FORMATION OF THE REGULAR SATELLITE SYSTEMS AND RINGS OF THE MAJOR PLANETS

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Abstract. In this paper, we apply the ideas presented by one of us (Prentice, 1978a, b) for the development of the proto-solar cloud into a system of Laplacian rings to the development of the proto-planetary clouds which ultimately led to Jupiter, Saturn and Uranus. We show that if one accepts this scenario – especially the idea of supersonic turbulence in the proto-planetary clouds – one can satisfactorily explain, on the basis of fixing a single adjustable parameter, both the geometric precession of the orbital radii of the regular satellite systems of these three planets and the chemical composition and mass distribution of these satellites. We suggest that thermal stirring in the proto-planetary cloud in the vicinity of the surface of the planet may be responsible for the smaller masses of some of the inner satellites as well as for the formation of the rocky rings of Uranus. The icy rings of Saturn are suggested to be the product of condensation processes in a continuous gaseous disc within the Roche limit of the planet.

1. Introduction

It is well known that Jupiter, Saturn and Uranus each possess an inner family of satellites which share many of the same regular features that are seen in the planetary system surrounding the Sun. The orbits are prograde and nearly circular and they lie practically in the equatorial plane defined by the axial rotation of the parent planet. In addition, the orbital radii R_n, when numbered towards the centre form a rough geometric sequence similar to the Titius-Bode law of planetary distances (ter Haar, 1967), viz.

$$R_n/R_{n+1} = \Gamma, \quad (n = 0, 1, 2, \ldots), \tag{1}$$

where Γ is approximately constant and equals about 1.65 for the Jupiter's Galilean system, 1.3 for Saturn's system and 1.46 for that of Uranus. This suggests that the same cosmogonic process may have been responsible for the origin of both planetary and satellite systems, a conclusion also reached by Alfvén and Arrhenius (1976). The question of the origin of these satellite systems has recently received a fresh impetus through the discovery of a system of rings around Uranus (Elliot *et al.*, 1977; Millis *et al.*, 1977; Bhattacharyya and Kuppuswamy, 1977; Persson *et al.*, 1978).

There is a school of thought which believes that orbital resonances play an important role in determining the present structure of the solar system (*cf.* Dermott, 1973). Cassini's division in Saturn's rings, for example, lies at a position corresponding to roughly half the orbital period of Mimas. Dermott and Gold (1977) have suggested that the structure of

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the Uranian rings can be explained in terms of small particles librating about stable orbital resonances produced by pairs of known satellites, notably Ariel-Oberon and Ariel-Titania. Goldreich and Nicholson (1977) have questioned this model, pointing out that the control of resonance orbits is far more complex than Dermott and Gold envisaged. In particular, it would appear that Miranda, rather than Ariel, plays the key role. Ip (1978) and Steigmann (1978) have shown that the resonance theory alone can account for the width of the Uranian rings and that even to account correctly for the observed orbital spacings it is necessary to invoke the existence of a hypothetical Uranian satellite having a mass comparable to that of Miranda and having an orbital radius of 1.03×10^{10} cm.

In this paper, we wish to suggest that the regular satellite systems and rings of the major planets were probably formed in a manner similar to that originally proposed by Laplace (1796) for the formation of the planetary system: namely, through condensation from a concentric system of gaseous rings which were shed during the primordial gravitational contraction of the envelopes of Jupiter, Saturn and Uranus. This work is an extension of the theory for the formation of the planetary system which one of us presented to the NATO Advanced Study Institute on the Origin of the Solar System in 1976; Stevenson, 1978). The larger density of Neptune suggests that this planet consists of the Astronomical Society of Australia (Prentice, 1977).

