ON THE ORIGIN OF COMETARY NUCLEI IN THE PRESOLAR NEBULA*

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Abstract. If we assume that the cometary nuclei originated by the gravitational instability of a dust layer, which formed in the equatorial plane of the outer parts of the presolar nebula (PSN) during a period of approximate equilibrium between gravity, centrifugal force, and the pressure gradient, a simple relation is derived between the PSN's temperature and the upper limit to the mass of the planetesimals. It contains, besides the density of the cometary nuclei ρ_p, only the fraction (by mass) of the condensable elements in the PSN, which became part of the dust particle disc, which, on the basis of available observational evidence on the solid particles in interplanetary and interstellar space and of theoretical considerations on the relationship between them and on the sedimentation process, is found to be of the order of $\gtrsim 10\%$; this estimate will require still further justification. Assuming a temperature in the range 15–20 K, an equatorial diameter of the PSN of 0.1 pc and $\rho_p \approx$ a few 0.1 g/cm^3, upper limits for the planetesimal's mass of $\approx 10^{18}$g and for their radius of ≈ 10 km are obtained (on the basis of conservation of circulation, of mass and of angular momentum in the differentially rotating disc), in fair agreement with observation. With the dispersion of those parts of the PSN – of an assumed original mass of 2–3$M_\odot$ –, which did not become part of the Sun or the planets, by the young Sun's activity, the planetesimals must have lost a large part of their gravitational binding energy and their orbits must have become so large (semimajor axis several 10^4 A.U. or more, if not negative), that stellar perturbations produced the distribution in configurational and in velocity space now observed.

1. Introduction

Kuiper's work of 1951 has revived the interest in gravitational instability of a presolar nebula (PSN) as one of the mechanisms leading to the formation of planetesimals. In 1973 Goldreich and Ward (G & W) have studied the gravitational instability of the dust disc, which in a rotating PSN should form under certain rather general conditions; G & W have discussed this process for the inner parts of the solar system where, presumably after the Sun had begun to shine, the terrestrial planets formed.**

In recent years the similarities between the chemical composition of cometary nuclei and the condensable constituents of dense interstellar clouds have become more and more apparent (see for instance the contributions by Donn, and Whipple and Lecar to the GSFC Symposium on Comets, 1974; and Shimizu 1977, to appear). Work by Everhart (1973) has established that the long period and "new" comets are most unlikely to have

* Paper dedicated to Professor Hannes Alfvén on the occasion of his 70th birthday, 30 May, 1978.
** The earlier work done since about 1950 in the U.S.S.R. is described in Safronov (1972).

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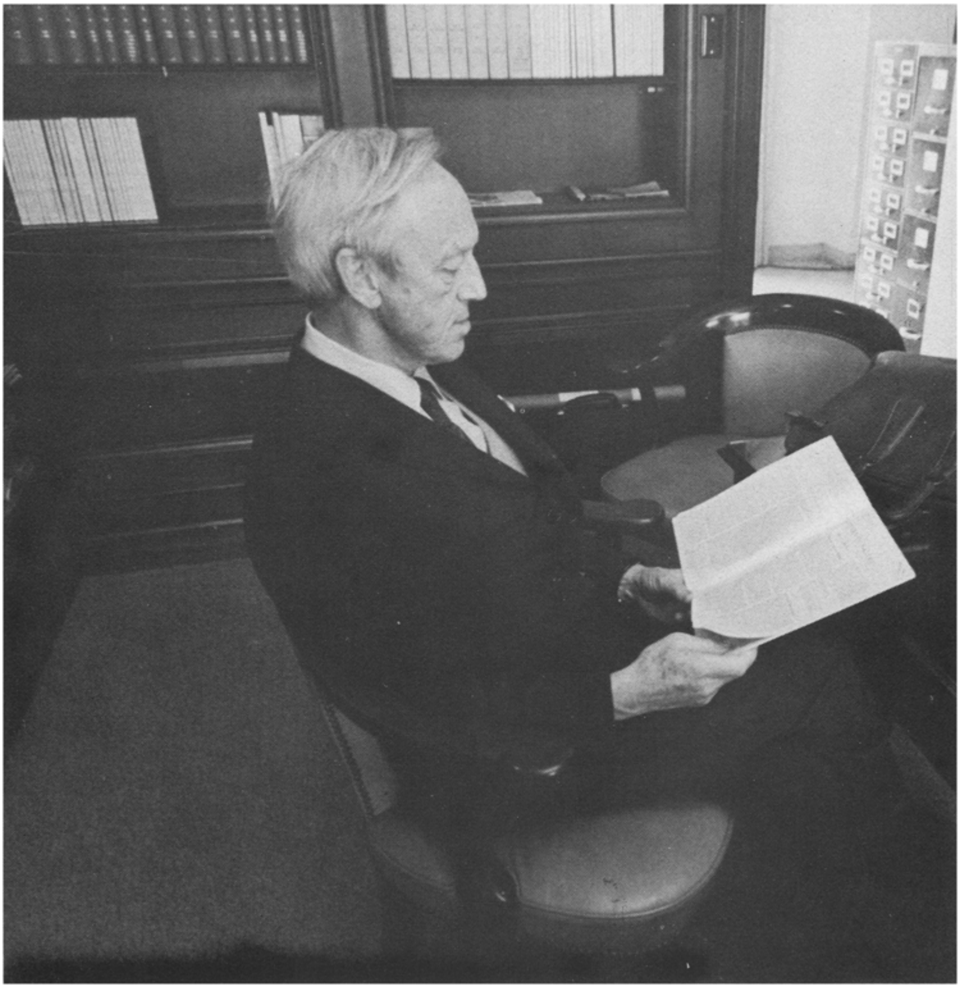

Professor Hannes Alfvén, at the Geophysical and Geochemical Conference held at the Lunar Science Institute in Houston, Texas, between January 10–12, 1973.

originated in the regions occupied by the planets. The alternative, an origin in the outer parts of the PSN means that "new" comets are likely to be almost untouched specimens of the original material from which the planets formed. This is, of course, the main reason, why cometary space missions are under active discussion (Biermann and Michel, 1977). The question how the cometary nuclei got into the present volume of the "Oort cloud" is evidently much more easily answered the further away from the center of gravity of the PSN they formed, particularly so if they formed before the Sun became luminous and active and the non-condensed part of the PSN was removed.*

* All values for the ratio $^{12}C/^{13}C$ determined so far point to an origin of the cometary nuclei 5×10^9 years ago (Vanýsek and Rahe, 1978).

