ON THE PROCESSES OF MASS TRANSPORT IN ROTATING GAS-DUST CLOUDS

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Abstract. A model of motion of gaseous matter flowing into a rotating gas-dust cloud from a central body is considered. The authors suggest an equation of matter transport and the boundary conditions for a number of cases, and also deduce relations governing the variations of concentration of the flowing matter particles in the pre-planetary gas-dust cloud, depending on distance and time. It is demonstrated that the motion of matter in rotating gas-dust clouds is governed by the classical law of mass transfer.

The results obtained confirm a physico-chemical hypothesis about the formation of the planetary systems which was suggested earlier, and which is based on diffusion mechanisms of the appearance of the primary rings of the condensed matter, the rings later becoming accumulated into planets and satellites.

When considering the problem of solar system planets formation the fact (Alfvén and Arrhenius, 1976) that they were formed of gas-dust cloud matter, rotating around the Sun and filling in all the space of the solar system is believed to be generally accepted. However, the opinions of astronomers about the protoplanet cloud evolution are considerably different. The stability of laminar rotation of the matter in a protoplanetary cloud (Safronov and Ruskol, 1957) was demonstrated, as disordered macroscopic motions, which appeared in the process of formation of a gas-dust cloud, died down rapidly and the rotation motion of the cloud approached laminary. Accordingly the dust attracted by the gas does not change the character of its movement. The particles and molecules of this cloud move around the Sun along Kepler orbits with the average angular velocity $w = (GMr^{-3})^{1/2}$, where G is the gravitational constant, M is the mass of the Sun, r is the distance to the centre of rotation.

It is generally known that for the development of gravitational instability (and as a result, the process of planet formation) it is necessary that the density of the cloud reaches a critical level in a particular place. However, studies have shown (Safronov and Ruskol, 1957) that under the influence of the vertical gravitational force towards the Sun such a density can not be reached.

In accordance with the physico-chemical hypothesis of the formation of planetary systems (Gladyshev, 1977), such a critical density of the matter within a cloud can be reached as a result of special periodic condensation of gaseous matter from a supersaturated condition. Moreover, dust particles settling slowly through the gaseous cloud towards the equatorial plane serve as centres of condensation enabling the formation of space–time precipitation.

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Taking all this into consideration, the aim of this paper is to establish the fact that the transfer of matter in near-solar space rotating with Kepler velocities around a central body can be described by classical diffusion equations.

A possible model for molecular (particle) motion of solar gas is considered in this paper: boundary conditions and matter transfer equations are proposed, and for the simplest cases as an illustration, a law is given for the distribution of solar gas molecules in a protoplanetary gas-dust cloud.

Transfer Equations and Boundary Conditions

We will consider the motion of solar gas molecules in a system of coordinates related to the Sun. An equation of continuity for the particle flow is of the form

$$\frac{\partial n}{\partial t} = -\nabla \mathbf{j}, \tag{1}$$

where n is the number of particles per unit of volume, and j is the particle flow. In turn j can be equal to (Frank-Kamenetsky, 1967)

$$\mathbf{j} = -\nabla(n \cdot D) + \mathbf{v}n + \frac{D}{kT} n \cdot \mathbf{F}, \tag{2}$$

where D is the coefficient of diffusion, $\mathbf{v}$ is the speed of the directed movement of molecules, and $\mathbf{F}$ is the external force acting upon a molecule. The first component of the equation gives a formulation of the flow related to a change of the number of molecules and of temperature with distance. The first component has a typical form with constant temperature and concentration of the molecules of the principle gas (a gas-dust cloud in our case) in which the movement of the molecules under consideration is taking place: $D\nabla n$. The second component characterizes the transfer of the convection matter due to the average speed of molecular movement. $\mathbf{v}$ is determined by the initial conditions of movement (by the speed at which molecules fly away from the surface of the Sun) and by the collisions with molecules and particles of the gas-dust cloud surrounding the Sun. The third component determines the part of the flow arising under the influence of external forces: gravity F_g, centrifugal forces F_c, Coriolis forces F_{Cor}, etc. We will consider the movement of molecules under the forces F_g, F_c, F_{Cor}.

We will consider the precise appearance and change with time (or with distance) of the second and third components of Equation (2). Let the speed of a molecule at the boundary a be v_a: $\mathbf{v}_a = \mathbf{v}(r = a)$. It can be seen from the symmetry of the problem that we will be interested in the radial and tangential components of velocity v_r and v_φ, we will take the component v_θ (θ is the angle counted from the axis of rotation of the Sun) as equal to zero.

Collisions of molecules with each other can be accompanied by various processes: deviation (dispersion) at a particular angle, disintegration into atoms, excitement of inner degrees of freedom, etc. We will consider the elastic impact of solar gas molecules with molecules rotating in Kepler orbits.