2. Shedding of the System of Gaseous Laplacian Rings

The densities and computed structures of Jupiter, Saturn and Uranus suggest that each of these planets consists predominantly of an envelope of H and He surrounding a small rock/ice core of mass $m_c \sim 10 M_\oplus$ (Podolak and Cameron, 1974; Stevenson and Salpeter, 1976; Stevenson, 1978). The larger density of Neptune suggests that this planet consists mostly of a rock/ice core. The existence of such central cores of roughly comparable mass is in accord with the view that the first stage in the formation of the major planets was the aggregation of such a mass of rock/ice planetesimals in gaseous rings, each of mass about $1000 M_\oplus$, which were shed by the contracting protosun at the orbits of these planets (Prentice, 1974, 1978a). Once these dense planetary cores have formed they can act as a gravitational sink for the residual gases in each of the rings, though no gravitational collapse can ensue until the cores have sufficiently cooled down. This takes some 10^6 yr to complete. During this time the gases of the outer rings evaporate out of the solar system. In a detailed calculation one of us has shown that at the orbit of Neptune all of the gas evaporates. Near Uranus nearly all, and near Saturn and Jupiter the proportion of gas remaining occurs almost exactly in the same ratio as the observed masses of these planets (Prentice, 1979).

Consider now the contraction of the primitive envelopes of Jupiter, Saturn and Uranus. If, as in the theory of planetary formation (Prentice, 1978a) one takes into account a supersonic turbulent stress $\langle \rho_t v_t^2 \rangle = \beta \rho G M(r)/r$ arising from the motions of overshooting convective elements, where ρ is the gas density, $M(r)$ the mass interior to radius r and

$\beta \sim 0.1$ is the so-called turbulence parameter, it is possible to show that the envelope becomes very centrally condensed (that is, acquires a very low moment-of-inertia coefficient $f = I/M_{\text{env}}R^2 \sim 0.01$–$0.02$, where M_{env} is the envelope mass and R its equatorial radius) and develops at its photosurface a very dense outer shell of non-turbulent gas (Prentice, 1973). When rotation is included, this outer shell evolves into a belt-like structure at the equator. If the turbulence is sufficiently strong, this belt is capable of storing so much non-turbulent gas (in order to withstand the great turbulence stress beneath the photosurface) that when the critically rotating configuration is reached at a radius R_0, determined by the condition that the centrifugal force at the equator for the first time balances the gravitational force, any further extrusion of material from the turbulent envelope causes its equatorial regions to withdraw discontinuously to a smaller radius. The cloud therefore literally sheds a belt of gas at radius R_0. The abandoned belt distributes itself uniformly about the circular Keplerian orbit R_0 to form a gaseous torus, or ring. The angular velocity and density distributions in this ring of mass m and temperature T_0, expressed in cylindrical polar co-ordinates (s, z) referred to the axis of rotation, are given by

$$\omega(s, z) \simeq \sqrt{GMR_0}/s^2, \quad \rho(s, z) \simeq \frac{\alpha m}{4\pi^2 R_0^3} \exp\left[-\tfrac{1}{2}\alpha\xi^2/R_0^2\right], \tag{2}$$

where $M = M_{\text{env}} + m_c$ is the total mass interior to R_0, $\alpha = \mu GM/\mathscr{R}T_0R_0$ and $\xi = [(s - R_0)^2 + z^2]^{1/2}$ is the distance from the circular orbit $s = R_0$.

After shedding its first ring, the turbulent cloud continues to contract, stabilizing itself rotationally by extruding further material to the equator under conditions of uniform rotation maintained by the supersonic turbulent convection, until the critical configuration is again reached, at a radius R_1, where a second ring is shed, and so on. The sequence of orbital radii R_n of the successively disposed rings satisfies the equation

$$R_n/R_{n+1} = [1 + m/M_{\text{env}}f]^2. \tag{3}$$

It follows from this equation that if the contraction of the envelope occurs more or less homologously, so that both m/M_{env} and f remain sensibly constant, then the sequence R_n is, on the mean, a geometric one, as required by the observational data.

The location of the first shed ring is determined by the initial spin angular momentum of the contracting envelope and defines the outer boundary of the regular satellite system. The radius R_0 would thus correspond to the orbits of Callisto, Rhea and Oberon, respectively. After a gaseous ring has been shed the various condensates in the gas, appropriate to the prevailing density and temperature, settle onto the central circular Keplerian orbit of radius R_n with angular velocity $\omega\,(=\sqrt{GM/R_n^3})$ to form a concentrated stream of satellitesimals. This circular orbiting stream of satellitesimals may then subsequently aggregate under the action of its own gravity into a single satellite mass. The latter process is expected to take place only as long as the gaseous ring remains intact and can act as a sink for the excess motions of the aggregating satellitesimals (Hourigan, 1977).

