THE FINE STRUCTURE OF THE SATURNIAN RING SYSTEM

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Abstract. A dust disc within a planetary magnetosphere constitutes a novel type of dust-ring current. Such an azimuthal current carrying dust disc is subject to the dusty plasma analog of the well known finite-resistivity 'tearing' mode instability in regular plasma current sheets, at long wavelengths. It is proposed that the presently observed fine ringlet structure of the Saturnian ring system is a relic of this process operating at cosmogonic times and breaking up the initial proto-ring (which may be regarded as an admixture of fine dust and plasma) into an ensemble of thin ringlets. It is shown that this instability developes at a rate that is many orders of magnitude faster than any other known instability, when the disc thickness reaches a value that is comparable to its present observed value.

1. Introduction

Among the most exciting and significant discoveries made by the recent Voyagers 1 and 2 encounters with Saturn was the overall fine structure of the ring system. The classical picture of a few more or less broad discs separated by gaps and discontinuities, based on Earth bound observations, has been replaced by the post encounter one wherein the ring system is seen to be composed of tens of thousands or more of extremely narrow ringlets, the narrowest of which have widths ≈ 1 km (e.g., see Lane *et al.*, 1981).

A few of these 'ringlets' are shown to be associated with spiral density waves that are similar to the waves believed to be responsible for the spiral structure of galaxies (Cuzzi *et al.*, 1981). A few others are shown to be associated with spiral bending waves (Shu *et al.*, 1983), of the type that are believed to be responsible for the warping of the discs of spiral galaxies (Hunter and Toomre, 1969). Both these types of waves are apparently excited by resonances with the various satellites of Saturn. Several other ringlets including a few isolated ones in the C-ring may also be associated with satellite resonances (J. B. Holberg, 1983, private communication; see also, Holberg *et al.*, 1982; or Esposito *et al.*, 1983).

However, all these account for less than one percent of all the observed ringlets. Clearly some other mechanism is required to explain the bulk of them. The hypothesis that 'shepherd' satellites were responsible for the confinement of the narrow rings of Uranus (Goldreich and Tremaine, 1979) gained credibility with the Voyager 1 discovery of two nearby small satellites on either side of the narrow F-ring of Saturn. Consequently, several authors (Lissauer *et al.*, 1981; Hénon, 1981) have postulated the existence of embedded moonlets in order to both confine the Saturnian ringlets and to clear the gaps between them. Incidentally, Dermott and Gold (1977) had proposed, in the Uranian context, that the rings could be held in place by a moonlet within the ring itself. In this

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case, the transfer of angular momentum between this moonlet and the ring particles would make the latter move in horseshoe orbits in the moonlet's frame. This proposal too could be extended to the Saturnian case (see, e.g., Burns, 1981).

Voyager 2 images were carefully searched for such moonlets, particularly in an optimal region in the Cassini division, where the moonlets were expected to have diameters of 20–30 km. None were found (Smith *et al.*, 1982). Excluding the unlikely possibility that these (icy) moonlets are too dark to be observed, we are forced to the conclusion that, in fact, they do not exist.

An alternative possibility is that the mechanism responsible for ringlet formation is intrinsic. Alfvén (1970) had suggested that inelastic collisions between particles orbiting a central gravitating body (e.g., the Sun or a planet) would cause these particles to form 'jet streams' (i.e., groups of particles whose orbits are nearly identical) as a first step toward formation of a single body (a planet or a satellite). Baxter and Thompson (1973) made a detailed analysis of this process using a Boltzmann-type formulation, and showed that this Alfvén focusing does indeed take place if the collisions are sufficiently inelastic. They also showed that this mechanism, which they called 'negative diffusion', would cause density perturbations to grow in an initially uniform particle disc around a central body, with a time scale of the order of the interparticle collision time scale. Although not mentioned by the authors at the time, such a process may be associated with the formation of the overall fine (ringlet) structure of the Saturnian ring system.

