ORBITAL STABILITY OF MASSIVE PROTOPLANETS IN THE TERRESTRIAL PLANET REGION OF THE SOLAR SYSTEM

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Abstract. A number of theories of the formation of the planets advocate that the terrestrial planets were originally of cosmic composition and that it is only subsequent evolution that has removed their volatile components. This paper shows that such protoplanets could have remained in the terrestrial planet region without significant changes occurring in their orbits for an acceptable time interval.

1. Introduction

The mass of material of cosmic composition with a non-volatile content of mass equivalent to that of a terrestrial planet is about that of Jupiter (see for example Williams, 1977). This fact naturally leads to the notion that all the planets may have originated from a family of gaseous protoplanets, each of cosmic composition and mass comparable to Jupiter (Kopal, 1972). Theories for the formation of such protoplanets have been proposed by McCrea (1960), Woolfson (1964) and Cameron (1978). Of course, to obtain the terrestrial planets from such gaseous protoplanets, some chemical segregation must occur (see Williams 1978 for a discussion) but this is of no direct concern in the present work.

If such protoplanets existed, then their increased mass makes it likely that the dynamics of the inner solar system was radically different from that existing at present. Indeed, a simple calculation indicates that the presence of forces in excess of 10^4 times the maximum force exerted by Jupiter on a present day terrestrial planet would be a common event. It is therefore pertinent to ask whether such a system of gaseous protoplanets could ever have co-existed. There are in fact two questions to be answered. First, does a gross instability exist where a protoplanet is forced from its original neighbourhood in a few orbits. If this is not the case, then secondly is there a progressive evolution of the orbits away from their initial positions occurring on a reasonable time scale.

Since we do not know the original configuration of the protoplanets the problem is ill-defined as posed. As numerical solutions to the equations of motion have to be found, it is also not practical to investigate a large number of possible starting configurations. We therefore investigate the configuration most likely to be unstable and hope to show that this is not unstable, hence proving that it is meaningful to consider a family of protoplanets in the terrestrial planet region. We will in fact simplify the problem further. Since the largest perturbations arise from the interaction between the two closest pairs (remembering that they have almost equal masses), we consider only such binary interactions. The two closest planets amongst the terrestrial planets are Venus and the Earth with a minimum separation of 0.28 AU. We therefore consider the dynamical evolution of the

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three body system consisting of two protoplanets in orbit about the Sun, each being of mass similar to Jupiter with one initially on an Earth-like orbit and one on a Venus-like orbit.

Although it is not necessary to assume that motion is confined to one plane, it is of course necessary to assume some initial configuration. Harrington (1972) has shown that the stability of such a three body system will not in general depend on the inclination of the orbits except for inclinations within a few degrees of 90°. We therefore consider planar motion only since this leads to some saving in computation time.

Such a configuration can be followed by integrating the equations of motion for about a thousand orbits. If no secular changes in the orbits emerge in such a period, then it may be reasonable to conclude that nothing drastic will occur in an interval of perhaps ten thousand orbital periods. For intervals longer than this some other technique must be employed in order to make deductions regarding the orbital behaviour of the system. We therefore consider relevant protoplanetary time scales. Segregation of the non-volatile elements takes 10^3–10^4 yr (McCrea and Williams, 1965). Tidal instability is fairly rapid (Donnison and Williams, 1975), although it should be borne in mind that the timescale given by this calculation is that for the first part of the escaping envelope to leave the protoplanet, dispersal of the whole protoplanet takes considerably longer. 10^4 yr is not therefore an unreasonable time to require protoplanets to remain in existence. It is therefore meaningful to proceed by integrating the equations of motion for about a thousand orbits.

2. Calculations

Let $\mathbf{r}_1$, $\mathbf{r}_2$ be the position vectors of the two protoplanets, of mass M_1 and M_2, relative to the Sun. Then $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$ is the vector from M_1 to M_2. Let C be the centre of mass of the Sun and M_1, and $\mathbf{r}_c$ its position vector relative to the Sun, then $\mathbf{r}_c = M_1/(M_1 + M_\odot)\mathbf{r}_1$. Also let $\mathbf{r}_{c2}$ be the position vector of M_2 relative to C, then

$$\mathbf{r}_{12} = \mathbf{r}_{c2} - \frac{M_\odot}{M_1 + M_\odot}\mathbf{r}_1$$

and

$$\mathbf{r}_2 = \mathbf{r}_{c2} + \frac{M_1}{M_1 + M_\odot}\mathbf{r}_1 .$$

It is easy to show that the equations of motion then take the form

$$\ddot{\mathbf{r}}_1 = -G(M_\odot + M_1)\left(\frac{\mathbf{r}_1}{r_1^3} + \frac{M_2}{(M_\odot + M_1)}\frac{\mathbf{r}_2}{r_2^3} - \frac{M_2}{(M_\odot + M_1)}\frac{\mathbf{r}_{12}}{r_{12}^3}\right) \tag{1}$$

and

$$\ddot{\mathbf{r}}_{c2} = -G\frac{(M_\odot + M_1 + M_2)}{(M_\odot + M_1)}\left(\frac{M_\odot \mathbf{r}_2}{r_2^3} + \frac{M_1 \mathbf{r}_{12}}{r_{12}^3}\right) \tag{2}$$