Cameron (1976, 1977) has reported that in the outer parts of the dust disc of his model of the early solar system (formed *after* the PSN was dispersed by hot coronal gases) planetesimals of 10^{16} g should have formed by gravitational instability (along the lines discussed by G & W), which from their chemical composition were destined to become cometary nuclei. It is the object of this note to show that planetesimals in the whole mass range of the observed cometary nuclei, up to 10^{18} g at least, could have formed under more general circumstances and over a large range of distances from the center of gravity of the PSN before the Sun came into existence.

Concerning the conditions for the sedimentation of dust particles towards a central plane and the formation of a dust disc we note first, that according to Zuckerman and Palmer (1972) the empirical data on the physical parameters of dense interstellar clouds together with those of the birth rate of stars strongly suggest that such clouds, if they collapse, are doing so at a rate that is much slower than the free fall rate.* More recent data on such clouds, tend to support this conclusion, though the evidence certainly cannot as yet be called conclusive (Smith, 1977; personal communications).**

It seems natural to interpret these observations by the retarding influence of the rotation of the fragments – the actual time scale of the "collapse" being determined by that of the loss of the fragment's angular momentun of rotation – and by the influence of the embedded magnetic fields. The probably considerable influence of ambipolar diffusion‡ resulting from the very low degree of ionization (some 10^{-8} at the densities considered here) makes it at least doubtful that the magnetic fields are the most important mechanisms determining the rate of the loss of angular momentum. If the influence of rotation prevails, a configuration with approximate balance between centrifugal force and gravity in its outer parts should result; it seems rather improbable that a mass fragment of several percent of the larger cloud, with an original radius of 0.2–0.3 pc, should not have got – by the internal turbulent motions in the larger clouds of some 10^4 cm/sec – enough angular momentum of rotation ($a^2\Omega$ of the order of 10^{22} cm^2/sec) for having at least in some approximation Keplerian motion of the fragment cloud's outer parts. This specific angular momentum could in any case be lost in the course of time (say after $> 10\ t_{ff}$ or some $2\pi/\Omega$), e.g., by turbulence resulting from special

* Time scale t_{ff} within < 4% given by $(\pi G\rho)^{-1/2} = 2/\omega_0$ (ρ mass density, $\omega_0^2 = 4\pi G\rho$).

** The observed mass of the dense interstellar clouds would seem to permit the formation, in our galaxy, of 10^2–10^3 stars of $1 M_\odot$ per year, against < 10 observed. Rather special initial conditions seem to be required for a star to be formed instead of a re-dispersion, by some instability, of the dense cloud, and still more restricted ones, referring to the initial distribution of mass, angular momentum and magnetic fields with appropriate symmetries, to permit the formation of a system with such regularities as found in ours.

‡ Biermann and Schlüter (1958), Mouschovias (1977), proposes that most clouds rotate slowly and that the retardation of the collapse is largely due to magnetic breaking of the collapse itself. A fast rotating dust globule is discussed by Millman (1977).

instabilities due to $\nabla\Omega$ (cf. Stewart, 1975)* or by the generation of torsional Alfvén waves. If on the other hand the influence of the embedded fields would initially be larger than that of rotation, one would have first condensation parallel to B towards a central plane and subsequently a disc of smaller radius (10^2–10^3 A.U.) more similar to the present PSN models (without magnetic fields) of Tscharnuter (personal communications), in the outer parts of which rotation would again balance part of the acceleration of gravity.

The PSN of few solar masses, considered here is a contracting fragment of a larger interstellar dense cloud ($M > 10^2 M_\odot$, radius $\lesssim 1$ pc) in a stage where the free fall collapse is arrested by the gas pressure nkT and centrifugal forces. So a marginally stable flattened object is obtained, whose further contraction time scale is appreciably longer than the free-fall time $t_{ff} \approx (\pi G\rho)^{-1/2}$ (ρ = mass density). In the absence of reliable detailed models and for the sake of clarity, we adopt for this PSN, at a stage long before the central object (Sun) has evolved, an oblate spheroid with the ratio of the half axes $a_1/a_3 = 3$. This is, on the one hand, thick enough so that the structural instabilities of a thin disc are not imminent; on the other hand, some convenient concepts of the flat disc can be used in fair approximation, such as surface density σ, balance between gravitational and thermal pressure along the vertical axis a_3, and vertical acceleration $4\pi G\rho z$ in dependence on distance z above the equatorial plane at nearly uniform density. The mean kinetic energies have to satisfy, at least in some approximation, the virial theorem. The rotational volume energy is chosen to be half the thermal one because a larger ratio is likely to imply instability (Ostriker, 1972). Under these conditions, the temperature of the gas (molecular hydrogen with 10 at. % He, so that the mean molecular weight is $\bar{m} = 2.4$) is the same as the one of the dust. The dust particles are heated mostly by stellar u.v. photons, the larger part of which diffuse into the PSN, optically thick at short wavelengths before being converted into i.r. photons, they cool by far-infrared emission, for which the PSN is optically thin (Drapatz and Michel, 1977). A good estimate for the dust temperature in the cloud is $kT = \bar{m} \times 0.6 \times 10^9$ erg or $T = 17$ K. Thus in the last end, it is the angular momentum of rotation and the interstellar radiation field which arrest the free-fall collapse of the PSN. To allow for the mass-loss in the later T-Tauri phase, which should lead to a sizeable decrease of the gravitational binding energy of the cometary nuclei and subsequently, by stellar perturbations, to the formation of Oort's Cloud, we consider a PSN of a few solar masses, e.g. $M = 5 \times 10^{33}$ g. Along the axis of rotation gravity is balanced by the gas pressure $\sigma_0^2 \approx (2/\pi G)\,(kT/\bar{m})\rho$ (Safronov, 1972), which can be written as $GM/a_1 \approx (a_1/a_3)\,(kT/\bar{m})$ or $(GM/a_1)\,v_s^{-2} \approx (a_1/a_3)/\gamma$. Then the following figures, which serve for numerical estimates, but which have the value of rough guidelines only, are in accord with the above model:

