PROTOPLANETARY CORE FORMATION BY RAIN-OUT OF IRON DROPS

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(Received 18 April; revised 8 June, 1978)

Abstract. Using the stochastic collection equation we find that the time scale for rain out of liquid iron in a Saturn mass protoplanet is rapid compared with other evolutionary time scales and hence iron protoplanetary core formation is inevitable. The survival of this core during subsequent protoplanetary evolution and the consequences of the rain-out on the evolution are also discussed.

1. Introduction

Models of the primitive solar accretion disk by Cameron (1978) indicate that giant gaseous protoplanets are formed by gravitational instabilities as the disk rotates around a central condensation. These protoplanets, consisting mostly of hydrogen and helium with a small quantity of heavy elements convectively mixed throughout as grains, continue to evolve; contracting and growing warmer much as a low mass star does. Evolutionary calculations of these objects done by DeCampli and Cameron (1978) show that if the mass of the protoplanet is less than a Jupiter mass, then the interior conditions require that the iron which would be present as clumped interstellar grains would melt and remain liquid for up to 10^5 yr. Other minerals would melt in protoplanets but, for simplicity, we will only consider iron. The melted iron drops would form a cloud and we would expect a rain-out of large iron drops after a period of coalescence. We shall show in a sample calculation that the coalescence and rain-out of iron drops in a protoplanet are very rapid compared to evolutionary time scales and hence that mass transport of iron to the center of a protoplanet is an efficient mechanism for core formation. After the addition of other possible mineral melts this protoplanetary core would either become a planetary core for the Jovian planets, or the planets themselves for the terrestrial planets following atmospheric mass loss by overflow of the inner Lagrangian point as calculated by DeCampli and Cameron (1978).

In Section 2 we will discuss the method used to model the rain-out of drops. The physics required for the modeling is described in Section 3; this includes drop velocities and collisional cross sections. A sample coalescence calculation for a pre-Saturn protoplanet and the relevant time scales are considered in Section 4. In Section 5 we will discuss survival of the core during subsequent protoplanetary evolution.

The Moon and the Planets **19** (1978) 443–456.

2. The Cloud Model

By the time a protoplanet forms from the accretion disk the interstellar grains would have clumped to some extent (Cameron, 1975). As the protoplanet continues its evolution the environment in its deep interior would be such that the clumped iron grains would melt and form droplets (DeCampli and Cameron, 1978). These droplets would grow by coalescence – e.g., a bigger drop which is falling faster than a smaller one would catch up with the smaller drop, coalesce, go even faster and thus potentially be able to coalesce at a greater rate with smaller drops. To model this process mathematically we use the stochastic collection equation which has been used very successfully to model the rain-out of water from warm clouds in the Earth's atmosphere. (Young, 1974; Ogura and Takahashi, 1973; Takada, 1971; Danielsen *et al.*, 1972.)

Given a number density function $f(x)$ where $f(x)\mathrm{d}x$ is the number of drops per unit volume in the size interval x to $x + \mathrm{d}x$ and x is the mass of a droplet, then the time evolution of the spectrum $f(x)$ by coalescence is given by the following stochastic collection equation

$$\partial f(x)/\partial t = \int_0^{x/2} f(x-x')V(x-x', x')f(x')\mathrm{d}x' - \int_0^{\infty} f(x)V(x, x')f(x')\mathrm{d}x' \qquad (1)$$

(Berry, 1967), where $V(x, x')$ is the collection kernel for drops with masses x and x'; it will be discussed in Section 3. The first integral is the gain of x-drops from drops whose masses sum to x and the second integral is the loss of x-drops due to collisions of drops with x to all other x' mass drops. The expression $\Delta t V(x, x')f(x')\mathrm{d}x'$ gives the probability that a particular x-drop will collect an x'-drop during the time interval Δt.

The cloud is assumed to be large enough so that the number density function $f(x)$ represents an average volume unit within the cloud. This requires that the extent of the cloud must be greater than the mean distance that a drop will fall during the time covered by the computation. This assumption will be examined later.

In contrast with the meteorological rain drop production we do not need to include the growth of drops due to condensation in a protoplanet. The drops form initially by melting of clumped interstellar grains and further growth is by coalescence only. In this paper we also did not include drop breakup from any cause. If we are allowed to use the analogy of rain drop formation in the Earth's atmosphere then Srivastava (1971) has shown that inclusion of drop breakup in the model only results in a 30% time delay in the growth of a distribution (containing mostly medium sized drops to large rain-drop sizes). Growth time from small to medium sized drops is affected very little. Further-more once a stationary distribution is reached (i.e., where the breakup rate is equal to the growth rate) the mean radius of the distribution differs by less than 5% from the results of a calculation which does not include drop breakup. These observations will not affect our conclusions. The mechanisms of drop breakup will be discussed in greater detail in the next section. We also neglect the growth of small drops due to wind shear (Manton, 1974; and Ryan, 1974). Inclusion of this effect (to be done in a subsequent paper) is expected to significantly deplete the small drop population and hence opacity sources in the protoplanet.