Equations for the conservation of energy and angular momentum can also be derived and these can be used to check the accuracy of the numerical solutions obtained for Equations

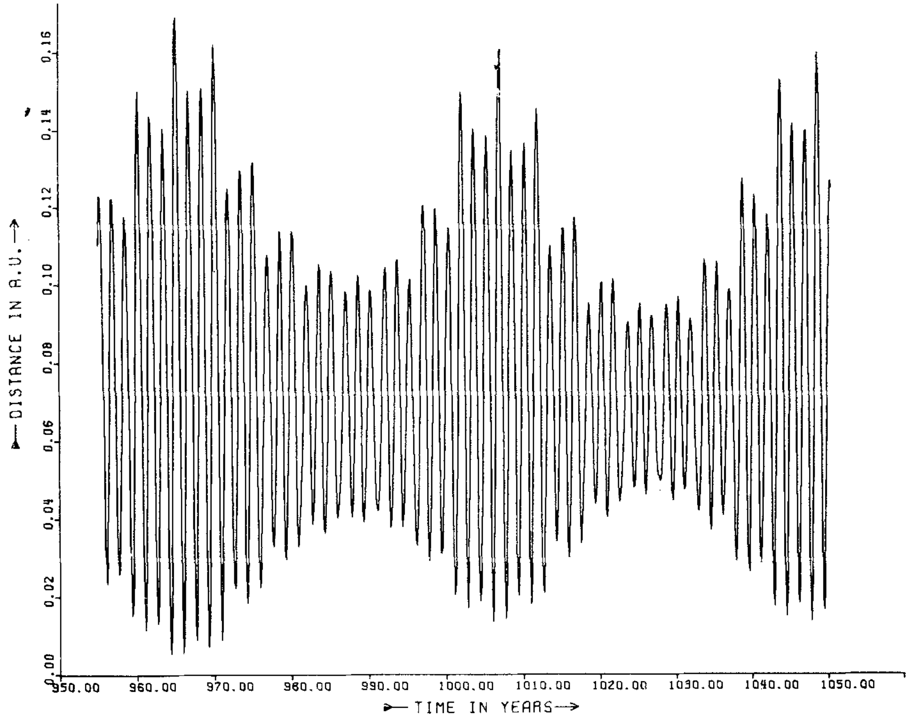

Fig. 1. Shows the variation of the position of the Proto-Earth from its initial position for the final hundred years of the period integrated.

(1) and (2). The energy equation is at least one order of magnitude more sensative than the angular momentum equation (Harrington, 1972) and we use it as the major test. It is given by

$$E = \frac{1}{2}\left(\frac{M_\odot M_1}{M_\odot + M_1}\right)\dot{r}_1^2 + \frac{1}{2}\left(\frac{M_\odot + M_1}{M_\odot + M_1 + M_2}\right)M_1\dot{r}_{c2}^2$$
$$- G\left(\frac{M_\odot M_1}{r_1} + \frac{M_\odot M_2}{r_2} + \frac{M_1 M_2}{r_{12}}\right). \qquad (3)$$

Equations (1) and (2) were integrated numerically using the standard fourth order Runge-Kutta step by step integration procedure. Since we are dealing with near circular orbits there is no advantage in using a variable steplength. As mentioned in the introduction M_1 and M_2 were taken to be equal, with the value $10^{-3} M_\odot$. As initial conditions M_1 and M_2 were taken to be moving on circular orbits at distance of 1 AU and 0.723332 AU from the Sun. The integrations were carried out for a period equivalent to a little over 1000 yr. For an acceptable numerical solution the calculated energy was required to remain constant to eight significant figures.

3. Results and Conclusions

It was found that there were no close encounters between any of the components and no tendency for any of the bodies to escape so that the presence of gross instabilities which tend to show up after a small number of orbits (see Harrington, 1972) was ruled out. In order to examine the possibility of any instability emerging after a longer period a plot showing the variations from the initial value of the separation of all three bodies from the centre of mass of the system against time was constructed. A segment of this plot for the protoplanet at the Earth's distance is shown in Figure 1 for the final hundred years of the period integrated. This shows clearly, as do all the other plots, that the perturbations are periodic in nature with no discernable secular changes present. The system is therefore stable over the period covered by the integrations and will not be disrupted within the period of interest.

As we have investigated the configuration most likely to produce instabilities, and found none, it is clear that no difficulty is encountered if massive protoplanets coexist in the terrestrial planet region. If any theory demands it, it is therefore acceptable to place protoplanets on circular terrestrial-like orbits before considering any further evolution of the protoplanet.

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