* There is as yet no really convincing proof, that a gradient of the angular velocity close to but (in its magnitude) smaller than that corresponding to purely Keplerian motion leads to turbulence; what has emerged so far tends to indicate, that, if – as one would suppose on the basis of very old well-known general arguments – an instability leading to turbulence arises, it should have a growth rate substantially slower than the inverse period of rotation ($> 10 t_{ff}$ in our model).

equatorial radius	a_1	$= 10^{17.2}$ cm
shorter half-axis	a_3	$= 10^{16.7}$ cm
average surface density	σ	$= 1/16\ \mathrm{g/cm^2}$
average specific potential energy	GM/a_1	$= 2.10^9$ erg/g
average specific rotational energy		$= 0.3\ 10^9$ erg/g
specific thermal energy $\times \frac{2}{3}$	$kT/\bar{m}$	$= 0.6\ 10^9$ erg/g
angular velocity of rotation	Ω	$= 2.5\ 10^{-13}$ rad/sec
sound velocity ($\gamma = 5/3$)	v_s	$= 10^{4.5}$ cm/sec
average density	$\bar{\rho}$	$= 10^{-18.0}\ \mathrm{g/cm^3}$
angular frequency of larger grains oscillating through the equatorial plane	$(4\pi G\bar{\rho})^{1/2} = \omega_0$	$= 10^{-12.0}\ \mathrm{sec^{-1}}$
free-fall time $t_{ff} = \left(\frac{32}{3\pi}G\bar{\rho}\right)^{-1/2}$	$\approx (\pi G\bar{\rho})^{-1/2}$	$= 2.10^{12}$ sec
period of rotation	$2\pi/\Omega$	$= 2.5\ 10^{13}$ sec

The scale given by the thermal and the rotational velocities and ω_0 is $\gtrless 0.4\,a_1$, such that the increase of the density inwards due to the approximately isothermal conditions should be moderate. This model, however, certainly ignores the larger density gradients, suggested by some computer studies (Tscharnuter, 1977). The description in terms of average properties should reflect at least approximately the conditions in the outer parts of the PSN, where comets have probably been formed.* Turbulence is disregarded, because it must have damped out before grains can separate from the gas phase (see also footnote p. 450).

In a rotating PSN in approximate Keplerian motion, at least in the outer parts, the time scale for sedimentation of the larger solid particles ($> 10^{-5}$ g) towards the central plane is of the order of some t_{ff} (whereas they spiral inwards much more slowly, due to that fraction of the gravitational acceleration's radial component which is not balanced by rotation). ** As will be shown in Section 3 solid particles with radii of $\approx 200 \mu$m will settle near the central plane in a time scale of the order of $4t_{ff}$.

These processes will be discussed in more detail in later sections of the present note in which we shall deal first with the mass range of the planetesimals resulting from the

* Comparing with Larson (1972) and Black and Bodenheimer (1976) it is seen, that a_1 is larger, by $\lessgtr 0.1$ in the logarithm, than $\alpha R_* \approx 0.8 R_*$ as defined by the latter authors (R_* is the Jeans length given by the mass, the thermal energy and the rotational energy per gram).

** Recent models of the interplanetary dust layer producing the zodiacal light suggest (Giese *et al.*, 1977) that particles with radii of some 10^{-3} up to some 10^{-2} cm, which are most likely to come from comets, constitute a major fraction of the mass of the solid interplanetary matter.

gravitational collapse of the dust disc, which owing to their small number density in space will not grow further by mutual collisions.

2. The Mass Range of the Planetesimals

In the present model the cometary nuclei are considered to be first-generation planetesimals, i.e. conglomerates of interstellar dust particles, originally suspended homogeneously in the PSN, then enriched in the equatorial plane by sedimentation in the gravitational field of the flattened nebula during a relatively long quasi-stable period of the PSN's contraction. The spheroidal model of the PSN described above is consistent with such a quasistable nebula.

Due to the dust enrichment, the regions around the equatorial plane become gravitationally unstable; the sedimentation process and the growth of the gravitational instabilities have been discussed in detail by Safronov (1972) and by Goldreich and Ward (1973).

The latter's work is based on the linearized theory of gravitational instability for uniform rotation of an infinite homogeneous system, as given by Chandrasekhar (1955) and in the work of Toomre (1964) and Goldreich and Lynden-Bell (1965) for the case also of non-uniform rotation. Safronov shows, that in the case of uniform rotation, gravitational instability will not arise from perturbations propagating in a plane perpendicular to the axis of rotation. Safronov also derives a gravitational instability condition for a flat system in non-uniform rotation, arriving at a critical density ρ_{cr}, which is minimal, when the wavelength λ of the perturbation is eight times the cloud thickness. His values of ρ_{cr} are up to 6 times greater than the critical densities of previous work for uniform rotation, its actual value being

$$\rho_{cr} = 2.1 \frac{M}{\frac{4}{3}\pi a^3} = 2.1\,\bar{\rho}.$$

Goldreich and Ward, and Safronov's results differ and from the papers it is not easy to see at what point their arguments diverge (Safronov's paper became accessible when G & W's paper had been essentially completed). For this reason we present here a simple derivation which leads in essence to the G & W formula for the maximum wave length (and mass) of a perturbation, producing a stable planetesimal. Since both theories agree in predicting gravitational instability in a sedimented dust disc during a comparatively stationary period of the PSN, we can content ourselves with the following conservation laws, applied to the separation of a cylindrical mass element from the differentially rotating PSN and the subsequent contraction to form a spherical planetesimal of mass density $\rho_p < 1\ \mathrm{g/cm^3}$.

