THE DEPENDENCE OF ASTEROID LIGHTCURVES ON THE ORIENTATION PARAMETERS AND THE SHAPES OF ASTEROIDS

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Abstract. The dependence of asteroidal light curves has been derived from the obliquity, aspect and phase angles. The effects of an angle variation are discussed taking into account the possible geometry of an asteroidal body.

The amplitude-aspect relation is discussed for different asteroidal shapes.

On the basis of this relation a graphical attempt to determine the value of the aspect from the light curves' amplitude is described.

1. Introduction

All the available information about the nature of the asteroids is stored in their light curves. The lack of imagery from spacecraft does not allow a direct analysis of the morphological and dynamical characteristics of the asteroids. An attempt in understanding the more important characteristics of these bodies has been undertaken by Dunlap (1971) with laboratory work on the shapes of asteroids.

The simulation carried out by Dunlap on 12 different rotating models gave the idea that an experimental approach could improve the knowledge of the role of geometrical and dynamical parameters in shaping the light curves of the asteroids. A numerical simulation has been carried out by Surdey and Surdey (1978). They simulated the rotation of an asteroid by a tri-axis ellipsoid model obtaining the asteroid's light curves which were then compared with Dunlap's laboratory results. The physical properties of the asteroids have been discussed by many authors (Veverka, 1981; Irvine, 1966). A method for determining the orientation of the asteroid's pole has been suggested by many authors. A review of this subject referring to the works until 1971 is given by Veselý (1971), while the most recent contributions are given by Taylor (1979) and Zappalà (1980).

This paper deals with the dependence of asteroidal light curves on the angular parameters of the asteroids. The authors describe the behaviour of the asteroid light curves while the values of the angular parameters, which characterize the orientation of an asteroid with respect to the observer, change. In fact, in literature works non-realistic values for the angular parameters in describing the variation of the light curves were chosen.

In this paper an analytical expression for the magnitude as a function of obliquity, aspect and phase angle, is derived for a complete rotational period of a regular shaped body.

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2. The Method

The light curve of an asteroid is the variation of the magnitude due to geometrical and physical effects. Geometrical effects depend on the shape of the asteroidal body and its orientation. Physical effects depend on the composition of the material and the surface morphology of the asteroid. In this work the physical effects are not taken into account. A general three-axes ellipsoid model in describing the shape of an asteroid is assumed.

The orientation of an asteroid with respect to the observer is defined by three parameters: the aspect angle, the obliquity angle and the rotation. The rotation is assumed to occur around the shortest axis of the ellipsoid and is fully described by an angle R as a function of time. The aspect angle is defined as the angle between the rotation axis and the asteroid-Earth direction (line of the sight). The obliquity angle is defined as the dihedral angle between the plane determined by the line of the sight and the axis of rotation, and by the plane perpendicular to the plane containing the phase angle (Figure 1).

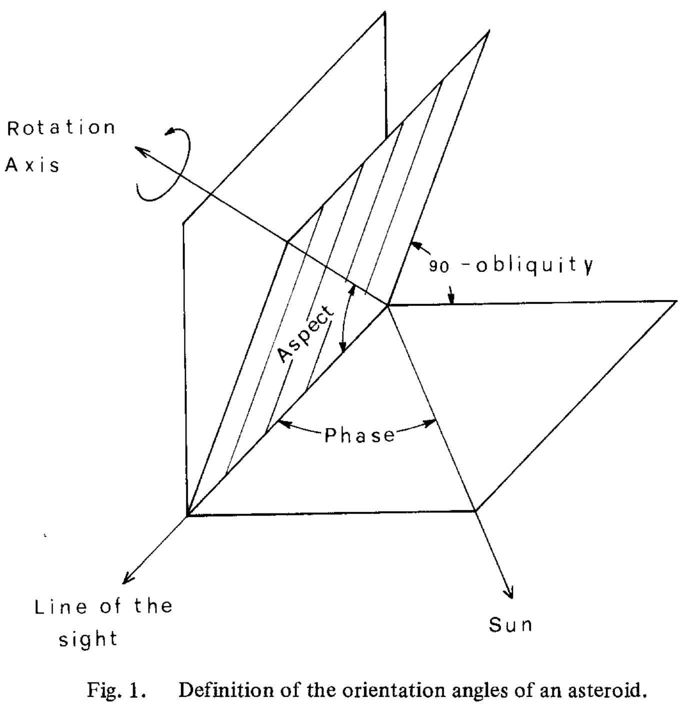

Fig. 1. Definition of the orientation angles of an asteroid.

