motion of $C1$. A similar approach which exploits the reversibility of the equations of motion has been already used by Horedt (1974) mentioned above. Note that he has obtained the negative result even for a fast mass increase by the factor 20, while Rabe (1954) has used only restricted analytical approximation. Hence, the Trojan problem as former satellites of Jupiter is open (Yoder, 1979).

To investigate this problem as the inverse one we suppose that it can be reduced to the elucidation of the possibility and the conditions of breakdown of libration motion which leads to a close encounter of m with $C1$ favorable to subsequent capture. A positive criterion of this event within the framework of the model under study will be considered below.

2. Statement of the Problem

The extreme case of the problem of the motion of a body of negligible mass in the commensurability 1/1 with $C1$ may be presented as the next. The initial state of the system into consideration is that of a constant mass ratio of the centers $C1$ and $C2$ and a constant circular orbit of $C1$ about the coordinate origin taken in $C2$. At the beginning the body of negligible mass m is in one of the triangular Lagrangian points L_4 or L_5 and its velocity is that of the circular motion of $C1$. The mass and separation variations are now introduced. Under the total mass $M = M_{C1} + M_{C2}$ and the angular momentum conservation their values satisfy the relation (see Papers 1, 2)

$$a = a_o(\mu_o/\mu)^2 = a_f(\mu_f/\mu)^2, \tag{1}$$

where subscript f refers to the steady state and subscript o refers to the initial state in the reverse problem which may not generally coincide with the steady one ($\mu_f \neq \mu_o$) when μ_o is taken from an interval $[\mu_f, \mu_*)$ where μ_* is the limiting value in the model considered. According to our previous results this value does not exceed several μ_f (Paper 1) and hereafter it is set equal to $3\mu_f$.

For the Eddington-Jeans law of mass loss

$$d\mu/dt \sim \mu^{(4+3/\alpha)} \tag{2}$$

the relative motion of $C1$ about $C2$ can be presented by a spiral

$$a = a_o(1 + C\phi)^{2\alpha/3}; \qquad \phi = (1/C)((\mu_o/\mu)^{3/\alpha} - 1); \tag{3}$$

where ϕ is a polar angle between the radius-vector of $C1$ and the abscissa and the parameters C and α are determined by a mass loss rate, $(dM_{C1}/dt)_o$, for μ_o, i.e. for the system in the initial state, and the time for a mass loss from μ_o to μ_f (cf. Paper 1). If the characteristic values of the steady-state parameters are taken as those of the Sun-Jupiter system ($a_f = 5.2$ AU, $\mu_f = 0.001$) then for $\alpha = 1$ the law of mass loss becomes

$$dM_{C1}/dt = -2.7 \times 10^{-5}(1/\nu)(\mu/\mu_f)^7 M_\odot \text{ yr}^{-1}, \tag{4}$$

where the parameter ν determines the time of the mass loss, t_f, from μ_* to μ_f and is given in revolutions of a spiral trajectory of $C1$, $t_f \approx \nu \cdot T_f/2$ (T_f is a period of $C1$ about $C2$ in the steady state). For $\nu = 300$ the initial rate of mass variation is approximately $10^{-7} M_\odot \text{ yr}^{-1}$ and the total time of mass loss is about 2000 yrs.

The equation of motion of m in the coordinate frame with the origin in $C2$ becomes

$$d^2\mathbf{r}/dt^2 = -(M_{C2} + m)(\mathbf{r}/r^3) - M_{C1}((\mathbf{r} - \mathbf{a})/\Delta^3 + \mathbf{a}/a^3), \tag{5}$$

where

$$\begin{aligned} r &= (x^2 + y^2)^{1/2} & \Delta &= ((x - x_{C1})^2 + (y - y_{C1})^2)^{1/2} \\ x_{C1} &= a\cos\phi, & y_{C1} &= a\sin\phi \end{aligned} \tag{6}$$

The initial conditions for $C1$ are determined by $\mu(0) = \mu_o$ then $\phi(0) = 0, x_{C1}(0) = a_o$, $y_{C1}(0) = 0$. As for m its starting position and velocity components for almost circular orbits are determined by two parameters; namely, the radius-vector modulus $r(0) = a_o/k$ (k is about unity) and the initial phase angle θ – i.e., the angle between the radius-vector of m and the positive direction of the abscissa.