As a result of the constancy of the circulation, if every volume element is in unperturbed circular Keplerian motion around the PSN's center at linear velocity V and angular velocity Ω, the *mean* angular velocity of rotation around the new center of the condensation will be (Safronov, 1972)

$$\omega = \tfrac{1}{2} \operatorname{rot} V = \frac{1}{2a} \frac{d}{da}(Va) = \tfrac{1}{4}\,\Omega$$

for $V \sim a^{-1/2}$ (a more slowly decreasing $V(a)$ would result in a larger ratio ω/Ω).

If we take this volume element to be a *rigid* cylinder of uniform surface density of dust σ_p, its angular velocity would be ω, if the diameter of the unstable disc corresponds to an instability wavelength λ_0 (the value of α indicates which fraction of the volume density will actually be incorporated into the planetesimal of mass M_{pl})

$$\alpha \int_0^{\lambda_0/2} r\frac{\Omega}{4} r 2\pi r \sigma_p dr = \frac{\pi}{128} \alpha\sigma_p \Omega \lambda_0^4 = \tfrac{1}{8} \Omega M_{pl} (\lambda/2)^2 \text{ g cm}^2 \text{ sec}^{-1}.$$

During contraction, the gases, mixed initially with the dust will diffuse out. Due to inelastic collisions of the dust grains among each other, only the total angular momentum has to be conserved, which is for the rigid sphere of radius R

$$\int_0^R 4\pi\rho_p r\sqrt{R^2 - r^2}\, r^2 \omega dr = \tfrac{8}{15}\pi\rho_p \omega R^5 = \tfrac{2}{5} M_{pl} \omega R^2,$$

so that by equating the angular momenta

$$\alpha\sigma_p \Omega \lambda_0^4 = \frac{1024}{15} \rho_p \omega R^5.$$

Also the masses of the cylinder and the sphere have to be the same: i.e.,

$$\alpha\sigma_p (\lambda/2)^2 = \tfrac{4}{3}\rho_p R^3.$$

The sphere should be stable against centrifugal disruption at its surface (as a result of angular momentum conservation, centrifugal forces are largest when the object is smallest). At a density of $\rho_p \approx 1$ or somewhat less cohesion by chemical interactions just starts to become operative; in any case, the disruptive forces (tending to prevent the formation of the planetesimal) have to be overcome by gravitation before the chemical forces begin to operate.

On the basis of existing stability criteria we put

$$\omega^2 \leqslant \epsilon \times 4\pi G \rho_p,$$

where the parameter ϵ is of the order of a few 0.1 (cf., for instance, Ostriker, 1972).* Then eliminating ω and R from these last three equations we get

$$(\lambda_0)^4 \leqslant \frac{2^{26}\pi^3}{5^6} (3^4 \times \epsilon^3) \times \frac{(G\sigma_P)^4}{(G\rho_p)^1} \times \frac{\alpha^4}{\Omega^6}.$$

If we identify λ_0 with G & W's λ, the two expressions for λ^4 are nearly identical for $\alpha = 1$, if $(3^4\epsilon^3)$ is (arbitrarily) put equal to unity (that is $\epsilon = 0.23$), an entirely possible choice.

* See, however, also p. 462, last section, first paragraph.

The maximum mass of a planetesimal multiplied with G becomes

$$GM_{pl} = G\alpha\pi\sigma_p(\lambda/2)^2 = \frac{2^{14}}{10^3} \times (3^2\epsilon^{3/2}) \times \frac{(\pi G\alpha\sigma_p)^3}{(\pi G\rho_p)^{1/2}} \times \frac{1}{\Omega^3}.$$

For the model proposed in the Introduction, we have, with $\alpha > 0.1$ (see next section), $\bar{\sigma}_p = Z\bar{\sigma} = 1/800\,\mathrm{g/cm^2}$ and $\Omega = 2.5 \times 10^{-13}$ rad/sec

$$GM_{pl} < \tfrac{1}{3} \times 10^{11} \times (10\alpha)^3,$$

$$M_{pl} < \tfrac{1}{2} \times 10^{18} \times (10\alpha)^3.$$

In order to remove the term Ω^{-3} in the expression for GM_{pl}, we consider the following substitutions:

(a) $$\Omega^{-2} = (\beta_1 \pi G\bar{\rho})^{-1},$$

(b) $$(a_1\Omega)^{-2} = \beta_2^2 \times \frac{v_s^2}{GM/a_1} \times \frac{1}{GM/a_1}.$$

Substitution of (a) leads to

$$GM_{pl} \approx G\alpha\pi\sigma_p(\lambda/2)^2 = \frac{2^{14}}{10^3} \times (3^2\epsilon^{3/2}) \times \left(\frac{\alpha\sigma_p}{\beta_1\bar{\rho}}\right)^{3/2} \times \frac{(\pi G\alpha\sigma_P)^{3/2}}{(\pi G\rho_p)^{1/2}}\ \mathrm{cm^3/sec^2}$$

which is $\sim a_3^{3/2}$ and to $(\pi G\bar{\sigma})^{3/2}$ or $(GM)^{3/2}/a_1^3$, with $\pi a_1^2\bar{\sigma} \equiv M$, that is

$$GM_{pl} \sim \left(\frac{a_3}{a_1}\right)^{3/2} \times \left(\frac{GM}{a_1}\right)^{3/2},$$

where the second term in models such as considered here is $\sim v_s^3$. This is more easily seen from substitution (b), which yields

$$GM_{pl} \leqslant \frac{2^{14}}{10^3}(3^2\epsilon^{3/2}) \times \frac{1}{(\pi G\rho_p)^{1/2}} \times \left(\frac{\sigma}{\bar{\sigma}}\right)^3 (\alpha Z)^3\ (\beta_2 v_s)^3.$$

Another formulation would be

$$M_{pl} \leqslant 4\pi\left(\frac{8}{5}\right)^3 \frac{(3^2\epsilon^{3/2})(\alpha\sigma_p)^3}{(\beta_1\bar{\rho})^{3/2}\rho_p^{1/2}}.$$

We note, that in our model $\beta_2 \approx 10^{+0.2}$ and, for $\rho_p = 1$, $(\pi G\rho_p)^{-1/2} = 2.2 \times 10^3$ sec, β_1 and β_2^2 of order 1 expresses the conditions, which specify our model (see preceding section).