In order to find the values of the magnitude for each R i.e. in any instant of the rotational period, the expression of the portion of the ellipsoid illuminated by the Sun as seen by the observer has been calculated.

The observer sees the projection of the illuminated surface on the plane perpendicular to the line of the sight. If the phase angle ϕ is 0, that projection is the ellipse obtained by the intersection of the ellipsoid and the cylinder tangent to the ellipsoid itself with generatrix parallel to the line of the sight. When the phase angle is $\phi \neq 0$ the illuminated portion of the ellipsoid will be seen by the observer reduced by the phase effect. In this case, the projection of that portion on the plane perpendicular to the line of the sight will be composed by the two semi-ellipses which can be respectively obtained intersecting the ellipsoid with the tangent cylinder (1) with generatrix parallel to the line of the sight, (2) with generatrix parallel to the Sun-asteroid direction.

The solar light reflected by an asteroid is proportional to the area of this section. This area is a function of R: varying R between 0 and 360 the values of the area range between the maximum ($S_{\max}$) and the minimum ($S_{\min}$). The amplitude of the light curve is defined as

$$A = 2.5 \log_{10} (S_{\max}/S_{\min}).$$

The area S is obtained as a function of the above defined geometrical parameters (aspect α, obliquity ω, rotation R) and of the phase angle ϕ starting from the expression of the ellipsoid as function of these geometrical parameters.

A reference frame with the origin in the center of the asteroid is assumed, the x axis directed toward the observer and coincident with the b semiaxis, the y axis overlapping the major axis a, and the z axis directed in the same way of the minor axis c.

The dependence on the obliquity angle ω can be outlined by means of a rotation around the x axis (Figure 2); the influence of the aspect angle α is obtained by a rotation around the y' axis (Figure 3); the rotation of the asteroid is described by a rotation R around the z'' axis (Figure 4).

The equation for the asteroid in the last reference is

$$\frac{x'''^2}{b^2} + \frac{y'''^2}{a^2} + \frac{z'''^2}{c^2} = 1,$$

where

$$\begin{pmatrix} x''' \\ y''' \\ z''' \end{pmatrix} = D \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

with

$$D = \begin{pmatrix} \sin\alpha\cos R & -\sin\omega\cos\alpha\cos R + \cos\omega\sin R & -\cos\omega\cos\alpha\cos R - \sin\omega\sin R \\ -\sin\alpha\sin R & \sin\omega\cos\alpha\sin R + \cos\omega\cos R & \cos\omega\cos\alpha\sin R - \sin\omega\cos R \\ \cos\alpha & \sin\omega\sin\alpha & \cos\omega\sin\alpha \end{pmatrix}.$$

In the original frame (x, y, z), the equation which defines the ellipsoid in any position will be of the form

$$B_1 x^2 + B_2 y^2 + B_3 z^2 + B_4 xy + B_5 yz + B_6 xz = 1,$$

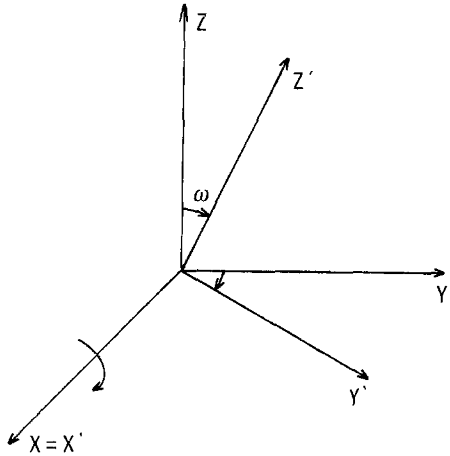

Fig. 2. The variation of the obliquity angle ω is obtained by means of a rotation around the x axis, which defines the frame (x', y', z').