To investigate the evolution orbits of small bodies we have to formulate a criterion which, being accomplished, would signify that a body in Trojan orbit in a system with $\mu_f \leqslant \mu \leqslant \mu_o$ has its origin into the Roche lobe of $C1$. This criterion would have a necessary but no sufficient nature within the domain of celestial mechanics. However, it should ensure, first, that a close approach of m to $C1$ would be obtained due to the variation both of μ and $C1$–$C2$ separation exclusively; and, secondly, that this approach would proceed along a trajectory favorable for a subsequent capture. Consider these two requirements in detail.

It is known that the real Trojans execute appreciable librations about either L_4 or L_5 and their trajectories do not differ qualitatively from the periodic orbits of commensurability 1/1 in the restricted problem of three bodies. Those have been studied by Rabe (1961) who has shown that inside a certain distance from the libration point (as $k \neq 1$) the starting position on the line passing through the coordinate origin and L_4 or L_5 gives rise to the tadpole-type orbit and an increase of k leads to a horseshoe orbit. Investigating the problem of the Earth's Trojans, Weissman and Wetherill (1974) have demonstrated that, for a further increase of k, the turning point of the head part of horseshoe orbit drops into the sphere of influence of $C1$; the orbital eccentricity increases; and a collision with $C1$ or an ejection from the system are virtually assured. Thus the closest approach can be obtained by means of the starting conditions only without any changes in mass of $C1$ and its separation.

It follows that the first general requirement of the above-mentioned criterion must include the initial values of k and θ well inside the limits of the periodic tadpole orbits of the system in steady state. It is worth mentioning that the instantaneous position of libration points does not coincide with that of the restricted circular problem as the motion of $C1$ proceeds along a spiral trajectory. However, this difference scaled by a does not exceed $2e$ (Rjabov, 1956) where e is orbital eccentricity of $C1$ given by $e \approx 0.1(1/\nu)(\mu/\mu_f)^3$ for $\mu < \mu_*$ and, thus, is negligible if the maximum value of k is equal to 1.052 which is the boundary value between tadpole and horseshoe orbits according to Rabe (1961). Hereafter, with no special mention, m starts at the positions of the circular libration points L_4 or L_5 $(k = 1)$.

The second part of our criterion constrains the approach trajectories of m. A capture of m in any satellite orbit of $C1$ in the reverse problem apparently is not real with no dissipative processes inside the Roche lobe of $C1$ – e.g., the gas drag or dynamic interactions with other small bodies already existing there. Moreover, within a certain limiting distance from a point mass the reverse problem does not appear to be equivalent to the direct one as $C1$-m interaction can not be described in a scope of a purely gravitational model. A temporary capture is not to be excluded as $C1$ gains its mass. However, any capture model would be occasional because of the limits of mass gain.

Hence, there is nothing for us left but to expose those cases of m's penetration into the Roche lobe of $C1$ which could be favorable for a subsequent capture. We assign to those the trajectories penetrating into the sphere of action of $C1$ of radius $R_a = a\mu^{0.4}$ on which surface the relative velocity of m would be less than the escape velocity

$$v_e = (2M_{C1}/R_a)^{1/2} = 2^{1/2} \times \mu^{0.3} V_{kC1}$$

(V_{kC1} is the Keplerian velocity of circular orbit around $C2$).

The choice of the sphere of action is due to the smallness of perturbations on the motion of m both from $C2$ inside this sphere and pointless mass of $C1$. Thus, if a body of negligible mass starting in the Trojan orbit satisfies the above-mentioned criterion while μ increases and $C1$–$C2$ separation decreases it is considered a virtual Trojan in the direct problem ($\mathrm{d}M_{C1} < 0$) within the framework of the model of two gravitational centers with variable separation.