The mathematical solution of Equation (1) is done by the method of Berry and Reinhardt (1974). Their numerical approach to the solution of the stochastic collection equation, which is described in detail in their paper, is very accurate. We will describe the method briefly. Following Berry and Reinhardt (1974) we discretize $f(x)$ by defining

$$r(J) = r_0 \exp\,[(J-1)/J_0]\,, \tag{2}$$

$$x(J) = \tfrac{4}{3}\pi\rho_L r_0^3 \exp\,[3(J-1)/J_0]\,, \tag{3}$$

where ρ_L is the density of the liquid drops ($\rho_L = 7\,\mathrm{g\,cm^{-3}}$) and $r(J)$ is the radius of drops in mass bin J with mass $x(J)$. For the calculation presented here $r_0 = 5 \times 10^{-4}$ cm (the smallest radius considered), J assumed values from 1 to 100. Hence, the maximum radius considered is 46.341 cm – the largest mass bin actually occupied was for $J = 91$, $r = 16.38$ and J_0 is a constant chosen below. We further define $G(J) = 3x^2 f(x)/J_0$ where $G(J)$ is the mass of drops per unit volume per unit logarithm of the radius ($G(J)\mathrm{d}J = xf(x)\mathrm{d}x$). By use of these definitions Equation (1) may be written as

$$\frac{\partial G(J)}{\partial t} = \frac{x(J)}{J_0}\left\{\int_1^{J_d}\mathrm{d}J'\,\frac{x(J)}{x(J_c)}\,G(J_c)\,\frac{V(J_c,J')}{x(J_c)x(J')}\,G(J') - \int_1^{J_m}\mathrm{d}J'\,G(J)\,\frac{V(J,J')}{x(J)x(J')}\,G(J')\right\}, \tag{4}$$

where

$$J_d = x/2 = J - J_0 \ln 2/3 = J - 2 \qquad \text{if} \qquad J_0 = 6/\ln 2,$$

$$J_c = J + \frac{2}{\ln 2}\ln\,[1 - 2^{(J'-J)/2}]\,,$$

and J_m is the maximum index corresponding to the largest mass bin occupied. The solution for $G(J)$ in Equation (4) as a function of time proceeds as follows:

(1) calculate the gain and loss integrals for all J, sum for each J and get GAIN + LOSS $= G'(J)$, the change in mass bin J per unit time;

(2) find the maximum relative change in $G(J)$, i.e. maximum $(|G'(J)/G(J)|)$;

(3) limit the forward time step such that any $G(J)$ is not changed by more than 5%. (Halving this to 2–1/2% made no difference in the calculation);

(4) reset $G(J)$ to $G(J) + G'(J)\Delta t$ where $\Delta t = 0.05/\text{maximum}\,[G'(J)/G(J)]$ and finally

(5) go to step 1.

Equation (4) requires the evaluation of the gain and loss integrals and consequently interpolations in the function G to calculate $G(J_c)$. To demonstrate the accuracy of

Berry and Reinhardt's (1974) integral evaluation and interpolations let us define L_b and L_e to be the liquid iron content ($L = \int xf(x)\mathrm{d}x$) at the beginning and end of a calculation respectively, then in the calculation presented below the error ratio $|L_b - L_e|/L_b$ was less than 5% over approximately 15 000 forward integrations of Equation (4).

The shape of the initial mass distribution was chosen to approximate that of cloud droplets in the Earth's atmosphere. A commonly used approximate initial mass distribution is the Pearson type III distribution (Berry and Reinhardt, 1974; or Leighton and Rogers, 1974). In terms of $G(J)$ this distribution may be written as

$$G(J) = \frac{3L(\nu+1)^{\nu+1}}{\Gamma(\nu+1)} s^{\nu+2} \exp\left[-(\nu+1)s\right], \tag{5}$$

where $s = x(J)/\bar{x}_{f,\text{init}}$, $\bar{x}_{f,\text{init}}$ = mean mass of the initial number distribution = $\int xf(x)\mathrm{d}x / \int f(x)\mathrm{d}x$, $L = \int xf(x)\mathrm{d}x$ = liquid iron content ($\mathrm{g\,cm^{-3}}$), the var $f(x) = 1/(\nu+1)$ and $\nu = 1$ here. Since some clumping of interstellar grains is expected, the mean radius of the mass distribution function was chosen as $10\,\mu$m. This is roughly the geometric mean of the interstellar dust distribution (Mathis *et al.*, 1977) and the mean radius of the distribution that Cameron (1975) calculates for grain clumping in the primitive solar nebula using optimistic clumping probabilities. The mass fraction of iron in the cloud was taken as 0.001.

We note here that the time evolution of Equation (4) scales inversely with L, the liquid iron content, i.e. if one changes the mass fraction of iron to 0.01 instead of 0.001 the time of growth to a certain size is less by a factor of 10. ($L \propto$ mass fraction of iron.)