More recently, Lin and Bodenheimer (1981) used a hydrodynamic treatment to show that collision-dominated particle discs around planets are unstable against a 'pinch' instability induced by viscous diffusion in the differentially rotating disc, which initiates the breakdown of the disc into an ensemble of thin ringlets. A very similar result was obtained independently by Ward (1981). The physical basis here is that, if one considers a ring element of such a disc, the viscous torque exerted on the outer edge of this element by the slower moving outer material will cause this ring element to lose angular momentum and move radially inward. The opposite process would take place at the inner edge, causing the ring element to move radially outward. Both authors suggested that the radial ringlet structure of Saturn's ring system was produced by this process. For the growth time, τ_g, of this instability, Lin and Bodenheimer (1981) obtain a value $\approx 10^3 D^2 \tau$ yr, where D (km) is the width of the ringlet and τ is the normal optical depth of the disc.

There is, however, another basic intrinsic mechanism for the break-up of a planetary disc into an ensemble of thin ringlets which would have been very efficient at cosmogonic times. In this note, we will investigate that mechanism in the context of the Saturnian ring system.

2. Dust Ring Currents

Particulate matter which is immersed in a planetary magnetosphere is charged to some electrostatic potential, whose sign and magnitude depend on the relative strengths of the charging currents (in particular the electron collection current and the photoemission current). While this charging has physical effects on the larger bodies, it can have both physical and dynamical effects on the smaller (micron-sized) bodies (e.g., see Mendis, 1981; Whipple, 1981; Grün *et al.*, 1983; Mendis *et al.*, 1983a; for detailed discussions). Of particular interest is the fact that the guiding centers of the charged dust grains, within a planetary magnetosphere, will move with an angular velocity Ω_G which is different from both the Kepler angular velocity Ω_K and the co-rotation angular velocity Ω_P. In fact, it is shown (Mendis *et al.*, 1982) that for Saturn, where the spin and magnetic moment vectors are strictly parallel, charged dust in the equatorial plane will move such that

$$\Omega_G^2 + \omega_0\Omega_G - (\omega_0\Omega_P + \Omega_K^2) = 0, \tag{1}$$

where $\omega_0 = -qB/mc$; q and m being the charge and mass of the grain, respectively, and B being the magnetic field strength. It was also found that for prograde grains in Saturn, which are shown to be negatively charged (Mendis *et al.*, 1982a; Hill and Mendis, 1982b)

$$\min(\Omega_P, \Omega_K) \leqslant \Omega_G \leqslant \max(\Omega_P, \Omega_K). \tag{2}$$

This differential motion between the charged dust (Ω_G) and the charge neutralizing ambient plasma (which is moving with angular velocity Ω_P) results in a novel type of dust ring current, whose magnitude is (Hill and Mendis, 1982b)

$$I(\text{esu}) = \frac{33.3}{\pi}\frac{\phi(V)}{a(\mu)}\int_{r_1}^{r_2} |\Omega_G - \Omega_P| r\sigma \, dr, \tag{3}$$

where a is the radius of the dust (all assumed to be spheres of the same size), ϕ is the grain potential, σ is the normal optical depth and r_1 and r_2 are the inner and outer radii of the ring.

Hill and Mendis (1982b) showed that in the F-ring, which is known to be largely composed of micron-sized grains, currents of the order of 10^5 A could be generated, which would substantially change the (dipole) magnetic field there. Also, Ip and Mendis (1983) discussed the consequences of the diurnal modulation of such a current in the inner D-ring on the global convection pattern in the equatorial outer ionosphere of Saturn.

Furthermore, it is not unreasonable to suppose that, at cosmogonic times, the entire proto-Saturnian ring was essentially a dusty plasma (e.g., Alfvén, 1981) with agglomeration up to the critical size allowed by the planetary gravitational shear taking place subsequently. In that case, the entire proto-Saturnian ring would have carried a large azimuthal ring current.

Such a current would be subject to the well-known finite-resistivity 'tearing' mode instability at long wavelengths. This instability would break-up the dust current disc along the current flow lines thereby forming an ensemble of ringlets. The physical basis of this mechanism is the mutual attraction between parallel currents which causes the growth of radial density perturbations in such a current disc. We propose that the present overall ringlet structure of the Saturnian ring system is a relic of this process operating at cosmogonic times.