3. Results of the Computer Simulations

The equation of motion (5) was integrated by the computation method combining the explicit two-step scheme and the implicit one of the second order (Potter, 1973). The

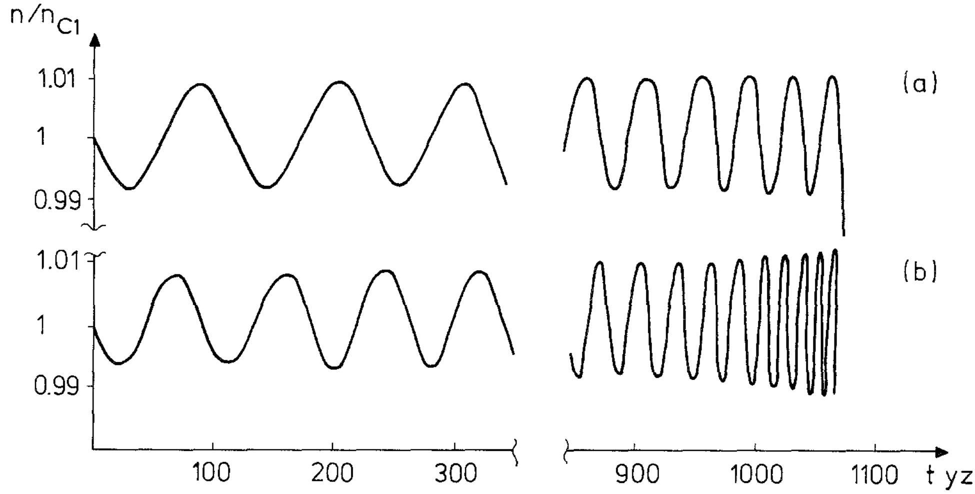

Fig. 1. Ratio of the mean motions of the librator and $C1$ as a function of time ($\mu_o = 1.0909\mu_f$, $\nu = 300, k = 1, \theta = 60°$): (a) m starts in L_4; (b) m starts in L_5.

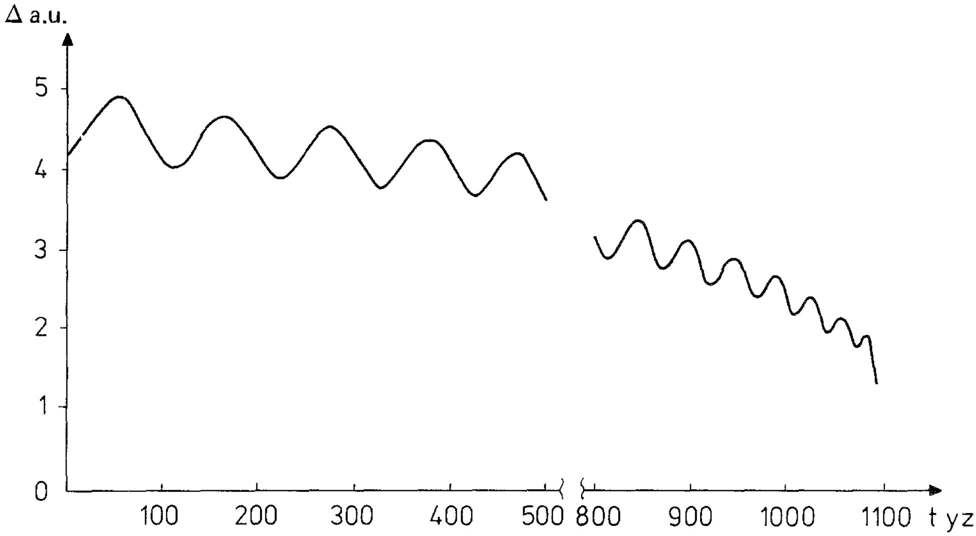

Fig. 2. Distance between the librator and $C1$ as a function of time ($\mu_o = 1.0909\mu_f$, $\nu = 300, k = 1$, $\theta = 60°$). Starting position coincides with L_4.

maximum time step-size was chosen to ensure adequate conservation of energy and momentum of the body m to a relative tolerance of 10^{-3}–10^{-4} for the period $\Delta t =$ 1000 yrs in the backward and forward integrations. Typical results for the body starting in L_4 and L_5 are presented in Figures 1 and 2. It can be seen that the body m maintains the librations as the mass of $C1$ increases and its separation with $C2$ decreases. The libration frequency increases as the period of $C1$ decreases and is different for the bodies starting from L_4 and L_5.