3. The Collection Kernel

COLLISIONAL CROSS-SECTION

The collection kernel $V(x, x')$ which contains most of the physics is more easily discussed in terms of the radii r_L and r_s where r_L and r_s are the radii of the large and small iron drops respectively. To clarify further discussion we will first consider the geometric collisional cross section. For this case

$$V(r_L, r_s) = \pi(r_L + r_s)^2 \,\Delta v,$$

$$\Delta v = |v(r_L) - v(r_s)| \qquad \text{and} \qquad v(r),$$

is equal to the velocity of drop r (discussed below). Since drop collisions actually occur in an atmosphere, it is more appropriate to use an aerodynamic collection kernel; or, more explicitly,

$$V(r_L, r_s) = \pi r_L^2 Y_c^2 \,\Delta v,$$

where Y_c is the linear collision efficiency. For the geometric case *only* $Y_c = 1 + r_s/r_L$ or Y_c is a linear function of r_s/r_L. In our sample calculation we used the aerodynamic collision efficiencies of Lin and Lee (1975). Figure 1 shows the collision efficiencies of iron drops in the same protoplanet environment which is used for the collection calculation of Section 4. The data of Lin and Lee were scaled to the protoplanet environment by calculation of the Reynolds number for a water drop in their environment and assuming that an iron drop with the same Reynolds number in a protoplanet has the same collision efficiency. Observation on Figure 1 shows that the aerodynamic collision efficiencies differ considerably from the geometric collision efficiencies (dashed curve).

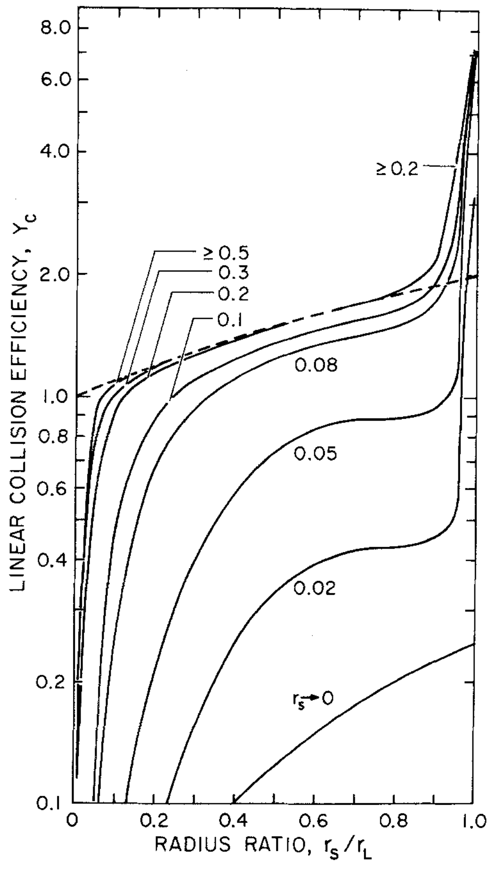

Fig. 1. Aerodynamic linear collision efficiencies for a Saturn mass protoplanet. The number associated with each line gives the radius of the larger drop in cm. Dashed curve shows geometric collision efficiencies.

The efficiency of collisions of drops with a small radius ratio ($r_s/r_L \leqslant 0.1$) is decreased because the smaller drop has a tendency to be pushed away from the larger collecting drop. For two nearly equal size droplets ($r_s/r_L \geqslant 0.8$) the collision efficiencies are greater than the geometric cross-section because, despite the fact that the collected drop is pushed away by gas flow in front of the collecting drop, the disturbed gases in back of the collecting drop drag the collected drop into its wake region. The lowest Reynolds number (for r_L) considered by Lin and Lee was 0.15. It is reasonable to assume that as the Reynolds number goes to zero a lower limit for Y_c is of order r_s/r_L, in other words the impact parameter has to be at least of order r_s. For a lower limit of Y_c, we chose $\frac{1}{4} r_s/r_L$. This is shown in Figure 1 with the label $r_s \to 0$.

IRON DROP VELOCITIES IN AN ATMOSPHERE

Before discussing the velocities of iron drops in a hydrogen and helium atmosphere it is instructive to determine the size range of interest.