As a result of chaotic collisions with the solar gas molecule the radial component of its speed will decrease. Its change will be described by a Langevin equation incorporating both the action of the fluctuating force from the environment and the stochastic resistance proportional to the speed of motion of the particle (Chandrasekhar, 1943; Isihara, 1971, Landau and Lifshitz, 1944). As a result for v_r we obtain

$$v_r = v_{r,a}\, e^{-t/\tau_D}, \tag{3}$$

where $\tau_D = mD/kT$, m is the mass of the particle (molecule). With $t \gg \tau_D$ a Maxwell distribution is established for the velocity v_r. For the tangential component velocity, considering the change of velocity after the elastic impact and ν collisions we get

$$v_\varphi = (v_{\varphi,a} - v_K) \cdot e^{-m_1\nu/(m_1+m_2)}, \tag{4}$$

where $v_K = (GMr^{-3})^{1/2} \sin\theta$ is the Kepler velocity, $v_{\varphi,a} = v_\varphi(r=a)\,(\nu = 0)$: $v = (GMr^{-3})^{1/2} \sin\theta$, m_1, m_2 – the masses of the particles.

If we should consider the particles as hard spheres with mass m, then for an elastic impact the number of collisions ν_0 in a unit of time is equal (Landau and Lifshitz, 1976) to

$$\nu_0 = 4d^2 \sqrt{\frac{\pi}{mkT}}\, p, \tag{5}$$

where d is the diameter of particles; and p the pressure in a gas-dust cloud. Furthermore, the coefficient of diffusion D is equal to

$$D = \frac{8}{3} \frac{kT}{\pi m \nu_0}. \tag{6}$$

We will evaluate the characteristic parameters in Equations (4)–(6). If we take the mass of solar gas molecules (hydrogen) as $m = 3.3 \times 10^{-24}$ g, $d = 3$ Å, $T = 100°$, $p = 10^{-11}$ atm, then $\nu = 3 \times 10^{-1}\ \mathrm{s}^{-1}$, $D = 10^{10}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, $l = 4$ km, $\tau_D = 3$ s. Other values for the parameters m, T, d will change the valuations insignificantly. It is interesting to observe that $\tau_D \approx 1/\nu_0 \approx 3$ s. During this time a molecule will travel $r^* = \sqrt{4D\tau_D} \approx 3.5$ km.

Thus, considering these processes the duration of which is more than $10\,\tau_D$, or which cover a distance greater than $10r^*$, then

$$v_r \simeq 0, \quad v_\varphi \simeq v_K \quad \text{when} \quad t > 10\tau_D \simeq \frac{10}{\nu_0} \quad \text{and} \quad r > 10r^*. \tag{7}$$

It should be noted that these relations (7) are correct if the density of the solar gas leaving the Sun is small in comparison to the density of the gas-dust cloud. If this condition is not met, a constant value for the velocity of motion will be reached more slowly. Nevertheless the magnitudes under consideration, which characterize the establishment of a stable condition, are much less than the magnitudes which characterize the processes in near-solar space.

In view of the relation (7), Equation (2) becomes considerably simpler. In fact, after a certain number of collisions with the molecules of the gas-dust cloud, the molecules of solar gas will rotate in Kepler orbits: $v_r = 0$, $v_\varphi = v_K$ and consequently, $F_g = -F_c$,

$F_{\mathrm{Cor}} = 0$ and the third component in Equation (2) is equal to zero. $n\mathbf{v}$ will be equal to $n\mathbf{v}_\varphi$. Moreover, the symmetry requires that $\partial n\mathbf{v}_\varphi/\partial\varphi = 0$ and the second component will also equal zero. Then, instead of (2) we have

$$\frac{\partial n}{\partial t} = \Delta_{r,\theta}(Dn), \tag{8}$$

where $\Delta_{r,\theta}$, r, θ are components of the Laplace operator, since n is only a function of r and θ. D also like T can be a function r and θ due to the compression in the gas-dust cloud. However, in order to simplify the problem we can consider $D = \mathrm{const}$ and we can separate D from the Laplace operator.

The initial conditions for n are

$$n(r, t) = n(r, 0) = 0 \qquad t = 0, \quad r \geqslant a; \tag{9}$$

where a is the distance from the centre of rotation to the boundary of the solar gas-dust cloud.

The boundary conditions at infinity are

$$n(r, t) = 0 \quad \text{when} \quad r \to \infty. \tag{10}$$

Conditions at the boundary of the solar gas-dust cloud separation are less defined. First of all, turbulent gas flows are possible on the surface of the Sun. They will die out either after being caught up in the rotational movement (Safronov, 1969) or after collision with the molecules and particles of the gas-dust cloud. A calculation of the size of such a turbulent zone is impossible without a knowledge of the concrete conditions.*

Secondly, the concentration of particles and molecules in the gas-dust cloud near the Sun changes gradually from zero to the characteristic level of the gas-dust cloud itself. Thus, the boundary has a certain extent. As a zero approximation we will consider the distance from the centre of rotation to be the boundary in our problem, when one can discount the turbulence of the initial flow of solar gas and when the concentration of molecules and particles of the gas-dust cloud reaches a level no longer disturbed by turbulent solar gas flows.