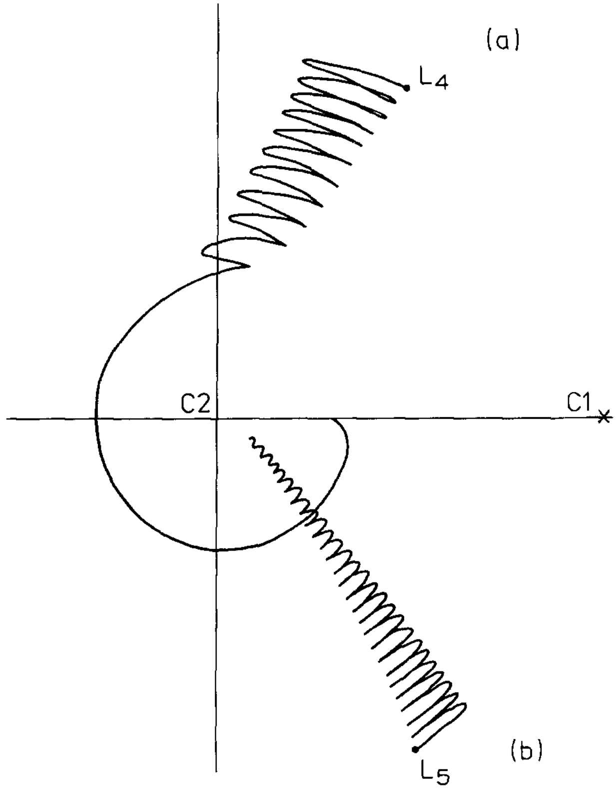

Fig. 3. Motion of small body in the coordinate frame rotating with $C1$ ($\mu_o = 1.009\mu_f$, $\nu = 200$): (a) m starts in L_4; (b) m starts in L_5.

On the contrary, the libration amplitude changes extremely slow and does not practically differ for both types of starting positions. At the same time, the distance between m and $C1$ at the moments of closest approach decreases as the libration frequency becomes greater. Figure 3 gives the additional data on the motion of m in the rotational frame of reference related with $C1$. This frame is not scaled by the separation value; hence, the motion of $C1$ proceeds along the abscissa to the origin of the coordinates. In this figure one can see that the trajectory of the body having its origin in L_4 looses and takes the form of the tadpole tail lengthened in the azimuthal direction and displaced to the coordinate origin in such a way that the turning points of the orbit move away from $C1$. Such a shape of trajectory arises because of the separation and mass variations unlike the case of the mass variation only (dotted line in Figure 4).

Any further removal of the outermost point of the orbit transforms it to horseshoe-type about both libration points. The situation is very similar to the case of the constant mass ratio and separation but with no shift of the starting position from the libration point as in Rabe (1961). In our experiment the following increase of μ and decrease of