A lower limit on the drop size can be found by consideration of surface effects during condensation. One can show (Rogers, 1976; p. 59; or Huang, 1963; p. 38) that drops with radii smaller than a critical radius

$$r_{\min} = \frac{2\sigma H\mu}{\rho_L k_B T \ln S}$$

are unstable towards evaporation. In the foregoing equation H is the mass of a hydrogen atom, μ is the molecular weight of the condensable, S is the ratio of the actual pressure to the saturation vapor pressure, σ is the surface tension of the fluid with density ρ_L, k_B is Boltzmann's constant and T is the temperature. According to DeCampli and Cameron's (1978) Saturn protoplanet models a typical value for the saturation ratio, S, is 3. Taking this value for S and a temperature of 1850 K we find $r_{\min} \simeq 10^{-7}$ cm. However since the minimum drop size considered in this paper is greater than 10^{-5} cm we will only display information for drops larger than 10^{-5} cm. At the other end of the scale, there are three mechanisms which limit drop growth; (1) aerodynamic instabilities (Pruppacher and Pitter, 1971; and Klett, 1971), (2) turbulence and (3) collision induced drop breakup. (Brazier-Smith *et al.,* 1972). Klett (1971) did a stability analysis of circular capillary-gravity waves in a drop and arrived at an upper limit for the aerodynamic instability of a drop as

$$r_{\max} = 1.84 \left[\frac{\sigma}{(\rho_L - \rho)g}\right]^{1/2},$$

where ρ is the density of the gas through which the drop is falling, g is the acceleration of gravity and σ is the surface tension. For $g = 1.2\ \mathrm{cm\,s^{-2}}$ (conditions similar to those in a protoplanet), $r_{\max} = 27$ cm. As mentioned earlier, drop breakup is only expected to effect the growth time from medium sized drops to large sized drops. Since large drops may conservatively be defined as those drops with radii $\gtrsim 5$ cm, the conclusions of this paper are not vitiated by excluding drop breakup from the model.

Fortunately for our purposes Beard (1976) has done a dimensional analysis of the empirical data for the fall velocities of water drops. This enables us to immediately apply his formulas to the calculation of iron drop terminal velocities in a protoplanet with an arbitrary atmosphere. Beard's formulas explicitly include the following effects:

(1) slip correction for when the drop size is on the order of the mean free path (Epstein regime), and

(2) flattening of larger drops as a function of the drops' surface tension, σ.

Beard's formulas are fitted only up to the maximum terminal velocity. Drops larger than this have essentially the same velocity due to flattening (Berry and Pranger, 1974).

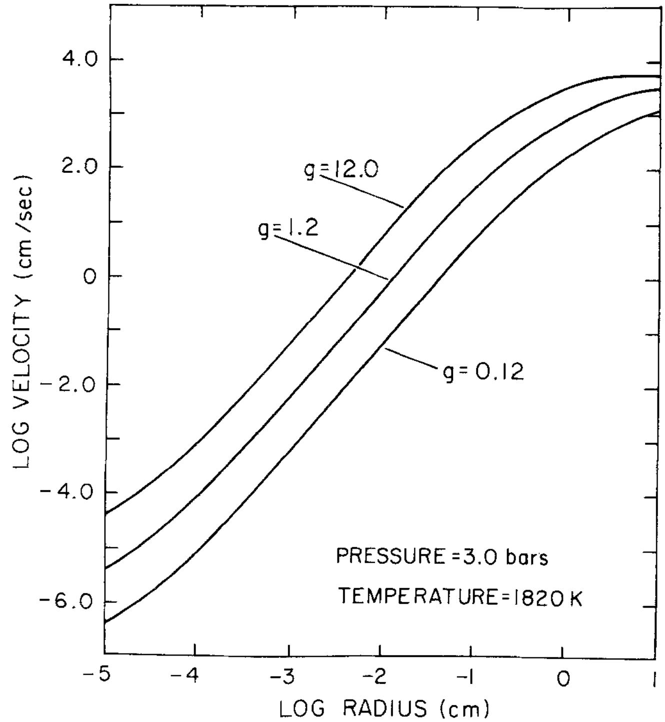

Fig. 2. Iron drop terminal velocities as a function of radius and the acceleration of gravity, g.

In Figure 2 we have plotted the terminal velocity of iron drops as a function of drop radius for 3 different values of the acceleration of gravity. Similarly, Figure 3 shows drop velocities at different pressures. The other parameters used for the plots are nominal values for temperature (1820 K), pressure (3 bar) and gravity ($1.2\ \mathrm{cm\ s^{-2}}$). The interpretation of Figure 2 is straightforward; however, Figure 3 requires an explanation. The central straight portion is essentially Stokes's law and merely demonstrates that viscosity is independent of pressure. At the lower end the drop size becomes comparable to the mean free path (Epstein regime) and the velocity is faster than Stokes's law due to boundary slippage. The amount of slippage is dependent upon the pressure. At the upper end of the scale, large drops have very high Reynolds numbers and hence the force on them is independent of the Reynolds number (Landau and Lifshitz, 1959; p. 169) and thus dependent upon the gas pressure. The leveling of the curves is of course due to drop flattening.