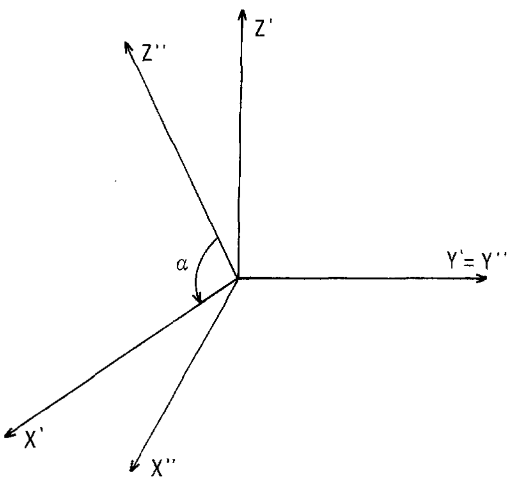

Fig. 3. The variation of the aspect angle α is obtained by means of a rotation around the y' axis, which defines the frame (x'', y'', z'').

where

$$B_1 = A_1^2 + A_2^2 + A_3^2,$$

$$B_2 = A_4^2 + A_5^2 + A_6^2,$$

$$B_3 = A_7^2 + A_8^2 + A_9^2,$$

$$B_4 = 2(A_1A_4 + A_2A_5 + A_3A_6),$$

$$B_5 = 2(A_4A_7 + A_5A_8 + A_6A_9),$$

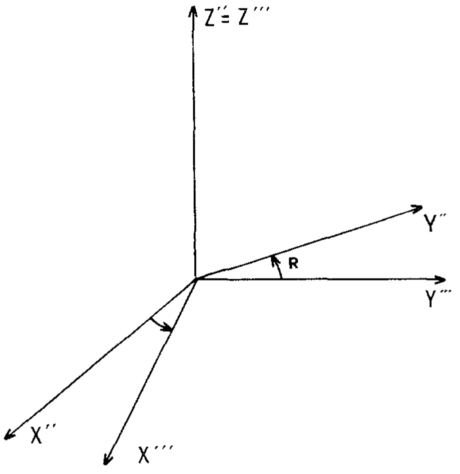

Fig. 4. The variation of the rotation angle R is obtained by means of a rotation around the z'' axis, which defines the frame (x''', y''', z''').

$$B_6 = 2(A_1A_7 + A_2A_8 + A_3A_9);$$

$$A_1 = \sin\alpha\cos R/b\ ,$$

$$A_2 = -\sin\alpha\sin R/a\,,$$

$$A_3 = \cos\alpha/c\,,$$

$$A_4 = (-\sin\omega\cos\alpha\cos R + \cos\omega\sin R)/b\,,$$

$$A_5 = (\sin\omega\cos\alpha\sin R + \cos\omega\cos R)/a,$$

$$A_6 = \sin\omega\sin\alpha/c\,,$$

$$A_7 = (-\cos\omega\cos\alpha\cos R - \sin\omega\sin R)/b\,,$$

$$A_8 = (\cos\omega\cos\alpha\sin R - \sin\omega\cos R)/a\ ,$$

$$A_9 = \cos\omega\sin\alpha/c.$$

The intersection of the ellipsoid with the tangent cylinder with generatrix parallel to the line of the sight gives

$$y^2(B_4^2 - 4B_1B_2) + z^2(B_6^2 - 4B_1B_3) + 2(B_4B_6 - 2B_1B_5)yz + 4B_1 = 0$$

i.e. the ellipse

$$a_1y^2 + b_1z^2 + c_1yz = 1,$$

where

$$a_1 = (-B_4^2 + 4B_1B_2)/4B_1\,,$$

$$b_1 = (-B_6^2 + 4B_1B_3)/4B_1\,,$$

$$c_1 = (4B_1B_5 - 2B_4B_6)/4B_1\,.$$

The intersection of the ellipsoid with the tangent cylinder with generatrix parallel to the direction object-Sun, forming an angle ϕ (phase) with x axis gives the ellipse