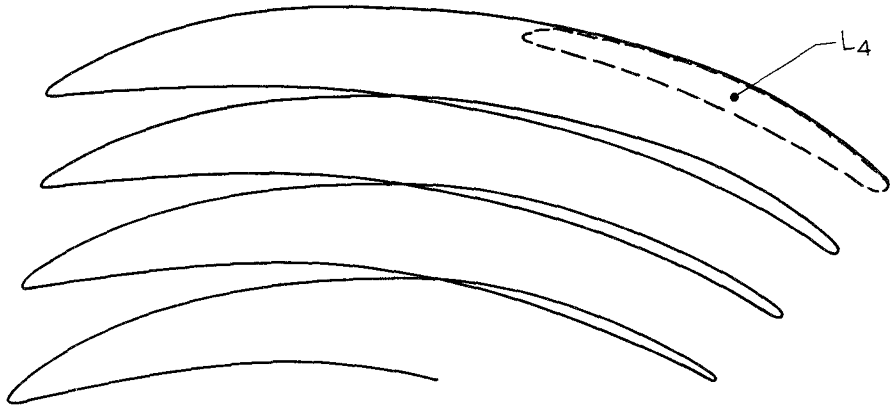

Fig. 4. Comparison of the orbits for a constant (broken line) and variable (solid line) separation ($\mu_o = 1.3636\mu_f$, $\nu = 300$, $k = 1$, $\theta = 50°$). For a constant separation the time interval shown is of 6 periods of libration when $C1$'s mass is increased in 6 times.

a leads to a close encounter with $C1$ in such a way that the osculating value of semimajor axis becomes greater than that of $C1$.

After the libration breakdown $C1$, moving in the inner orbit, reduces a phase difference with respect to m within a single revolution (Figure 5). On approaching $C1$ brakes m, diminishing the angular momentum of the latter to yield a decline of its semimajor axis and rise of the eccentricity. As a result, their orbits intersect and m drops into the sphere of action of $C1$ along the parabolic trajectory with a relative velocity equal to 0.8 of the escape one on the surface of this sphere. At this moment the mass of $C1$ is $2.15\mu_f$ and mass gain rate is $3.7 \times 10^{-5} M_\odot \text{ yr}^{-1}$. With μ_o changed in the computer simulation these values of $C1$'s mass (hereafter called critical mass μ_c) and entrance velocity do not vary considerably: e.g., the latter is equal to 0.7–0.85 of the escape one in all the simulation runs.

The critical mass can depend on the mass gain rate, the former diminishing with an increase of the latter. For example in other computations repeated under the same conditions save ν value equal to 200 or 100 (instead of 300) the critical mass turns to be $2.0\mu_f$ and $1.48\mu_f$, respectively.

Rather different is the trajectory of m originating in L_5 (Figure 3b). In this case it becomes a head of tadpole moving to the coordinate origin. From Rabe (1961) and Weissman and Wetherill (1974) we know a considerably lower sensibility of the orbit head to any perturbation in comparison with its tail. This property was encountered also in our simulations and explains the fact that an entry of the forward turning point of the orbit into the sphere of action of $C1$ proceeds under considerably greater critical mass.

In most of the runs the breakdown of librations of the body starting in L_5 occurs with the mass greater than the limiting value μ_* by which we constrain the applicability of the model. Only with the more considerable shift of the starting position from L_5 one

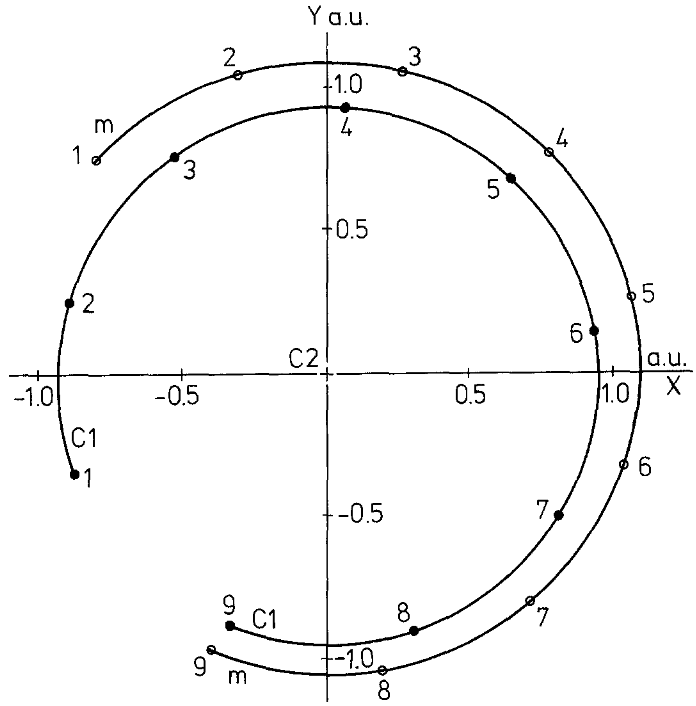

Fig. 5. Approach trajectories of m and $C1$ after breakdown of the librations (• indicates position of $C1$, ○ indicates position of m). Dots mark every 0.1 yr. The least corresponds to the distance of $0.93\ R_a$ (R_a is a radius of sphere of action).

