# LUNAR CORE DETECTION USING TWO SURFACE MAGNETOMETERS AT VERY LOW FREQUENCY

FLOYD HERBERT

*Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ, U.S.A.*

(Received 16 November, 1979)

**Abstract.** A method of analysis is developed for combining the measurements of two surface magnetometers so as to deduce empirically the very low frequency electromagnetic transfer function of the Moon. The method is expected to be useful in determining the presence of a lunar metallic core by making usable for this purpose simultaneous magnetic field measurements by the Apollo 15 and 16 surface modules.

## 1. Introduction

Whether the Moon possesses a metallic core is still very much an open question. Axial moment of inertia data gives a value for the mean moment of inertia coefficient of $0.3904 \pm 0.0023$ (W. L. Sjogren, personal communication), which is a value quite suggestive of the presence of a core. Seismic data suggest a core of radius about 170–360 km (Latham *et al.*, 1978). Current electromagnetic sounding analyses based upon the power spectral densities of signals at Explorer 35 and Apollo 12 suggest an upper bound on a highly conductive core of about 400 km radius (Wiskerchen and Sonett, 1977).

One limitation of electromagnetic sounding is the length of available magnetic field time series data. Only by choosing very low excitation frequencies (corresponding to very long data lengths) can core detection be accomplished. Core detection requires frequencies at least as low as $10^{-5}$ Hz, and thus depending on the time series analysis alogarithm to be used, the time series length may be as short as $10^5$ s. Conservatively one prefers, however, at least 50 or more hours.

Data swaths this long are rare in the overlap of Apollo 12 and Explorer 35 measurements, and thus one would like to augment the data available from this source with additional information. By the time of emplacement of later lunar surface magnetometers, however, Explorer 35 was not functioning, and hence a method of analysis not requiring data taken by a lunar orbiter is required.

In the next section a method is described which allows analysis of the dipolar component of the lunar induction using only measurements of two surface magnetometers. This is possible because the very low frequency of the relevant signal components limits the lunar response to the dipole partial wave. Because only surface magnetometers are necessary, the simultaneous measurements mady by the Apollo 15 and 16 surface modules can be utilized. Thus additional low frequency information concerning the lunar transfer function is thereby made applicable to the problem of determining the size and existence of a metallic lunar core.

*The Moon and the Planets* 23 (1980) 127–131. 0165–0807/80/0231–0127$00.75.

## 2. Theory

The theoretical basis for the analysis method developed in this paper is the spherically symmetric plasma (SSP) confinement model of Sonett *et al.*, (1972). In this approximation the confinement of the induced magnetic field fluctuations on the day side of the Moon by Lenz' law currents flowing in the incoming solar wind is assumed (for simplicity) to hold over the entire lunar surface. Approximate calculations of the modification of the transfer function due to the lack of confinement on the anti-solar side of the Moon (Schubert *et al.,* 1973) have shown that for the case of small cores (400 km or less) as contemplated here in the light of prior analysis (Wiskerchen and Sonett, 1977) the error made by using the SSP approximation at low frequency is not large.

The additional approximation made in the present work consists of restricting the analysis to low enough frequency that the SSP response is dominated by the dipole term. In such a case the magnetic field fluctuations at angular frequency $\omega$ detected at the surface are given by

$$\begin{aligned} b_r &\approx B_0 c(\omega) \sin\theta \sin\phi \, \mathrm{e}^{-i\omega t}, \\ b_\phi &\approx B_0 c(\omega) \cos\phi \sin\phi \frac{a}{2} \frac{\mathrm{d}G_1(a)}{\mathrm{d}r} \mathrm{e}^{-i\omega t}, \\ b_\theta &\approx B_0 c(\omega) \cos\phi \frac{a}{2} \frac{\mathrm{d}G_1(a)}{\mathrm{d}r} \mathrm{e}^{-i\omega t}, \end{aligned} \tag{1}$$

where $a$ is the radius of the Moon, $B_0$ is the magnitude of the solar wind forcing field, $\theta$ and $\phi$ relate to the coordinates of the surface magnetometer in the scattering angle frame of reference (see Sonett *et al.,* 1972) and $G_1(r)$ is the radius-dependent factor of the dipole term in the multiple expansion for the pseudopotential describing the lunar response (see Sonett *et al.,* 1972). The quantity $c(\omega)$ is given by

$$c(\omega) = \frac{3\lambda}{2\pi a} j_1 \left(\frac{2\pi a}{\lambda}\right), \tag{2}$$

where $\lambda = 2\pi v_{ph}/\omega$, $v_{ph}$ is the phase velocity of the incoming wave and $j_1$ is the spherical Bessel function of order unity. In the long-wavelength limit $\lambda \gg a$ and $c(\omega) \approx 1$.

When isochronous data from both Apollo 15 and 16 are used, the three equations (1) yield a set of six simultaneous equations in six unknowns: $B_0$, $\mathrm{d}G_1(a)/\mathrm{d}r$, and the values of $\theta$ and $\phi$ corresponding to each site.