3. The 'Tearing' of the Proto-Ring

For a quantitative application of this process to the Saturnian ring system, we use the analysis of Furth *et al.* (1963) concerning finite-resistivity instabilities in a sheet pinch, with the appropriate modifications for a two-component dusty plasma containing negatively charged dust and positive ions. For simplicity we assume that there are no free electrons.

If we follow Furth *et al.* (1963), the basic requirements for the excitation of the tearing mode are

$$\alpha^{-4} < S < \alpha^{-4}|G|^{-5/2} \tag{4}$$

and

$$\alpha < 1; \tag{5}$$

where $\alpha = a/\lambda$, a = disc thickness, λ = perturbation wavelength and $S = \tau_R/\tau_H$, where τ_R and τ_H are, respectively, the resistive diffusion time and the hydromagnetic transit time, given by

$$\tau_R = \frac{4\pi a^2}{c^2\eta} \tag{6}$$

and

$$\tau_H = \frac{a}{v_A} = \frac{a(4\pi\rho)^{1/2}}{B}. \tag{7}$$

In this latter equation ρ is the distributed dust mass density and η is the resistivity (in esu).

We may also write the resistivity as

$$\eta = \chi\eta_c, \tag{8}$$

where η_c is the classical resistivity and χ $(1 \leqslant \chi \leqslant n_i\lambda_D^3)$ is the possible turbulent enhancement of the resistivity in the neutral sheet due to the large current density.

Considering the two-component (positive ions and negative dust) dusty plasma and assuming (with obvious notation) the charge neutrality condition

$$z_d n_d + z_i n_i = 0, \tag{9}$$

we have η_c given by (Spitzer, 1967)

$$\mathbf{P}_{id} = \eta_c |z_i z_d| e^2 n_i n_d (\mathbf{v}_d - \mathbf{v}_i), \tag{10}$$

where $\mathbf{v}_d$ and $\mathbf{v}_i$ are the velocities of the dust and ions, and $\mathbf{P}_{id}$ is the momentum transfer rate (per unit volume) from the dust to the ions, which is given by (Banks an Kockarts, 1973).

$$\mathbf{P}_{id} = n_i \frac{m_i m_d}{m_i + m_d} \bar{\nu}_{id}(\mathbf{v}_d - \mathbf{v}_i) f\left(\frac{v}{s}\right), \tag{11}$$

where

$$v = |\mathbf{v}_d - \mathbf{v}_i|, \tag{12}$$

$$s = \left[2k\left(\frac{T_i}{m_i} + \frac{T_d}{m_d}\right)\right]^{1/2}, \tag{13}$$

$$f\left(\frac{v}{s}\right) = \tfrac{4}{3}\pi^{1/2}\left[\left(\frac{s}{v}\right)^3 \operatorname{erf}\left(\frac{v}{s}\right) - \frac{2}{\pi^{1/2}}\left(\frac{s}{v}\right)^2 \exp\left(-\frac{v^2}{s^2}\right)\right], \tag{14}$$

$$\bar{\nu}_{id} = \frac{4n_d}{3}(2\pi)^{1/2}\left[\frac{(m_i+m_d)z_i z_d e^2}{m_i m_d}\right]^2\left[\frac{kT_i}{m_i}+\frac{kT_d}{m_d}\right]^{-3/2} \ln\Lambda, \tag{15}$$

$$\Lambda = \frac{\mu g^2 \lambda_D}{|z_i z_d| e^2}, \tag{16}$$

$$\lambda_D = \left[4\pi e^2\left(\frac{z_i^2 n_i}{kT_i} + \frac{z_d^2 n_d}{kT_d}\right)\right]^{-1/2}; \quad \left|\frac{z_i e\phi}{kT_i}\right| < 1; \quad \left|\frac{z_d e\phi}{kT_d}\right| < 1, \tag{17}$$

and

$$g = \int\int |\mathbf{v}_i - \mathbf{v}_d| f_i(\mathbf{v}_i) f_d(\mathbf{v}_d)\, d\mathbf{v}_i\, d\mathbf{v}_d \approx \left(\frac{8k}{\pi}\right)^{1/2}\left(\frac{T_i}{m_i}+\frac{T_d}{m_d}\right)^{1/2} \text{ if } \frac{v}{s} \ll 1. \tag{18}$$

In the above equations μ represents the reduced mass of the system and ϕ is the grain potential in esu.