would expect to enter into the sphere of action of $C1$. Hence it seems that the orbits about L_5 turn to be more stable in the model presented. The significance of this result compared with the different numbers of asteroids around L_4 and L_5 will be discussed below.

The computer runs under initial conditions of the extreme case mentioned above demonstrate a possibility of the penetration of m into the sphere of action of $C1$ from the most stable positions (inverse problem). However, those do not give rise to the libration motion the steady state of the system being achieved as m is in the triangular point at $\mu = \mu_f$. On the contrary, if one sets $\mu_o \neq \mu_f$ some libration parameters of interest – first of all, the asymmetry of libration amplitude (it seems to be reduced in $C1$'s direction from the equilibrium point compared with the opposite one) – would be obtained by the forward computation either in the interval $\mu_o \geqslant \mu \geqslant \mu_f$ and several libration periods in addition at the steady state or in the interval $\mu_c \geqslant \mu \geqslant \mu_f$, i.e. from the starting position on the surface of the sphere of action obtained previously in the backward computation.

The procedure is quite a natural one as there is not any prescribed value of μ_o in the model and the backward integration with any μ_o gives almost the same values both of μ_c and the relative velocity of m on the sphere of action of $C1$.

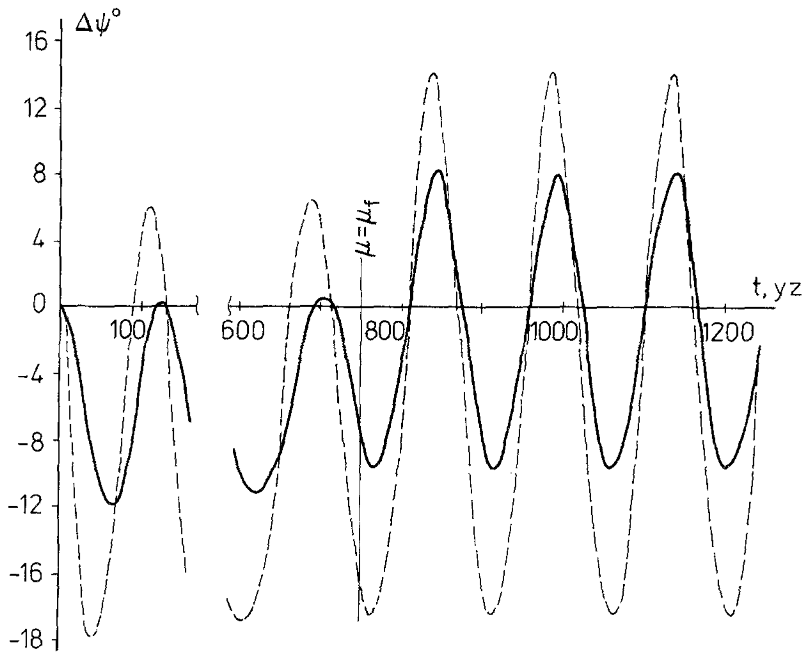

Fig. 6. Direct problem. The libration variable as a function of time ($\mu_o = 1.0909\mu_f$, $\nu = 300$, $\theta = 60°$). Solid line is for $k = 1$, broken line is for $k = 1.01$.