We note particularly, that the mass of the presolar nebula has dropped out, though its order of magnitude still comes in via the condition of sufficient optical thickness (see next section).

The value of M_{pl} thus derived is an upper limit for the planetesimals mass; the actual masses will ordinarily be smaller, also further fragmentation of the planetesimals original mass during the final contraction cannot be excluded. Our treatment, of course, does not show that the disc becomes gravitationally unstable upon dust sedimentation; gravitational instability of the dust disc has been demonstrated by previous work, however, in which the results of Safronov and G & W agree.

An interesting aspect in the above formula is, that M_{pl} depends in the rotating PSN predominantly on the local column density of dust σ_p and the average (smeared out) density of matter $\bar{\rho}$ in the PSN. In our approximately uniform, spheroidal model for the early PSN (favored by stability considerations) this implies that planetesimals should not be too different all through the PSN, viz. cometary nuclei would be similar in size and formation mechanism to the progenitors of the outermost major planets, though the expected moderate increase (inwards) of σ may also be of some consequence.

The largest radii of cometary nuclei determined so far with some accuracy (largely from the observed gas output) are around 2–3 km and refer to moderately bright comets. The brightest comets ever observed appear to have been (intrinsically) brighter by about a factor $> 10^1$, which would suggest a range in R (upward), corresponding to a factor 4.

Concerning the planetesimals' state of rotation the following can be said. From the interpretation of how the limits λ_0 and M_{pl} arise we conclude, that comets with masses near the upper limit are rotating almost as fast as permitted by the gravitational binding force, though this is likely to be a fraction only of the forces of cohesion ($\rho_p GM_{pl}/a$ of order $< 10^6$ erg/cm^3 $\approx 10^{18}$ eV/cm^3). They should thus spin with an angular velocity of rotation of the order of a few 10^{-4} rad/sec at most, with $\omega \sim R$ for smaller* masses, in the same sense as that of the revolution around the PSN's center of gravity. For bodies or irregular shape and surface properties as are to be expected, one can, however, make at least plausible that the reaction forces, due to the gas production (discussed by Whipple first in 1950), are likely to change the state of rotation drastically during the first or the first few approaches to the Sun,** during which volatile constituents should prevail in the evaporating gas and the reaction forces should be larger than in later stages.

Still the question remains to be answered which proportion of the dust column density can be embodied into the planetesimal. The answer requires consideration of the time necessary for sedimentation as compared to the characteristic time for the growth of the instability, break up of the dust disc, and orbital decay of grains due to drag.

3. The Sedimentation Process

It is the aim of this section to show, that those solid particles, which settle fastest near the equatorial plane (of size $v_s\bar{\rho}/\omega_0\rho_p$) do so in a time scale of several free fall times (several $(2/\omega_0)$ sec) only, and to discuss empirical and theoretical reasons supporting the

* In the very few cases in which the observation of spiral structures in the coma permitted an estimate of the period of rotation, values of ω of the order of 1/3 or 1/2 10^{-4} rad/sec have been found.

** A net torque of $> 10^{16}$ dyn $\times$ cm acting over $\approx 10^7$ sec on a body with a moment of inertia of some 10^{26} g cm^2 (as might be appropriate for comet Bennett) would change ω by some 10^{-4} rad/sec.

view, that such particles contributed substantially -10% or so by mass, by order of magnitude – to the solid matter in the dense interstellar cloud, which became the PSN.

Whereas gravitational attraction in the radial direction is compensated by the centrifugal force in all isothermal models of a differentially rotating PSN, the vertical gravitational forces have to be balanced by the gas pressure. If the nebula is sufficiently thin, the gravitational field of an infinite disk holds. For this case the acceleration of a test particle by a sheet of mass density ρ and thickness dz is independent of its distance z and given by $2\pi G\rho dz$. The integrated acceleration by all mass sheets to both sides of the midplane in the symmetrical isothermal disc nebula has to be compensated by the pressure gradient

$$2\int_0^z 2\pi G\rho dz = -\frac{1}{\rho}\frac{dP}{dz} = -\frac{kT}{\bar{m}}\frac{d\ln\rho}{dz}.$$

As long as $\omega^2 = 4\pi G\rho$ can be approximated by $\omega_0^2 = 4\pi G\bar{\rho}$, we get the relations *

$$\omega^2 z \approx -\frac{kTd\ln(\rho/\rho_0)}{\bar{m}dt} \qquad \rho/\rho_0 \approx e^{-(z/h_1)^2}$$

the scale height h_1 being given by

$$h_1^{-2} = \frac{\gamma\omega^2}{2v_s^2} \gtrapprox 7\times 10^{-34} \quad h_1 \lessapprox 3,7\times 10^{16}\ \mathrm{cm}.$$

Near the outer boundary, the gravitational acceleration approximates $2\pi G\sigma \approx 8/3\times 10^{-8}\ \mathrm{cm/sec^2}$ and we obtain the outer scale height h_2 by

$$|\nabla\ln\rho| \to \frac{2\pi G\sigma}{kT/\bar{m}} = \frac{40}{9}\times 10^{-17} = 1/2.25\times 10^{16}\ \mathrm{cm} \equiv 1/h_2\ .$$

Comparing with our values of a_3 (Section 1) of 5×10^{16} cm we see that h_1 is $\approx 3/4$ of a_3 and $h_2 < 1/2$ of a_3. We conclude that $\bar{\rho}$ should be, within the limits of our crude model, a fair approximation to ρ_0 (though necessarily $\rho_0 > \bar{\rho}$), and that a rectangular density profile should be a tolerable approximation to the true profile. The present approximately uniform PSN model of spheroidal shape implies a slight dependence of the thickness h on distance a from the nebula's center

$$h = 2a_3\sqrt{1-\frac{a^2}{a_1^2}}.$$

In a multicomponent mixture, a uniform distribution of both dust and gas is not an equilibrium distribution. The heavier component rather has a much smaller equivalent width (scale height) as indicated formally by the dependence of h on the sound velocity v_s

$$\frac{h_p}{h} = \sqrt{\frac{\bar{m}}{m_p}}.$$

* These are approximations to the exact solution (Spitzer, 1942, 1968).