$$a_2 y^2 + b_2 z^2 + c_2 yz = 1,$$

where

$$a_2 = B_1 C_1^2/C_3^2 + B_2 - B_4 C_1/C_3,$$

$$b_2 = B_1 C_2^2/C_3^2 + B_3 - B_6 C_2/C_3,$$

$$c_2 = 2B_1 C_1 C_2/C_3^2 - B_4 C_2/C_3 + B_5 - B_6 C_1/C_3.$$

The area of each ellipse is given by

$$S_i = \{(a_i \cos^2 q_i + b_i \sin^2 q_i + c_i \cos q_i \sin q_i) \times$$
$$\times (a_i \cos^2 q_i + b_i \sin^2 q_i - c_i \sin q_i \cos q_i)\}^{-1/2}, \qquad i = 1, 2$$

where

$$q_i = \tfrac{1}{2} \arctan \{c_i/(a_i - b_i)\}.$$

The illuminated surface seen by the observer will be (for $\phi \leqslant 90$)

$$S = (S_1 + S_2)/2.$$

3. Results

The role of the geometrical parameters which defined the asteroid's orientation is discussed on the basis of the amplitude of the light curves vs aspect plot. In Figure 5 such a relation for different phase angles assuming three-axial asteroid model with $a/b = 1.29$ and $b/c = 1.72$ is reported.

Each plot represents a family of curves obtained varying the obliquity values. The amplitude variation with the aspect for any value of obliquity and phase shows that the amplitude of an asteroid's light curve grows with the aspect.

The effect of obliquity is negligible for small phase angles while it is predominant for phases larger than 30.

The higher the phase angle the larger the maximum aplitude of the light curve will be. In Figure 6 the amplitude-aspect relation is calculated for biaxial objects characterized by different values of the a/b ratio (obliquity is 0 and the phase angle is 7).

An asteroid with spherical ($a = b = c$) or disc shape ($a = b > c$) has flat light curves (Figure 6, curve with $a/b = 1$).

The amplitude of a light curve grows according to a/b ratio. The three axes ellipsoid models (Figure 7) show a similar trend in the amplitude-aspect relation, but the influence of the b/c ratio is outlined by lower amplitude values for intermediate aspect angles in comparison with the correspondent amplitude values of biaxial model ellipsoid. In Figure 8 a family of curves obtained for a three axis model with $a/b = 2.25$ and for a set b/c ratio for a phase angle of 7 is shown. In Figure 9 the same curves are reported for phase angle of 18.5. The comparison between the two plots shows that for the same object the phase variation implies variations of the amplitude when the aspect is changing.