Figure 6 shows the libration variable $\Delta\psi = 60°$-(arg(m)-arg($C1$)) as a function of time for $k = 1$ and $k = 1.01$. Here the origin of the time axis corresponding to $\mu = \mu_o$ is shifted approximately at 1200 yrs from the moment of m's ejection from the sphere of action. One can see a drift of the maximum value of $\Delta\psi$ into the positive region at the subsequent librations as μ diminishes from μ_o to μ_f and then a rather abrupt enhancement up to the constant amplitude due to the halt of unwinding motion of $C1$. Figure 6 shows an obvious asymmetry of the libration with a period of 144 yrs determined by the steady state parameters μ_f and a_f. As for the magnitudes of libration amplitudes and their asymmetry those are relatively small due to the negligible eccentricity ($e \sim 0.001$) of the orbits under consideration depending on the initial conditions and the parameters of the mass loss law.

Note that the values of period and amplitude are nearly constant (the latter is within $0\overset{\circ}{.}01$ accuracy) during the computation including about 20 libration periods in the steady state.

4. Discussion of the Results

To estimate the results obtained for the cosmogonic implications one needs to consider first the significance of the 'capture' criterion for small bodies starting as librators in the reverse problem. In the forward computation this criterion has to be related with the processes which would transform the satellite orbits about the center of $C1$ to the escape ones.

The possibility of such processes can be qualitatively revealed as follows. Inside the Roche lobe of $C1$ any particle of small mass m could undergo the closest approach to the core of $C1$ in the peri-apsis of its orbit either as a result of an interaction with other similar bodies the number of which could be large enough according to the model or because of a large inclination to the plane of rotation of $C1$ when the particle's orbit could be transformed in an elongated ellipse within a time-interval much shorter than the evolution of the system.

Analogous process has been studied by Lidov (1961) in the Earth-its satellite-the Sun system.

Assumed mechanisms of the escape of small bodies from the Roche lobe of $C1$ are much more complicated in comparison with that through the inner Lagrangian point L_1 without any additional energy (Papers 1 and 2). The ejection of body was simply due to mass loss of $C1$ and the unwinding of the satellite orbits into an instability region nearest to the Roche limits.

Both possibilities would be realized within the framework of the restricted problem but it ought to consider also the nonzero probability of the 'catastrophic' events inside the Roche lobe of $C1$, e.g. the collisions of the small bodies followed by a scattering of the fragments some of them being supplied by additional kinetic energy necessary for an ejection from the sphere of action. For all the mechanisms a favorable factor is to be the mass loss of $C1$. Its value at the moment of an escape from the satellite orbit (critical mass) could vary with the mass loss rate but according to the numerical experiments it would be rather less than $\mu_* \sim 3\mu_f$ – i.e., within constraints of our previous simulations of the terrestrial planet formation (Paper 1). Thus the results of the present study fit well in a mutual evolution of the system into consideration which includes now both the inner planets and the asteroid belt structure with Trojan group.

The data of computer simulation can be directly compared to the real Trojans in some more aspects. First, we should point out the asymmetry of Trojan population around L_4 and L_5 which has found no explanation so far. Actually it is impossible to prefer any triangular libration point on purely dynamical ground in case of Jupiter's circular orbit even with a considerable mass loss. Meanwhile, according to Yoder (1979; and references quoted there) there seem to be three times as many Trojans at L_4 than at L_5; and this calls for an explanation within any theory of Trojan origin either on the cosmological ground, or because of the dynamical effects during their evolution.

According to Everhart (1973) who has calculated a great number of orbits near 1/1 commensurability any triangular libration point has no visible priority either because of an exchange between groups, or the libration breakdown (e.g., under the influence of Saturn).

At the same time our computations reveal that the asymmetry of Trojan population could be determined by the properties of the libration motion of small bodies in a system