Hence, if the interstellar cloud fragment has contracted to a metastable PSN of spheroidal shape and uniform mixing ratio, the dust particles will adjust to their equilibrium value h_p by sedimentation toward the midplane (the spiralling inwards, discussed below, will increase the local Z value). In the case of a rectangular distribution of matter which is not changed by the sedimentation process, analytical expressions for the change of the dust density with time can be found, always assuming that the ratio of dust to total density $\rho_s/\rho < 1$.

As discussed by Safronov, the motion of grains in the corresponding gravitational field with drag by the uniform gas density is determined by the differential equation for damped harmonic oscillation

$$\frac{d^2 z}{d(\omega_0 t)^2} + \left(\frac{\bar{\rho}}{\rho_p}\right)\left(\frac{v_{th}}{\omega_0 r}\right)\frac{dz}{d(\omega_0 t)} + z = 0,$$

$$= \frac{d^2 z}{d(\omega_0 t)^2} + 2\frac{r'}{r}\frac{dz}{d(\omega_0 t)} + z = 0,$$

with

$$r' = \frac{1}{2}\frac{v_{th}}{\omega_0} \times \frac{\bar{\rho}}{\rho_p} \approx \frac{2 \times 10^{-2}}{\rho_p}\,\mathrm{cm}, \quad v_{th} = \left(\frac{8}{\gamma\pi}\right)^{1/2} v_s\,.$$

For $r = r'$ the speed of sedimentation is fastest and the differential equation has the simple solution

$$\frac{z}{z_0} = (1 + \omega_0 t)\,\mathrm{e}^{-\omega_0 t}$$

$$= 10^{-2.5}\ \text{resp.}\ 10^{-3.3} \quad \text{for} \quad \omega_0 t = 8\ \text{resp.}\ 10.$$

It separates the solutions of oscillatory character (for $r > r'$) from those of exponential character (for $r < r'$). The existence of such large grains has been suggested by the interpretation to measurements of both interstellar clouds (Rowan-Robinson, 1975) and cometary dust (Finson and Probstein, 1968; Ney, 1974). Meteor showers, associated with Earth orbit crossing comets are further witness for grains up to 1 mm in size (Whipple, 1976). Most recently, the new measurements of the properties of the zodiacal light (Fechtig, 1976) and their interpretation (Giese *et al.*, 1977) strongly suggest the presence, in considerable number, of particles with radii of some 10^{-2} cm in interplanetary space, which in view of their small numbers in the asteroidal belt and their probably fluffy character (Giese *et al.*, 1977) are most likely to come from comets.

For grains with $r > r'$, the amplitude of the oscillatory solution, with the argument

$$\omega' t = (\omega^2 - b^2)^{1/2} t, \quad b = \frac{1}{2}\frac{\rho}{\rho_p}\frac{v_{th}}{r},$$

decreases as

$$\frac{z}{z_0} \sim \mathrm{e}^{-bt} = \mathrm{e}^{-(r'/r)\omega_0 t},$$

which, for instance for $r = 2r'$ and $\omega_0 t = 10$ resp. 20 leads to $10^{-2.2}$ resp. $10^{-4.3}$. For grains with $r < r'$ the solution rapidly approaches the asymptotic expression *

$$\frac{d \ln z}{d(\omega_0 t)} = -\frac{1}{2}\frac{r}{r'};$$

for $r = r'/\sqrt{2}$ it is

$$\frac{z}{z_0} = 1.21\, e^{-0.41\,\omega_0 t} - 0.21\, e^{-2.414\omega_0 t}$$

$$\approx 1.21\, e^{-0.41\omega_0 t} \qquad (\text{for } \omega_0 t > 1)$$

$= 10^{-3.52}$ for $\omega_0 t = 20$ (as compared to $e^{-7.07} = 10^{-3.07}$ for the asymptotic solution). The range around r', for which the relative compression amounts at least to a factor in the range $10^{2.5}$ – 10^3 (for $\omega_0 t$ in the range 15–20), corresponds therefore to a factor of order e ($e \approx 2\sqrt{2}$). For a simple power law in the number densities of grain sizes, which would be in agreement with presently available data on interstellar extinction ($r_{\min} = 100$ Å) and which holds before sedimentation started and with $r_{\max} \approx r'$ we may assume (e.g., Biermann, 1967; Barbieri *et al.*, 1974; Mathis, 1977)

$$\frac{dn_{po}}{dr} = \frac{Z\rho r^{-4}}{\frac{4}{3}\pi\rho_p \ln r'/r_{\min}} = 0.5 \times 10^{-21}\, r^{-4}\ \text{cm}^{-4}.$$

The evidence on the size distribution of the interplanetary dust particles (Fechtig, 1976; Giese *et al.*, 1977) – according to which a larger fraction of the total mass is between 10^{-3} and some 10^{-2} cm radius than between 10^{-5} and 10^{-3} cm which is unlikely for the interstellar particles – could then indicate (see, however, the second equation below) that part of the particles with $r \lessapprox 10^{-3}$ cm has been collected by the larger ones in the process of sedimentation.

One of the competing processes, the orbital decay, viz. the spiraling of grains toward the PSN's axis as a result of gas drag, has been formulated by Whipple (see G + W, equation 12). The orbital decay time is for the present model at $a = 10^4$ A.U. (for $r \lessapprox 2r'$)

$$\tau_{\text{orb}} = \frac{3\rho a^2}{\rho_p v_s r} = \frac{2 \times 10^{12}}{r}\ \text{sec.}$$

For $r \approx r'$ the time scale τ_{orb}, becomes 10^{14} sec or 5 times $20\omega_0^{-1}$. Hence, the dust disc should become somewhat smaller than the PSN, by perhaps 20% and the surface density to be used in the expression for GM_{pl} (given in Section 2) by perhaps 30–50%. Since GM_{pl} is $\sim \sigma_p^3$, this correction (upwards) is not trivial.