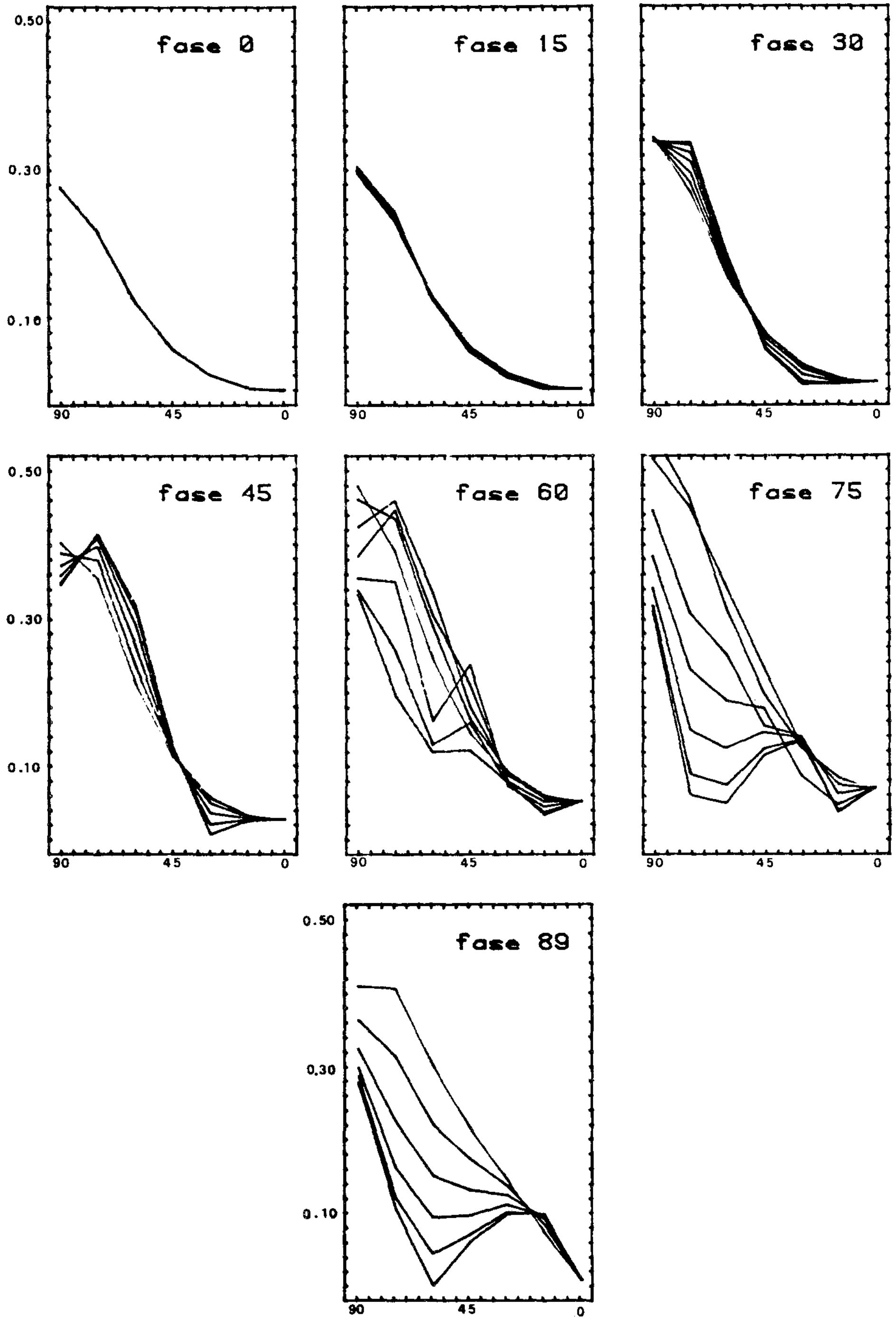

Fig. 5. Amplitude vs aspect plots for different phase angles (fase) assuming a three axial asteroid model with $a/b = 1.29$ and $b/c = 1.72$. Each plot represents a family of curves obtained varying the obliquity angle values in the range 0–90 (step 15).

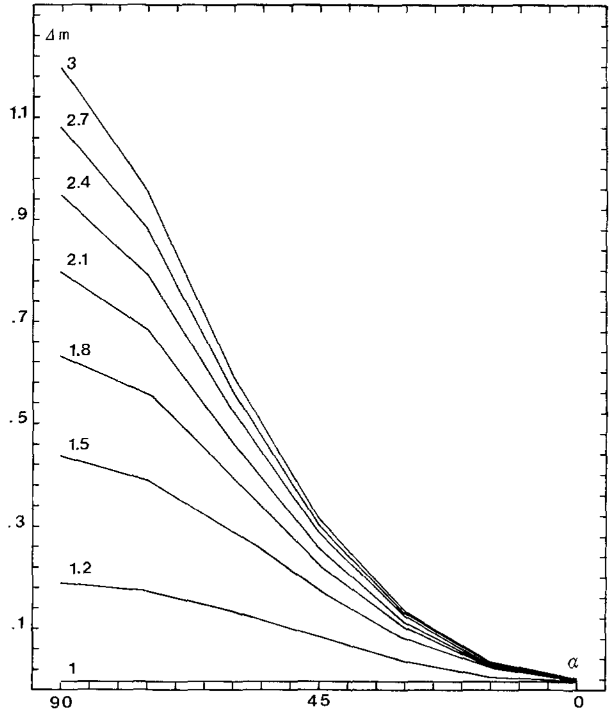

Fig. 6. Amplitude vs aspect plot for biaxial models with different values of a/b ratio ($\omega = 0, \phi = 7$).