If we combine the two tangential power spectral densities, $B_\phi^2 = b_\phi^* b_\phi$ and $B_\theta^2 = b_\theta^* b_\theta$ from either site we obtain the total tangential power

$$\begin{aligned} B_{\tan}^2 &= B_\theta^2 + B_\phi^2, \\ &= B_0^2 c(\omega)^2 \left(\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r}\right)^2 \left(\cos^2\theta \sin^2\phi + \cos^2\phi\right), \end{aligned} \tag{3}$$

$$= B_0^2 c(\omega)^2 \left(\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r}\right)^2 \left(1 - \sin^2\theta \sin^2\phi\right).$$

However,

$$B_r^2 = B_0^2 c(\omega)^2 \sin^2\theta \sin^2\phi; \tag{4}$$

so that

$$B_{\tan}^2 + \left(\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r}\right)^2 B_r^2 = B_0^2 c(\omega)^2 \left(\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r}\right)^2. \tag{5}$$

Thus we find that

$$\left(\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r}\right)^2 = \frac{B_{\tan}^2}{B_0^2 c(\omega)^2 - B_r^2}. \tag{6}$$

Since $\mathrm{d}G_1(a)/\mathrm{d}r$ has only radial dependence it is independent of the site from which the data is taken, and we can equate values for the two sites and obtain

$$B_0^2 = \frac{1}{c(\omega)^2} \frac{{}^{(1)}B_{\tan}^2\, {}^{(2)}B_r^2 - {}^{(2)}B_{\tan}^2\, {}^{(1)}B_r^2}{{}^{(1)}B_{\tan}^2 - {}^{(2)}B_{\tan}^2}, \tag{7}$$

where the superscripts in parenthesis distinguish measurements made at the two sites. Combining Equations (6) and (7) we find that

$$\left(\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r}\right)^2 = -\frac{{}^{(1)}B_{\tan}^2 - {}^{(2)}B_{\tan}^2}{{}^{(1)}B_r^2 - {}^{(2)}B_r^2} \tag{8}$$

The potential conflict between the signs of the two sites of Equations (7) and (8) that might arise due to the presence of noise in the data (such as selective perturbation of local permanent fields by varying solar wind momentum flux) may serve as a guide to data quality. Another possible indicator of data reliability is the value of $\sin^2\theta \sin^2\phi$ which can be obtained from Equation (4) by eliminating $B_0^2 c(\omega)^2$ as obtained from Equation (7). If values for this derived quantity fall above unity very often, one might conclude that the quality of the data will not support the analysis.

Since we are primarily interested in the asymptotic low-frequency behavior of the transfer function as influenced by a possible iron core, one might approximate the correct $G_1(r)$ by the function corresponding to the simpler case of an infinitely conductive core imbedded in an insulating mantle. Comparisons with prior calculations of this simple case (e.g., Schubert *et al.*, 1973) of a two-layer Moon yield

$$\frac{a}{2}\frac{\mathrm{d}G_1(a)}{\mathrm{d}r} \approx (1 + b^3/2a^3)/(1 - b^3/a^3), \tag{9}$$

where $b$ is the core radius. Justification for Equation (9) is based on the expected steep jump in conductivity between a rocky mantle and metallic core, simulating a two-layer Moon. Substitution of Equation (8) into (9) yields

$$b \approx a\left(\frac{q-1}{q+\frac{1}{2}}\right)^{1/3}, \tag{10}$$

where $q$ is the right-hand side of Equation (8). Equation (10) yields the core size directly if the data contained in $q$ are drawn from the frequency range where the full transfer function is asymptotically approaching a constant value.

The term "asymptotic" is used loosely here, inasmuch as the true function behavior more nearly resembles a plateau. At extremely low frequencies (far beyond our capacity for measurement) the skin depth in the iron core rises to a value comparable to the core radius and the transfer function declines to unity.

## 3. Error Sources

The most significant source of errors in the magnetometer data is likely to be fluctuations of the local permanent fields forced by variations of the solar wind momentum flux. The permanent field at the Apollo 16 site is large and consequently does not have to vary by much of its total magnitude in order to produce a significant source of fluctuation power unrelated to the external solar wind field.

Another source of spurious signal power could be the fringing field from the diamagnetic response of the tail cavity behind the Moon. This source has been suggested as a contaminant in the traditional type of transfer function measurements (Wiskerchen and Sonett, 1977) but is possibly not as significant as solar wind interaction with local fields.

The misestimation by SSP theory of the lunar response due to lack of confinement on the night side could also cause some inaccuracies, but as was mentioned before, the error is not expected to be large. On the other hand, the induction effect sought is also small. Estimation of the correction for a given trial model using asymmetric theory, as was done by Wiskerchen and Sonett (1977) would probably be sufficient to eliminate this error source.

## 4. Conclusions

The small amount of data available for the difficult task of searching for the lunar core can be augmented by making use of the overlapping Apollo 15 and 16 magnetometer data. While the usual task of data matching, degapping and cleaning is likely to be as difficult as for the traditional method of analysis, the simple theory presented in the present work can be used quite easily to reduce statistical uncertainties or even extend the low frequency limit of the existing transfer function estimates. Some data quality indicators exist which may give confidence to the results derivable from the present work's method of analysis. Consequently we expect this method to be of use in the sounding of the lunar interior.

## Acknowledgements

This work was supported by NASA grant 7020. This work was greatly aided by numerous conversations with C. P. Sonett.

## References

Latham, G. V., Dorman, H. J., Horvath, P., Ibrahim, A. K., Koyama, J., and Nakamura, Y.: 1978, *Proc. 9th Lunar Planet. Sci. Conf.*, 3609–3613.

Schubert, G., Sonett, C. P., Schwartz, K., and Lee, H. J.: 1973, *J. Geophys. Res.* **78**, No. 13, 2094–2110.

Sonett, C. P., Smith, B. F., and Colburn, D. S.: 1972, *Proc. 3rd Lunar Sci. Conf.* 2309–2336.

Wiskerchen, M. J. and Sonett, C. P.: 1977, *Proc. 8th Lunar Sci. Conf.* 515–535.