During the sedimentation toward the midplane these grains should thus grow by sticking collisions with smaller grains, which settle more slowly. For any realistic grain

* The equivalent solution for the case of spherical symmetry without rotation has been given by Alfvén (1977).

size distribution in accord with interstellar extinction the largest part of the solid mass of original density $Z\rho = 2 \times 10^{-20}$ g/cm^3 occurs in grains smaller than r', so that the growth law becomes (Safronov, 1972)

$$r = r_1 + \frac{Z\rho}{4\rho_p}(z_1 - z);$$

r_1 being the original grain radius at height z_1 ($\leqslant a_3 = 5 \times 10^{16}$ cm) above the plane. However, the growth rates will be somewhat faster by the spiralling inwards and, for particles with $r > r'$, by the oscillatory component of the motion, for which the increase in r may be of the order of several to 10μm. This point seems of interest in connection with the structure of the interplanetary particles ("fluffy particles") proposed by Giese *et al.* (1977), which would seem to be a natural result of the agglomeration of smaller particles by the large ones in the process of sedimentation.

Particle growth by mutual agglomeration as a result of Brownian motion is given by

$$\frac{dr}{dt} = \frac{1}{4\rho_p}\int_{r_{\min}}^{r}\sqrt{\frac{8kT}{\pi m}} \times m \times \frac{dn}{dr}dr = \sqrt{\frac{kT}{12\pi^2\rho_p\rho_p}}\frac{Z}{\ln r'/r_{\min}}$$
$$= 0.97 \times 10^{-20}\ \text{cm/sec}$$

where

$$m = \frac{4}{3}\pi\rho_p r^3.$$

Thus, during one oscillatory period $2\pi/\omega_0$ the growth in radius does not exceed 0.6×10^{-7} cm and the effect is negligible even for the smallest grains, as long as the dust density has not increased by more than a factor 10.

According to Safronov, the critical density ρ_{cr} at which the equatorial disc becomes unstable, is given by

$$\rho_{cr} = 2.1\,\bar{\rho},$$

where $\bar{\rho}$ is the distributed gas density (10^{-18} g/cm^3 in our model). The density increase must be due to dust enrichment in the midplane by a factor of 50–60 (with $Z \approx 2\%$).

Due to sedimentation the number density of grains of radii $r < 1/2\ r'$ in the size interval dr changes at constant surface density of particulate matter according to

$$\frac{dn_p}{dr}dr = \frac{dn_{po}}{dr}\frac{z_0}{z}dr = \frac{dn_{po}}{dr}e^{\frac{1}{2}\omega_0(r/r')r}\,dr,$$

because for an initially uniform matter distribution, the density of particles of a certain size r remains uniform during sedimentation within the distance z from the midplane and

$$z = z_0 e^{-\frac{1}{2}\omega_0 t(r/r')}$$

with $z_0 \lesssim a_3\sqrt{1-(a^2/a_1^2)}$ in the spheroidal model. This entails an enhancement in the particulate matter density around the midplane, due to the smaller particles, given by

$$\rho_s = \int_{r_{\min}}^{1/2\,r'} \frac{4\pi}{3} r^3 \rho_p \frac{dn_p}{dr} dr = \frac{Z\rho}{\ln r'/r_{\min}} \int_{r_{\min}/r'}^{1/2} e^{1/2\,\omega_0 t (r/r')} d \ln \frac{2r}{r'}$$

$$\approx \frac{Z\rho}{\ln r'/r_{\min}} \frac{4}{\omega_0 t} e^{1/4\,\omega_0 t},$$

where the approximate solution to the integral exponential function holds for large values of $\omega_0 t$ ($\frac{1}{2}\omega_0 t \gg 1$) with the above power law in size distribution.

It follows that a stable period of about $t = (>10) - 20\omega_0^{-1} < 2 \times 10^{13}$ sec is required, so that sedimentation under non-turbulent conditions in the PSN can induce gravitational instability of the dust disk. Initially only particles in the size interval some $10^{-3} < r <$ some 10^{-2} cm are enriched sufficiently in the midplane to be incorporated in the first condensation of the planetesimal. Hence, the grain size distribution of this first condensation is *not* expected to be the same as the original interstellar grain size distribution.

The question arises, which fraction of the dust's column density σ_p will be incorporated into the planetesimal. Because the thickness of the dust disk is still appreciably greater than the instability wavelength within the plane, $\lambda \approx 10^{11}$ cm, as shown above, this fraction might seem to depend on the extension λ_z of the instability region in z direction.

One recognizes, however, from the formula for ρ_s, that the dust disk becomes quickly unstable also in the vertical direction, once $\rho_s > \rho$ because from then on ω_0 is not constant any more. The particulate density increase by sedimentation rather as follows (differentiation of the equation for ρ_s w.r.t. time)

$$\frac{d\rho_s}{dt} \approx \omega_0 \rho_s = \sqrt{4\pi G}\, \rho_s^{3/2},$$

which formally leads to singularity ($\rho_s \to \infty$) within $t = 1/\sqrt{4\pi G\rho} = 10^{12}$ sec. Obviously, a strong density increase in the midplane will disturb the pressure equilibrium, and also the gaseous disk will be somewhat influenced dynamically in z direction within a period given essentially by the height of the nebula a_3 and the speed of sound, $a_3/v_s = 1.6 \times 10^{12}$ sec.

Despite the fact, that there is now a very dense dust disk in the midplane, the overall gravitational field changes only little, so that not a collapse of the gases but only a slight rearrangement in the pressure and density profile will ensue. The gas contains still most of the smaller particles, which determine according to the above size distribution the surface area for energy exchange.

These considerations about an isothermal PSN hold, if the nebula is optically thin to infrared radiation and the relaxation time for energy exchange between dust and gas is short as compared to $1/\omega_0$. The optical thickness in the vertical direction is given by

$$a_3 \int_{r_{\min}}^{r'} \pi r^2 Q_{\text{abs}} \frac{dn}{dr} = 7 \times 10^{-2},$$

with the infrared absorption efficiency (Drapatz and Michel, 1977) $Q_{\text{abs}} =$

8.4×10^{-3} $(r/\lambda^2) = 84r$ at the wavelength corresponding to the radiation maximum at $20K$ $\lambda = 0.2/T = 10^{-2}$ cm.