Although many of the parameters in the proto-Saturnian dust disc are unknown, it seems reasonable to assume that $v/s \lesssim 1$, which makes $f(v/s) \approx 1$. Using this approximation, and noting that $m_d \gg m_i$, $z_i = 1$, $T_d = 0$, and

$$z_d e = \frac{R_g \phi(V)}{300}, \tag{19}$$

from Equations (10) and (11) we obtain

$$\eta_c = \tfrac{4}{900}(2\pi)^{1/2}\frac{|\phi(V)| e R_g}{m_i}\left(\frac{kT_i}{m_i}\right)^{-3/2} \ln\left[\frac{1200}{|\phi(v)| e^2 n_i^{1/2} R_g}\left(\frac{kT_i}{\pi}\right)^{3/2}\right]; \tag{20}$$

and, using (20), we get the inequality (4) as

$$\frac{\lambda^4 \chi}{c_1 y z}\ln c_2 z < a^5 < |G|^{-5/2}\frac{\lambda^4\chi}{c_1 y z}\ln c_2 z, \tag{21}$$

where $c_1 = 1.20 \times 10^{-33}$, $c_2 = 1.52 \times 10^{-3}$, $y = B/m_i$,

$$z = \left(\frac{T_i^3}{\phi^2 n_i R_g^2}\right)^{1/2}, \quad \text{and} \quad S = \frac{a}{\chi} c_1 y z (\ln c_2 z)^{-1}.$$

Also, according to Furth *et al.* (1963), the quantity G occurring in Equation (21) is given by

$$G = -\tau_H^2 \frac{1}{\rho_0}\frac{\partial}{\partial y}(\rho_0 \dot{v}_0), \tag{22}$$

where $\dot{v}_0$ is the zero-order acceleration of the plasma. In the case of the proto-Saturnian ring,

$$G \approx \tau_H^2 \frac{1}{\rho_0}\frac{\rho_0 \dot{v}_0}{a} \approx \frac{a}{R}\left(\frac{v_c}{v_A}\right)^2, \tag{23}$$

where v_c and v_A are, respectively, the co-rotation and Alfvén speeds and R is the radial distance. The proto-Saturnian disc is well within the plasma pause of the magnetosphere. So $v_c < v_A$. Also $a \ll R$. Consequently, $G \ll 1$, as is required by inequality (21).

The values to be used for many of the quantities (e.g. T_i, ϕ, R_g, n_i, etc.) are highly uncertain, at cosmogonic times. We will, however, proceed by using guesstimates that are not unreasonable. Assume the magnetic field of Saturn has not changed significantly since cosmogonic times, we get $B \approx 0.21\ (R_s/R)^3$. Taking $R \approx 2R_s$ as a representative value in the ring system, we get $B \approx 0.025\Gamma$. We will also assume that the ring current was sufficiently strong to distort the dipole field so that the transverse component near the ring at that point also $\approx 0.025\Gamma$. For the grain radius R_g we will assume the value of 0.5μ, which seems to be the dominant size of the present day tenuous rings (particularly the F-ring) and the electrically controlled radial spokes (e.g., see Hill and Mendis, 1982a). We will also assume that the dominant ions are protons (i.e. $m_i = 1.67 \times 10^{-24}$ g), and that $kT_i \approx 10$ eV (corresponding to present day thermal plasma; e.g. see Mendis *et al.*, 1983b). The values of n_i at cosmogonic times would, of course, have been much larger than the present values ($\sim 1\ \text{cm}^{-3}$). We will take $n_i \approx 10^4\ \text{cm}^{-3}$ as a rather arbitrary value. If the dust grains could be considered as isolated, it is easy to show that $\phi \approx -40$ V (Mendis *et al.*, 1982), but it is reasonable to assume that the dust density was then sufficiently large, that even with such ion densities there would not have been sufficient plasma to charge all the grains to such a potential. From the charge neutrality condition (9), we get $|\phi_d| \approx 30/n_d$. With $n_d \approx 10^2\ \text{cm}^{-3}$, corresponding to an optical depth ≈ 0.1, we get $|\phi_d| \approx 0.3$ V. Substituting these values in the inequality (21) we get

$$a > \frac{\chi}{\alpha^4}\frac{1}{c_1 y z}\ln c_2 z \approx 37.4\frac{\chi}{\alpha^4}. \tag{24}$$