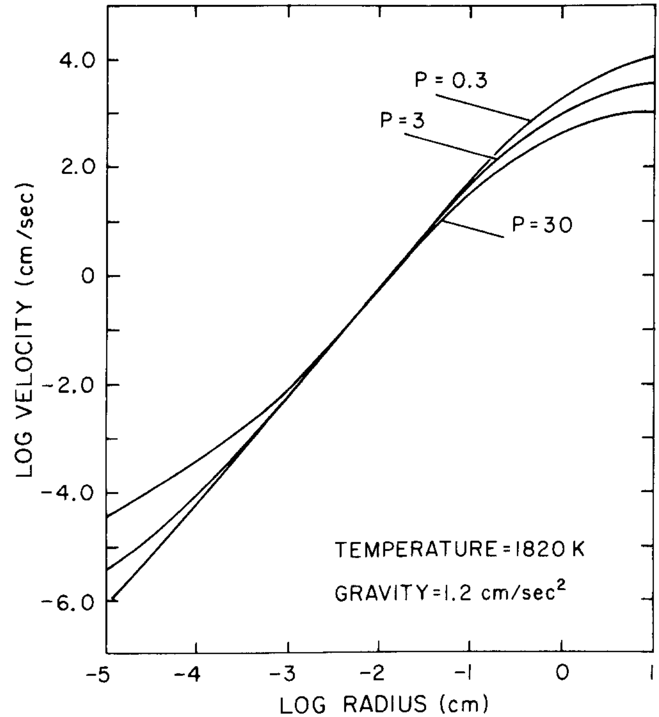

Fig. 3. Iron drop terminal velocities as a function of radius and pressure, P.

4. Results and Discussion

In Figure 4 we have plotted the results of our integration of Equation (4). On the vertical axis $G(x)$ is the mass (grams) of drops per cubic meter per unit logarithm of the radius (hence, the area under the curve is proportional to the mass of drops at that radius). Each curve in Figure 4 represents a snapshot of the mass spectrum of drops in an arbitrary (assuming boundary conditions are satisfied) cubic meter of the cloud at a particular time.

The environment of the cloud where the coalescence occurs is very similar to that obtained by DeCampli and Cameron's (1978) evolutionary calculations of an isolated protoplanet where the mass of the protoplanet is one Saturn mass. We chose the following parameters: pressure = 3.0 bar, gravity = 1.2 cm s^{-2}, temperature = 1820 K (melting point of iron) and the mass fraction of iron = 0.001. Under these conditions the maximum terminal velocity of drops is approximately 32 m s^{-1}. This velocity is obtained by drops larger than 7.3 cm in radius.

Observation of Figure 4 shows that the initial distribution with an approximate mean radius of 10 μm grew by coalescence to a distribution with a mean radius of 8 cm in 80 yr. The dashed line with spikes shows the growth of the radius of the mean mass of the distribution, $r_g = (x_g/\frac{4}{3}\pi\rho_L)^{1/3}$ where $x_g = \int x^2 f(x)/L$. The bimodal character of the final

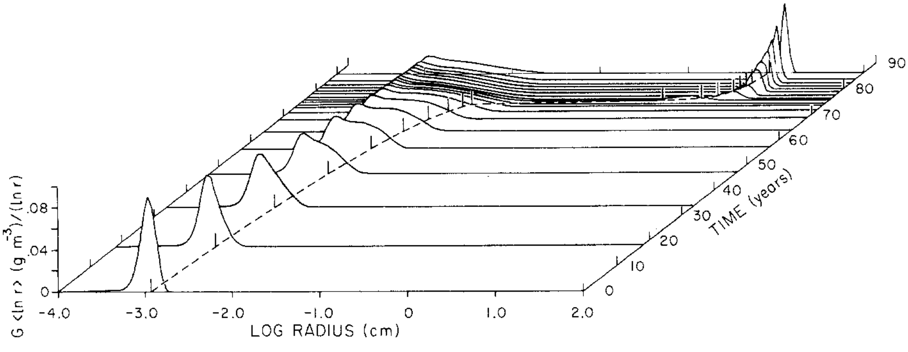

Fig. 4. Mass spectrums of iron drops in a Saturn mass protoplanet as a function of time. Dashed curve with vertical spikes is r_g (see text).

distribution is a result of the aerodynamic collection kernel (collisional cross section times relative velocity).

We are now in a position to determine the validity of the boundary conditions. As mentioned earlier the stochastic collection equation is valid as long as the distance that an average drop falls (during computation time) is small compared to the size of the cloud (6.3×10^{10} cm in a Saturn mass protoplanet [DeCampli and Cameron, 1978]). Taking 'small' to mean one-tenth of the cloud size we may write

$$6.3 \times 10^9 \text{ cm} = \int_0^{t_a} v(r_g)\mathrm{d}t, \tag{6}$$

(6)

where $v(r_g)$ is the velocity of the radius of the mean mass and t_a is the allowable growth time in 1/10 of the cloud. Simple trapezoidal of integration of Equation (6) gives $t_a = 75$ yr and correspondingly $r_{g,a} = 1.0$ cm and $v(r_{g,a}) = 850 \text{ cm s}^{-1}$. Drops larger than 1 cm in radius will of course form and rain out from the lower parts of the cloud, but the time scale is not as well defined. At the rate of 850 cm s^{-1} a 1 cm radius drop will fall from the top layer of the cloud in 2 yr. Since 1 cm is considerably less than 27 cm (aerodynamic instability maximum drop radius) we do not expect drop breakup from any cause to play a significant role in the time scale of growth to 1 cm in radius.