The energy transfer rate from gas to dust by collisions is per volume element

$$\dot{E} = n\langle v\rangle \int \pi r^2 \frac{dn_p}{dr} dr \times c_v(T_g - T_p);$$

so that the thermal relaxation time

$$\tau_{th} = \frac{nc_v T_g}{\dot{E}} \approx \left[\langle v\rangle \int \pi r^2 \frac{dn_p}{dr} dr \right]^{-1} \leqslant 3 \times 10^{10} \text{ sec}$$

is indeed the shortest time scale.

The quantity α introduced in Section 2 should, on the basis of the considerations presented in this section, have a value in the range (1 or a few) 10%, a value near 10% being more probable.

4. Discussion

We conclude that, once sedimentation has accumulated enough particulate matter so that it exceeds the original gas density, in the midplane within an "induction period" of about 15–20ω_0, further rapid density increase occurs by self-contraction. The requirement is only that the gas contains an adequate proportion of grains of sizes around $r' \approx 200 \mu$m. The induction period associated with sedimentation of about 5 or 6×10^5 yr depends on $\rho^{-1/2}$, the gas density being given essentially by geometric and overall stability constraints. So this result might be fairly insensitive to the finer details in model assumptions.

The final gravitational collapse of the dust layer of half-width $z_{r'}$ is arrested not by grain–grain collisions (the collision probability is too low for $r > \approx 1 \mu$m) but rather by gas friction. But the damping time constant is then only of the order of the sedimentation time constant, so that the final motion dies out much more rapidly.

Hence, the grains forming the dust disk are not expected to have been subject to any larger temperature changes during the formation of the dust layer or the planetesimal. Even during the stagnation of the gases after the free-fall of the interstellar cloud fragment to form the stationary PSN, only minor temperature changes are possible, as can be estimated from the strong shock wave approximation

$$\Delta T \leqslant \frac{\gamma - 1}{2k} \frac{1}{2} m_{H_2} v^2 = \frac{(\gamma - 1) m_{H_2}}{2k} \frac{GM}{a} = 27\,\text{K}.$$

A general grain temperature of about 20 K seems to be realistic as evidenced not only by calculation (Drapatz and Michel, 1976/77), but also by recent measurements on the infrared emission from dense cloud regions around the center of the galaxy (Rouan *et al.*, 1977; Nishimura *et al.*, 1977).

Consequently, the chemical composition, even of ice mantles, normally quite temperature sensitive, will remain unmodified and the same as the one of the original interstellar grains. Because of their larger surface area, however, ice mantles, deposited on the refractory cores in the interstellar cloud, are to be associated mostly with the smaller grains, which in our model remain suspended in the gas of the PSN. As "young" comets produce upon approach to the Sun large amounts of smaller grains as well and a mass ratio of evaporable compounds to refractory dust near unity, we have to conclude, that further material might accrete to the planetesimals during or after their formation. That this is at least in some measure to be expected, can be seen as follows. Particles in the size range around $r'/4$ fall towards the central plane with $d \ln z/d(\omega t) \approx 1/8$; in $20\omega_0^{-1}$ sec they will have changed their distance from the plane by a factor of $e^{-2.5}$ or $10^{-1.1}$; this group of particles alone will thus contribute to the overall density of the condensable material more than its original total amount. The planetesimals (cometary nuclei) formed as described above will thus move, as long as the PSN exists – i.e., for $\approx 10^{14}$ sec, in a dusty environment of $\approx 10^{-19}$ g/cm^3, with a differential speed of rotation around the centre of a few 10^4 cm/sec. Thus a gain of $\approx 10^{-1}$ g/cm^2 is certainly entirely to be expected, and it seems clear that this estimate could be seriously on the low side.

This leads to the question of the formation of the Oort cloud. The planetesimals have in our model on an average a total energy corresponding to $\lessapprox 10^4$ A.U. distance from the centre. If, as we assume, the unused part of the PSN is of the same order as that used for the formation of the Sun and the planets, then the dispersal of the remaining PSN will result in a near-zero total energy for many of the planetesimals, which then should get in much larger orbits. Those with orbits with $a \gtrapprox$ some 10^4 A.U., will easily be perturbed, by stars passing nearby, such that any trace of their original orbital characteristics should be lost after some 10^9 yr (cf. Oort, 1950; the note in proof contains an important correction to the original estimate of the expected change of the total energy).

The main hypothesis of this paper is that the contracting PSN reached – at least in the outer parts – an approximately stationary state, permitting the formation of an equatorial dust layer, the gravitational instability of which leads to the formation of the cometary nuclei as planetesimals. The available time scale for this to happen should be determined by the gradual loss of angular momentum by turbulence and/or magnetic fields, which should permit a quiescent period of at least a period of revolution (0.8 m.y. in our model). It has been shown, that such a model leads to a mass range of the cometary nuclei consistent with observation.

We wish to add that the upper mass limit derived is proportional to $\epsilon^{3/2}$ and to the inverse root of the density, which on the basis of the properties of terrestrial snow, might be as small as 0.1 g/cm^3 and still be capable of withstanding centrifugal disruption, such that, at least for comets consisting largely of the ice of ordinary water, a larger upper mass-limit would result. Thus the previous estimate for $M_{pl} < \frac{1}{2} \times 10^{18} \times [(10\alpha)^3/\rho_p^{1/2}]$ g $\approx 10^{18}$ (arrived at with the assumption that cohesion is only due to gravitation and not to chemical forces) might be on the low side and so the value for the cometary radii, derived from $M_{pl} = \frac{4}{3}\pi\rho_p r^3$.

Empirical checks on the model proposed here are expected mostly from more detailed studies of comets, the only unmodified survivors of these planetesimals. The presently available – even though sparse – evidence appears to be at least not in contradiction to it.

The comparison of the ice mantle composition and mass, and the conclusions on the chemical evolution in interstellar clouds will be the subject of a forthcoming paper. .

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