Also the characteristic growth time τ for the tearing mode is given (cf. Furth *et al.*, 1963) by

$$\tau = \left(\frac{\alpha}{S}\right)^{2/5}\tau_R \approx 3.8\times10^{-8}\,\frac{\alpha^{2/5}}{\chi^{3/5}}\,a^{8/5}. \tag{25}$$

Using $\chi \approx 10^3$ as an arbitrary value ($\lesssim n_i\lambda_D^3 \approx 10^8$) we get $a > 0.37$ km. What this means is that if a dusty plasma disc was accreted in the equatorial plane of Saturn at cosmogonic times, by some process such as the one suggested by Alfvén (e.g. Alfvén, 1981), then if we take $\chi \approx 10^3$, and $\alpha \approx 1$ the dust disc will begin to tear when $a \approx 0.37$ km. If χ was smaller, say $\approx 3\times10^2$, the tearing will take place already when $a \approx 0.1$ km which in fact corresponds to the observed thickness of Saturn's rings. Also, if we take these latter values (i.e. $a \approx 0.1$ km and $\chi \approx 300$) the disc will tear into ringlets with spacing $\gtrsim 0.1$ km, with a characteristic time $\tau \lesssim 3\times10^{-3}$ s.

4. Discussion

As we pointed out earlier, the numerical values of most of the paramenters used in this calculation are highly uncertain, a situation that is unfortunately inherent in all cosmogonic calculations. So the numerical values we obtain are not to be taken at face value. However, the significance of this calculation is to show that, once we recognize the fact that the proto-Saturnian disc was a dusty plasma, it was subject to an intrinsic instability which broke it up into thin ringlets in an incredibly short time, with any reasonable set of values of the physical parameters. This time is orders of magnitude smaller than the time for the 'pinch' instability induced by viscous diffusion (Lin and Bedenheimer, 1981) which is of the order of years, for any reasonable value of the ringlet width and normal optical depth. Furthermore, the disc thickness at which this break-up process is initiated is of the order of its present thickness, for reasonable values of the physical parameters of the system.

We therefore believe that this 'tearing' instability in the current carrying proto-Saturnian dust disc, is what caused it to break up into the ensemble of thin ringlets that we see today. Also as was pointed out by Mendis *et al.* (1982), the relative motion between the charged dust of different sizes which have different specific charges would result in the agglomeration of the dust into larger particles, up to some critical value, within these ringlets. Once the bodies are sufficiently large, as in the main ring system, that electrodynamic forces cease to play any significant role, the 'negative diffusion' associated with highly inelastic collisions (Baxter and Thompson, 1973), or the angular momentum transport associated with the viscous shear between neighboring ringlets (Lin and Bodenheimer, 1981; Ward, 1981) could be responsible for maintaining the subsequent integrity of the ringlets.

Although we have confined our analysis to the Saturnian ring system, which is the most well studied yet, we believe that the same process is also responsible for the observed fine (ringlet) structures of Uranus and Jupiter. While stellar occultation observations show that Uranus has at least 9 rather widely separated thin ringlets (with perhaps many more in between), careful computer processing of the Voyager 2 images of the bright component of the Jovian ring has recently shown that it too has a fine ringlet structure (Haemmerle *et al.*, 1982).

In conclusion, we stress that our present calculation needs to be refined in several ways. A more realistic calculation should consider a three component dusty plasma (i.e. charged dust, positive ions and electrons) and not the simpler two-component one that we have considered here. Also, the effect of the component of the magnetic field normal to the dust disc on the tearing instability has to be considered. Finally, the difficult problem of a full non-linear analysis will have to be faced. Our analysis, in common with all others, is linear and as such it merely indicates a tendency for the process to take place. Only a proper non-linear analysis will show if the process will proceed or be arrested.