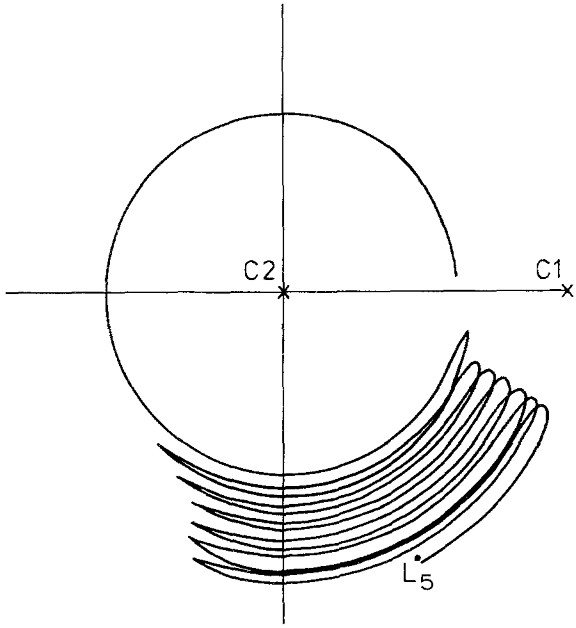

Fig. 7. Capture event from the origin in L_5 due to a shift of the starting position beyond the region of periodic tadpole type orbits ($\mu_o = 1.3636\mu_f$, $\nu = 300$, $k = 1.0533$, $\theta = 60°$). Critical mass $\mu_c = 1.72\mu_f$.

with variable separation (Figure 3), the symmetry breaking being in favor of L_4. Naturally, this result is valid under the starting conditions considered in the inverse problem. Beyond certain limits one would also obtain a 'capture' from L_5 as a mirror image of that from L_4 (Figure 7). In Figure 7 it is seen that this event becomes possible when a starting orbit is of full tadpole type about L_5. However, in this case $k = 1.0533$ – i.e., exceeds the limiting value $k = 1.052$ which according to Rabe (1961) restricts the existence of tadpole periodic orbit in the Sun–Jupiter system. It is advisable to consider our results to be in favor either of the greater possibility of an escape in libration orbit about L_4 or the greater stability of motion about L_5 under variable $C1 - C2$ separation.

Within the framework of our model other peculiarities of Trojan orbits may be explained: namely, a restriction of libration amplitudes compared with the maximum allowed (Yoder, 1979) and their asymmetry. If the body m is in a triangular point or its nearest region before the steady state realize the following decrease of $C1$'s mass will permit m to be in a closer approach to $C1$ during each subsequent libration which amplitude to $C1$ direction will be permanently smaller than to the opposite one, regardless of the m's position in the orbit at the moment of the end of the mass loss.

One could relate the problem of Trojan origin with that of the unique large eccentricity and inclination of Pallas orbit; those seem to exclude any possibility of accumulation of such a large body in the unperturbed solar nebula (Whipple *et al.*, 1972). On the assumption that only its closest approach to a much more massive body could give rise to a similar orbit, the authors of the work just mentioned have tried to correlate the present orbital elements of Pallas with its eventual approach to Jupiter located at 5.2 AU from the Sun; but their solution has been negative.

Within the framework of our model, their idea becomes more plausible if one assumes that this event could happen when the Protojupiter was more close to the Sun – i.e., Pallas – might be in close relation with Trojans (it is strengthened by the similarity of observed photometry). A small shift in its starting conditions has not however, resulted, in a capture in 1/1 resonance, but in the close encounter with the Proto-jupiter; Pallas having gained a large eccentricity and inclination and remained in an inner orbit.

To conclude, we wish to emphasize that the mechanism considered here cannot be regarded as unique for an explanation of the afore-mentioned parameters of Trojan motion, because of limits of the 'capture' criterion and certain simplification of the problem. However, parameters obtained in our computer simulations were not included in the initial conditions and could be considered as the corollaries from the model under study.

5. Conclusions

The study of the evolution of Trojan type orbits in the scope of the model of two gravitational centers with variable separation reveals a possibility that they could have originated as former satellites of Jupiter. This result, together with those of Papers 1 and 2 on the computer simulation of formation of the inner planets of the solar system and the origin of the asteroid belt structure, completes mainly the description of formation of this part of the solar system. The integrity of data obtained indicates on the fruitful approach to this complicated problem within the domain of celestial mechanics and on the effectiveness of pure gravitational processes to explain many features of the present resonant structure both of asteroid belt and interplanetary spacings.

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