In summary, drop growth takes approximately 75 yr and after growth some rain from all of the cloud will be transported to the center of the protoplanet within two years. This calculation should be regarded as approximate since we have ignored such effects as the variation of gravity, drop breakup (for larger drops), wind shear growth (for smaller drops), and the inherent spherical geometry. Nevertheless, the time-scales should be of the right order. The actual formation of the core will be discussed in the next section.

5. Protoplanetary Core Formation and Subsequent Evolution

Time Scale for Core Formation

In the previous sections we have shown that in a protoplanet the rain-out of iron drops is an efficient mechanism for mass transport of iron to the center of the planet. Thus rain-out of iron leads to the formation of a protoplanetary core. Certain silicates would either rain-out with the iron (mixed in the initial clumped grains as solids) or possibly rain-out later depending upon the exact evolution of the protoplanet. Since droplet growth is so rapid compared with the evolutionary time scale we must conclude that a large cloud with high iron abundance throughout, such as used in the previous section, cannot actually exist in an evolving protoplanet. As the protoplanet evolves the cloud volume would expand slowly outward and rain-out would occur shortly after liquid iron appeared in each layer. Excluding convection, significant rain-out would continue as long as the cloud is expanding and iron remains liquid (up to 10^5 yr in a Saturn mass protoplanet; cf. DeCampli and Cameron, 1978). On the other hand convection (which is expected) would act to bring fresh grains into even a small spherical cloud at the center, and the rain-out would essentially be finished in a time interval which is a few times the convective turnover time scale, say 400 yr. DeCampli and Cameron (1978) give 60 yr as the convective turnover time in a Saturn mass protoplanet.

DeCampli and Cameron note further that since over 99% of the gravitational energy released by the infall of iron drops goes into viscous heating of the gases in the cloud layer, then the luminosity of a protoplanet during rain-out is approximately 10 times greater than it would be without rain-out. This excess luminosity, which would have serious consequences on evolutionary calculations demonstrates that grain rain-out must be included in the calculations for a correct picture of their evolution. The sudden energy release may lead to a substantial mass loss at the surface of the protoplanet, it would also almost guarantee convection in the cloud layer. Because of the higher temperatures produced, this excess luminosity also greatly complicates the time scale for core formation. More careful modeling will be required, but the time scale is expected to be somewhat greater than the 400 yr arrived at above.

SURVIVAL OF A CORE

After rain-out the protoplanet is expected to evolve until the envelope surrounding the core is no longer saturated with iron vapor (DeCampli and Cameron, 1978). The question arises, will the core survive during subsequent evolution? Since less than 1% of the gravitational energy released during the dropfall is released upon impact, heating at the core envelope boundary is expected to be minimal and thus energy transport is expected to revert back to radiation rather than convection a short time after rain-out ceases.

Under these assumptions the evaporation would be governed by diffusion. The rate of change of the radius of a sphere is given by the following expression

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{C}{r}, \tag{7}$$

$$C = (S-1)\left[\frac{\mu L_{\mathrm{h}}^2 \rho_{\mathrm{L}}}{K N_0 k_{\mathrm{B}} T^2} + \frac{N_0 k_{\mathrm{B}} T \rho_{\mathrm{L}}}{\mu D P_{\mathrm{s}}(T)}\right]^{-1}, \tag{8}$$

where L_{h} is the latent heat of vaporization, ρ_{L} is the liquid density, N_0, Avogadro's number; k_{B}, Boltzmann's constant, μ, the mean molecular weight of the liquid, T, the temperature; K, the coefficient of thermal conductivity of the atmosphere; D, the diffusion coefficient of iron vapor in the atmosphere; and $S = P/P_{\mathrm{s}}(T)$, where P is the pressure at the core envelope boundary and $P_{\mathrm{s}}(T)$ is the saturation vapor pressure for iron at temperature T. (Cf. Leighton and Rogers, 1974; or Rogers, 1976; p. 69ff.) Dimensionally Equation (7) goes to $\tau = R^2/C$ where τ is the life time of an object with radius R. For an Earth sized core, even under the extreme conditions of $T = 5000$ K, and $P = 3$ bar, $\tau \sim 10^{13}$ yr. (See Appendix for details on the calculation of C). Hence a large protoplanet core would survive evolution after the rain-out is completed.

Conclusions

On the basis of a Saturn mass protoplanet we have shown that the rain-out of iron, using the stochastic collection equation, is a rapid and efficient mechanism for protoplanetary core formation. Further if the core envelope boundary is not convective the core would survive protoplanetary evolution after the rain-out and core formation has occurred. We also note that the energy released as large drops fall through the atmosphere has a substantial effect on protoplanetary evolution.