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References

Alfvén, H.: 1970, *Astrophys. Space Sci.* **6**, 161.
Alfvén, H.: 1981, *Cosmic Plasma*, D. Reidel Publ. Co., Dordrecht, Holland.
Banks, P. M. and Kockarts, G.: 1973, *Aeronomy* (Part B), Academic Press, p. 155.
Baxter, D. C. and Thompson, W. B.: 1973, *Astrophys. J.* **183**, 323.
Burns, J. A.: 1981, in J. K. Beatty, B. O'Leary and A. Chaikin (eds.), *The New Solar System* Cambridge Univ. Press, p. 129
Cuzzi, J. N., Lissauer, J. J., and Shu, F. H.: 1981, *Nature* **292**, 703.
Dermott, S. F. and Gold, T.: 1977, *Nature* **267**, 590.
Esposito, L. W., Cuzzi, J. N., Evans, D. R., Holberg, J. B., Marouf, E. A., Tyler, G. L., and Porco, C. C.: 1983, in T. Gehrels (ed.), *Saturn* Univ. of Arizona Press (in press).
Furth, H. P., Killeen, J., and Rosenbluth, M.: 1963, *The Physics of Fluids* **6**, 459.
Goldreich, P. and Tremaine, S.: 1979, *Nature* **277**, 97.
Grün, E., Morfill, G. E., and Mendis, D. A.: 1983, in A. Brahic and R. Greenberg (eds.), *Planetary Rings*, D. Reidel Publ. Co., Dordrecht, Holland (in press.)
Haemmerle, V. R., Danielson, G. E., and Currie, D. G.: 1982, *Bull. Amer. Astron. Soc.* **14**(3), 747.
Hénon, M.: 1981, *Nature* **293**, 33.
Hill, J. R. and Mendis, D. A.: 1982a, *J. Geophys. Res.* **87**, 7413.
Hill, J. R. and Mendis, D. A.: 1982b, *Geophys. Res. Lett.* **9**, 1060.
Holberg, J. B., Forrester, W. T., and Lissauer, J. J.: 1982, *Nature* **297**, 115.
Hunter, C. and Toomre, A.: 1969, *Astrophys. J.* **155**, 747.
Ip, W. -H. and Mendis, D. A.: 1983, *Geophys. Rev. Lett.* **10**, 207.
Lane, A. L., Hord, C. W., West, R. A., Esposito, L. W., Coffeen, D. L., Sato, M., Simmons, K. E., Pomphrey, R. B., and Morris, R. B.: 1981, *Science* **215**, 537.
Lin, D. N. C. and Bodenheimer, P.: 1981, *Astrophys. J.* **248**, L83.
Lissauer, J. J., Shu, F. H., and Cuzzi, J. N.: 1981, *Nature* **292**, 707.
Mendis, D. A.: 1981, 'The Role of Electrostatic Charging of Small and Intermediate Sized Bodies in the Solar System', in F. D. Kahn (ed.), *Investigating the Universe* D. Reidel Publ. Co., Dordrecht, Holland, pp. 353–384.
Mendis, D. A., Houpis, H. L. F., and Hill, J. R.: 1982, *J. Geophys. Res.* **87**, 3449.
Mendis, D. A., Ip, W. -H., Hill, J. R., Goertz, C. K., and Grün, E.: 1983a, in T. Gehrels (ed.), *Saturn* Univ. of Arizona Press (in press).
Mendis, D. A., Hill, J. R., and Houpis, H. L. F.: 1983b, *J. Geophys. Res.* **88** (Supplement), p. A929.
Shu, F. H., Cuzzi, J. N., and Lissauer, J. J.: 1983, *Icarus* **53**, 185.
Smith, B. A. *et al.*: 1982, *Science* **215**, 499.
Spitzer, L.: 1967, *Physics of Fully Ionized Gases*, Interscience Publishers (4th printing).
Ward, W. R.: 1981, *Geophys. Rev. Lett.* **8**, 641.
Whipple, E. C.: 1981, *Reports on Progress in Physics* **44**, 1197.