At the Sun–cloud boundary ($r = a$) a flow $\mathbf{j}_a$ of solar gas molecules comes from the Sun due to processes within the Sun. Moreover, the magnitude of the flow $\mathbf{j}_a$ can have a constant radial component $\mathbf{j}_0(\mathbf{r}/r)$, and also a component $\mathbf{j}_\rho$ determined by the centrifugal forces on the surface of the Sun lying on the plane of rotation and perpendicular to the surface tangent: $\mathbf{j}_\rho = A'w^2 a \sin\theta = j_1 \sin\theta$, where A', j_1 are certain constants, θ is the angle which can be counted from the axis of rotation.

Then for the radial component of the particle flow at a distance a we get

* As an estimate of the maximum size of this layer we can use the results of calculations (Safronov and Ruskol, 1957) which have shown that the possible zone of turbulence is less than 0.01 AU, – i.e., less than one million kilometres. We will note that the radius of the Sun is 0.7 million kilometres, the nearest planet, Mercury is 0.39 AU or 60 million kilometers from the Sun.

$$j_{a,r} = [\mathbf{j}_a(\mathbf{r}/r)] = j_0 + j_1 \sin\theta, \tag{11}$$

where j_0 and j_1 are magnitudes which were previously unknown.

In the case where the speed of rotation w of the surface of the Sun depends on the angle θ, then Equation (11) becomes more complicated.

Thus, an analysis of the transfer processes in a cloud rotating near the Sun shows that the transfer of matter is described by Equation (8) with boundary conditions given by Equation (11).

Addendum

Solution of simplified equation for transfer of matter in near-solar space.

Let the transfer of molecules of solar gas be defined (cf Equation (8) in this article) by the equation

$$\frac{1}{D}\frac{\partial n}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial n}{\partial\theta}\right) \tag{1-1}$$

Let the initial and boundary conditions be given by the equations (see (10-11)):

$$n = 0 \quad t = 0 \quad r \geqslant a$$

$$n = 0 \quad r \to \infty$$

$$j_{a,r} = -D\frac{\partial n}{\partial r} = j_0 P_0 + \tfrac{2}{3} j_1 (P_0 - P_2) = A_0 D P_0 - A_2 D P_2, \tag{1-2}$$

where $P_0 = 1, P_2 = 1 - \frac{3}{2}\sin^2\theta$ are Legendre polynomials and A_0, A_2 are constants.

We will introduce a Laplace transformation n_p of the function n. Then a general solution of Equation (1-1) can be written as

$$n_p = \frac{1}{\sqrt{r}}\sum P_k(\theta)\left[C_{1k} I_k\left(\sqrt{\frac{P}{D}}r\right) + C_{2k} K_k\left(\sqrt{\frac{P}{D}}r\right)\right], \tag{1-3}$$

where I_k is a Bessel function of the imaginary argument, K_k is a McDonald function (Carslaw and Jaeger, 1959; Smirnov, 1954).

Applying boundary conditions, we find that $C_{1K} = 0$ and $k = 0.25$ and 2.5. With the appropriate calculations we get

$$n_p = \frac{a^2}{Pr} e^{-(PD^{-1})^{1/2}(r-a)}\left[\frac{A_0 P_0}{1 + \sqrt{(P/D)}\,a} - \frac{A_2 P_2 \alpha a^2}{r^2}\right], \tag{1-4}$$

where

$$\alpha(z) = \frac{z^2 + 3z + 3}{z_a^3 + 4z_a^2 + 9z_a + 9}, \tag{1-5}$$

and

$$z = \sqrt{(P/D)}r, \quad z_a = \sqrt{(P/D)}a.$$

To calculate n according to the value n_p it is convenient to consider two cases: $(P/D^{-1})^{1/2}$, $r < 1$ (the range of large times) and $(P/D^{-1})^{1/2}$, $a > 1$ (the range of small times) (Carslaw and Jaeger, 1959).

In the first case we get:

$$n = \frac{a^2}{r}\left[A_0P_0 - \tfrac{1}{3}P_2A_2\frac{a^2}{r^2}\right]\Phi^*\left(\frac{r-a}{2\sqrt{Dt}}\right) \qquad (1\text{-}6)$$

where $\Phi^*(x) = 1 - \Phi(x)$, $\Phi(x)$ is a function of errors.

In the second case ($a^2 > D/P$) we obtain:

$$r = \frac{2a}{r}\sqrt{Dt}\,[A_0P_0 - A_2P_2]\Phi_1^*\left(\frac{r-a}{2\sqrt{Dt}}\right) \qquad (1\text{-}7)$$

where Φ_1^* is the integral of the error function.

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