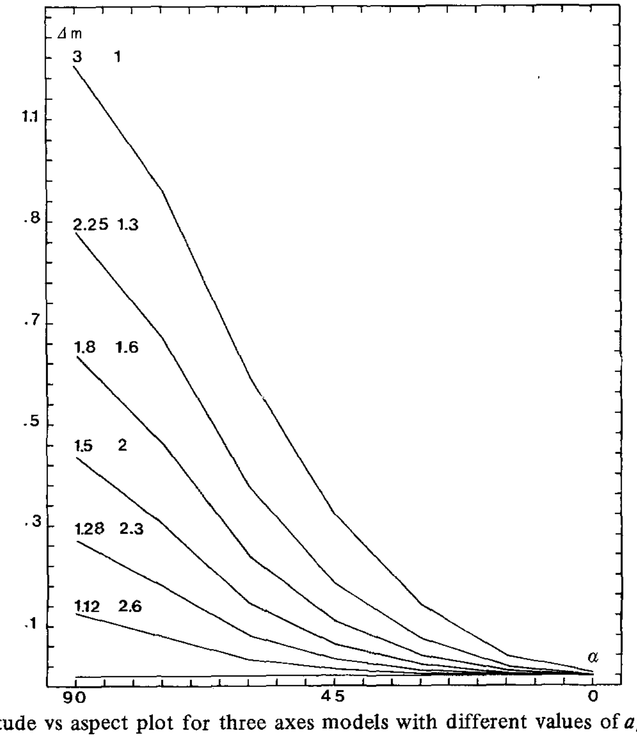

Fig. 7. Amplitude vs aspect plot for three axes models with different values of a/b (first digit) and b/c ratios ($\omega = 0, \phi = 7$).

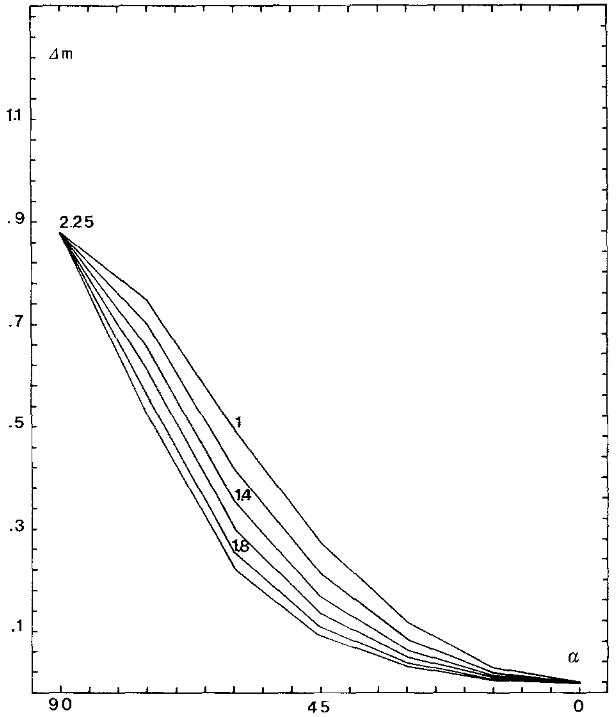

Fig. 8. Amplitude vs aspect plot for three axes models with $a/b = 2.25$ and different b/c values. ($\omega = 0, \phi = 7$).