A subsequent paper will deal with protoplanets of different masses, rain of various possible mineral melts, different initial drop spectrums, and the effect of wind shear on drop growth.

Appendix

In this appendix we will give the details of the calculation of various microscopic parameters such as viscosity, diffusion coefficient, etc., for a molecular hydrogen and helium gas mixture.

Preliminary Definitions

Define $W = X_m + Y_m$ where $X_m = X/A_{\mathrm{H}_2}$, $Y_m = Y/A_{\mathrm{He}}$, X is the hydrogen mass fraction, Y is the helium mass fraction (chosen as 0.78 and 0.22, respectively), A_{H_2} and A_{He} are the hydrogen and helium molecular weights respectively.

Diffusion Coefficient

Let $R_{m\mathrm{H}_2} = X_m/W$ and $R_{m\mathrm{He}} = Y_m/W$. Then the diffusion coefficient of iron into a molecular hydrogen and helium mixture is

$$D = (R_{m\mathrm{H}_2}/D_{\mathrm{H}_2-\mathrm{Fe}} + R_{m\mathrm{He}}/D_{\mathrm{He}-\mathrm{Fe}})^{-1},$$

where D_{H_2} and D_{He} are the binary diffusion coefficients of iron into H_2 and He respectively (Chapman and Cowling, 1970; p. 354). D_{H_2-Fe} and D_{He-Fe} are obtained by using a power law fit of binary diffusion coefficients of noble gas data, e.g. $D_{1-2} = pA_2^q$ where D_{1-2} is the diffusion coefficient of molecule 2 in a molecular gas of element 1, p and q are fitted constants and A_2 is the atomic weight of molecule 2. As a result of the fitting procedure, $D_{He-Fe} = 0.629$ and $D_{H_2-Fe} = 0.640\,\mathrm{cm^2\,s^{-1}}$ at standard conditions (Chapman and Cowling, 1970; p. 263). Both coefficients were adjusted for temperature and pressure by use of the formula

$$D(T,P) = D_{sc}\left(\frac{T}{273}\right)^{1.75}\left(\frac{10^6}{P}\right),$$

where D_{sc} is the diffusion coefficient at standard conditions (*American Institute of Physics Handbook*, 1972; pp. 2–250).

Viscosity

Assuming that the molecular collision cross sections for hydrogen and helium are the same, one can show (Reif, 1965; p. 473f) that the average viscosity for a mixture of hydrogen and helium can be expressed as

$$\eta_{av} = (\eta_{H_2} X_m + \eta_{He} Y_m)/W,$$

where η_{av} is the average viscosity of the gas and η_{H_2} and η_{He} are the viscosities of hydrogen and helium at the correct temperature which are obtained from the Sutherland interpolation equation

$$\eta_i = \eta_{0,i}\,\frac{273 + t_i}{T + t_i}\left(\frac{T}{273}\right)^{3/2}.$$

where $\eta_{0,H_2} = 8.51 \times 10^{-5}$ poises and $\eta_{0,He} = 1.842 \times 10^{-4}$ poises, $t_{H_2} = 70.6\,\mathrm{K}$ and $t_{He} = 97.6\,\mathrm{K}$ (*Smithsonian Physical Tables*, 1956; p. 331).

Thermal Conductivity

We used the following formula for the thermal conductivity:

$$K = \tfrac{1}{4}(9\gamma - 5)\eta c_v,$$

where η is the viscosity, $\gamma = c_p/c_v$, c_p and c_v are the specific heats (per gram) at constant pressure and volume of the hydrogen and helium gas mixture (*Smithsonian Physical Tables*, 1956; p. 142).

Mean Free Path

Again, assuming that the collisional cross sections of hydrogen and helium molecules are the same, one can show (Reif, 1965; p. 471) that the average mean free path may be expressed as

$$l_{av} = (l_{H_2} X_m + l_{He} Y_m)/W,$$

where $l_{H_2} = 16 \times 10^{-6}$ cm and $l_{He} = 25.25 \times 10^{-6}$ cm are the mean free paths of hydrogen and helium, respectively, at 0.9868 bar and 273 K. Further correction for temperature and pressure give

$$l = l_{av}\left(\frac{\eta}{\eta_{273}}\right)\left(\frac{0.9868 \times 10^6}{P}\right)\left(\frac{T}{273}\right)^{1/2},$$

where η_{273} is the viscosity of the mixture at 273 K (Beard, 1976).

Other Numerical Constants

The value for the latent heat of iron, L_h, was taken as 7.23×10^{10} erg gm^{-1} (*Smithsonian Physical Tables*, 1956; p. 165). The value for the surface tension of liquid iron was taken as $\sigma = 1780$ dyn cm^{-1} (*Handbook of Chemistry and Physics*, 1974; p. F-29).