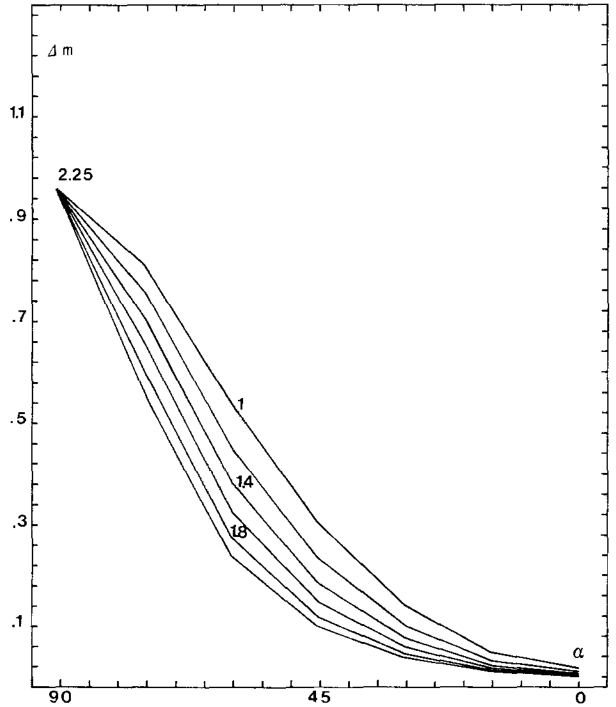

Fig. 9. Amplitude vs aspect plot for three axes models with $a/b = 2.25$ and different b/c values. ($\omega = 0, \phi = 18.5$).

4. Conclusions

In this paper an analytical expression to calculate the amplitude of an asteroid's light curve has been discussed. The expression for the amplitude has been obtained as a function of the angular parameters which define the orientation of the asteroid with respect to the observer: aspect, obliquity and phase angles.

The influence of such parameters on the shape of the light curve has also been discussed.

The previous experimental (Dunlap, 1971) and theoretical (Surdey and Surdey, 1977) works in the same field present results referring to a set of angular parameters which are non-realistic with respect to the observational procedures. In fact they discuss the amplitude-aspect relation for a three axes asteroid model choosing high phase values and few aspect and obliquity values.

In our approach the observational reality is taken into account with the choice of small phase angles (similar to the values which characterize opposition periods) and a larger set of the orientation parameters.

In that way we find that the obliquity angle influence in shaping the light curves is negligible for phase angles lower than 30. This implies that the characterization of the light curves in the observational phase range is quite independent on the obliquity.

The influence of the shape of the object, supposed to be a biaxial or a three axes regular ellipsoid, on the amplitude-aspect relation has been investigated varying the ratio between the ellipsoid semiaxes. The comparison of this kind of result obtained with different phase angles may allow the theoretical determination of the semiaxes ratio. In fact, if we use diagrams similar to these in Figures 6, 8 and 9 as an abacus, it is possible, knowing the amplitude of the light curve, to find all the intersections with the different plotted curves and to find the corresponding aspect value. Taking into account that the orbital motion of the asteroid and of the Earth determine at any time the direction of the line of the sight (which is a function of the inclination between the ecliptic and the orbital plane of the asteroid) and considering that the rotation axis of the asteroid will have the same orientation along its orbit, it is possible to argue that at a different time the aspect angle of the object will assume different values.

Using known values of amplitude at different phases (by the telescopic observations) and choosing the curve characterized by the a/b ratio which corresponds to the difference in the aspect due to the phase variation in the hypothesis of a biaxial object (Figure 6), we may have a first estimation of that ratio. Entering such estimated a/b value into the families of curves for three axes models (Figures 8 and 9) it is possible to obtain with the same procedure an estimate of the b/c ratio.

That method gives only theoretically the general solution of the problem to individuate the geometrical and angular parameters which define the shape and the orientation of regular asteroidal objects.

The variation of the aspect values with the phase are small and the observational case are in the same range of the instrumental errors. Moreover the hypothesis of regularly shaped objects is too restrictive: in fact, the variations due to the surface morphology of the objects are probably one order of magnitude larger than the purely geometrical ones.

The proposed model helps in describing the behaviour of the light curves from a general point of view.

A use for these results is described by Barucci *et al.* (1982) as a test for experimental work done in simulating asteroidal light curves in laboratory.

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