Acknowledgements

I wish to acknowledge many helpful conversations with Drs A. G. W. Cameron and W. DeCampli of the Harvard Astronomy Department. I greatly appreciate the use of Berry and Reinhardt's (1974) waterdrop collection program which was sent to me by Dr E. X Berry of Atmospheric Research and Technology, Sacramento, CA. I also wish to thank a referee, Dr George Rybicki, for some very important comments on the aerodynamic collisional cross-sections. This research has been supported by NASA grant number NGR 22-007-269.

References

American Institute of Physics Handbook, 1972, 3d edition, McGraw Hill, New York.

Berry, E. X: 1967, 'Cloud Droplet Growth by Collection', *J. Atmospheric Sci.* **24**, 688–701.

Berry, E. X and Pranger, M. R.: 1974, 'Equations for Calculating the Terminal Velocities of Water Drops', *J. Appl. Meteorol.* **13**, 108–113.

Berry, E. X and Reinhardt, R. L.: 1974, 'An Analysis of Cloud Drop Growth by Collection: Part I. Double Distributions', *J. Atmospheric Sci.* **31**, 1814–1824.

Beard, K. V.: 1976, 'Terminal Velocity and Shape of Cloud and Precipitation Drops Aloft', *J. Atmospheric Sci.* **33**, 851–864.

Brazier-Smith, P. R., Jennings, S. G., and Latham, J.: 1972, 'The Interaction of Falling Water Drops: Coalescence', *Proc. Roy. Soc. London A.* **326**, 393–408.

Cameron, A. G. W.: 1975, 'Clumping of Interstellar Grain during Formation of the Primitive Solar Nebula', *Icarus* **24**, 128–133.

Cameron, A. G. W.: 1978, 'Physics of the Primitive Solar Accretion Disk', *The Moon and the Planets* **18**, 5.

Chapman, S. and Cowling, T. G.: 1970, '*The Mathematical Theory of Non-Uniform Gases*', University Printing House, Cambridge.

Danielsen, E. F., Bleck, R., and Morris, D. A.: 1972, 'Hail Growth by Stochastic Collection in a Cumulus Model', *J. Atmospheric Sci.* **29**, 135–155.

DeCampli, W. M. and Cameron, A. G. W.: 1978, 'Giant Gaseous Protoplanets', submitted to *Icarus,* April 1978.

Klett, J. D.: 1971, 'On the Breakup of Water Drops in Air', *J. Atmospheric Sci.* **28**, 646–647.

Handbook of Chemistry and Physics, 1974 57th Edition, Chemical Rubber Company, Cleveland.
Huang, K.: 1963, *Statistical Mechanics*, John Wiley and Sons, New York.
Landau, L. D. and Lifshitz, E. M.: 1959, *Fluid Mechanics*, Addison Wesley, Reading, Mass.
Leighton, H. G. and Rogers, R. R.: 1974, 'Droplet Growth by Condensation and Coalescence in a Strong Updraft', *J. Atmospheric Sci.* **31**, 271–279.
Lin, C. L. and Lee, S. C.: 1975, 'Collision Efficiency of Water Drops in the Atmosphere', *J. Atmospheric Sci.* **32**, 1412–1418.
Manton, M. J.: 1974, 'The Effect of a Simple Shear Flow upon the Coalescence Rate of Water Droplets', *Geophys. Fluid Dynamics* **6**, 83–89.
Mathis, J. S., Rumpl, W., and Nordsieck, K. H.: 1977, 'The Size Distribution of Interstellar Grains', *Astrophys. J.* **217**, 425–433.
Ogura, Y. and Takahashi, T.: 1973, 'The Development of Warm Rain in a Cumulus Model', *J. Atmospheric Sci.* **30**, 262–277.
Pruppacher, H. R. and Pitter, R. L.: 1971, 'A Semi-Empirical Determination of the Shape of Cloud and Rain Drops', *J. Atmospheric Sci.* **28**, 86–94.
Reif, F.: 1965, *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, New York.
Rogers, R. R.: 1976, *A Short Course in Cloud Physics*, Pergamon, New York.
Ryan, B. F.: 1974, 'Growth of Drops by Coalescence: The Effect of Different Collection Kernels and Additional Growth by Condensation', *J. Atmospheric Sci.* **31**, 1942–1948.
Smithsonian Physical Tables, 1956, Smithsonian, Washington, D.C.
Srivastava, R. C.: 1971, 'Size Distribution of Raindrops Generated by their Breakup and Coalescence', *J. Atmospheric Sci.* **28**, 410–415.
Takeda, T.: 1971, 'Numerical Simulation of a Precipitating Convective Cloud: the Formation of a "Long-Lasting" Cloud', *J. Atmospheric Sci.* **28**, 350–376.
Young, K. C.: 1974, 'A Numerical Simulation of Wintertime, Orographic Precipitation: Part I. Description of Model Microphysics and Numerical Techniques', *J. Atmospheric Sci.* **31**, 1735–1748.