3. Chemical Condensation of the Gaseous Rings

Let us now assume that Equation (3) holds for the satellite systems of Jupiter, Saturn and Uranus. We can then determine the mean mass m of each of the gaseous rings shed at the present orbital positions of the regular satellites, by putting M_{env} equal to the observed planetary mass M minus that of the central core $m_c \sim 10 M_\oplus$, $f = 0.02$, and taking R_n/R_{n+1} equal to the observed mean ratio. Next we construct a numerical model of a fully rotating chemically uniform envelope of mass M with turbulence parameter β and parameter α adjusted to yield these values of f and $\langle R_n/R_{n+1}\rangle$. This enables us to compute the temperature of each gaseous ring at its moment of detachment, as well as both the composition and mass of the condensing species. We assume the envelopes to have roughly a solar composition, in accord with the observations of Jupiter and Saturn (Newburn and Gulkis, 1973) and use the equilibrium chemical condensation sequences computed by Lewis (1972a, b). The present atmosphere of Uranus is much too cold to yield any indication of its original compositional state (Stevenson, 1978). Although we have assumed that the majority of the condensates in the outer protosolar gaseous rings aggregated into the planetary cores, thereby draining the gas of heavy elements, a reenhancement of these elements into the captured envelope will occur later if the outer mantle of the core becomes vaporized during the contraction phase (cf. ter Haar and Wergeland, 1947; Podolak and Cameron, 1974).

The results of the calculations are shown in Table I and Figure 1. Considering first Jupiter, the agreement between predicted and observed satellite compositions and masses is very good. Io and Europa have densities of 3.5 g cm^{-3}, consistent with a rocky composition, and this agrees with the condensation temperature computed in Figure 1. Similarly Ganymede and Callisto are thought to contain greater proportions of ice, increasing in that order, and this is also in accord with the temperature curve. The composition of the Galilean satellites is therefore consistent with a picture in which the protojovian envelope contracted almost homologously, shedding rings of roughly equal mass at approximately geometrically spaced distances, and with temperatures T_n behaving as $T_n \propto R_n^{-1}$. This latter law is a general feature of uniform gravitational contraction which is also reflected in the composition of the planetary system (*cf.* Kaula, 1976).

The close agreement we have obtained for the mass distribution of the Galilean satellites comes from the choice $f = 0.02$ for the envelope moment-of-inertia coefficient and hence from the value chosen for the turbulence parameter $\beta = 0.1$. Choosing a larger f (smaller β) results in a correspondingly larger condensate mass ($m_{\text{con}} \propto m \propto f$, using Equation (3)), as well as an increased envelope surface temperature T_e (see below). The best fit to the mass distribution of the planetary system is obtained with the choice $f = 0.01$ (Prentice, 1978a). Once f is set, however, both the mass distribution and chemistry of the planetary/satellite system are determined. In that respect, it is noteworthy that the modern Laplacian theory is capable of accounting simultaneously both for the mass and chemical composition of these systems with the choice of only a single parameter, namely the turbulence parameter β.

Table I

Theoretical masses and compositions of the regular satellites

Planet	Satellite spacing ratio $\langle R_n/R_{n+1}\rangle_{obs}$	Turbulence parameter β	Planetary envelope mass $M_{env}(M_\oplus)$	Gaseous ring mass m(g)	Condensate mass per ring (10^{24}g)		Observed satellite mass (10^{24}g)
Jupiter	1.65	0.100	310	1.0×10^{28}	Rock	50	Io 90, Europa 50
					Rock + H_2O ice	120	Ganymede 150, Callisto 110
							Janus 0.01, Mimas 0.04
Saturn	1.30	0.080	85	1.4×10^{27}	Rock	6	Enceladus 0.08, Tethys 0.6
							Dione 1.2, Rhea 2.7
					Rock + H_2O ice	16	–
Uranus	1.46	0.084	5	1.2×10^{26}	Rock	0.6	Miranda 0.1?
					Rock + H_2O + NH_3 ice	1.6	Ariel 1.3, Umbriel 0.5
					Rock + H_2O + NH_3 + CH_4 ice	2.1	Titania 4.4, Oberon 2.5

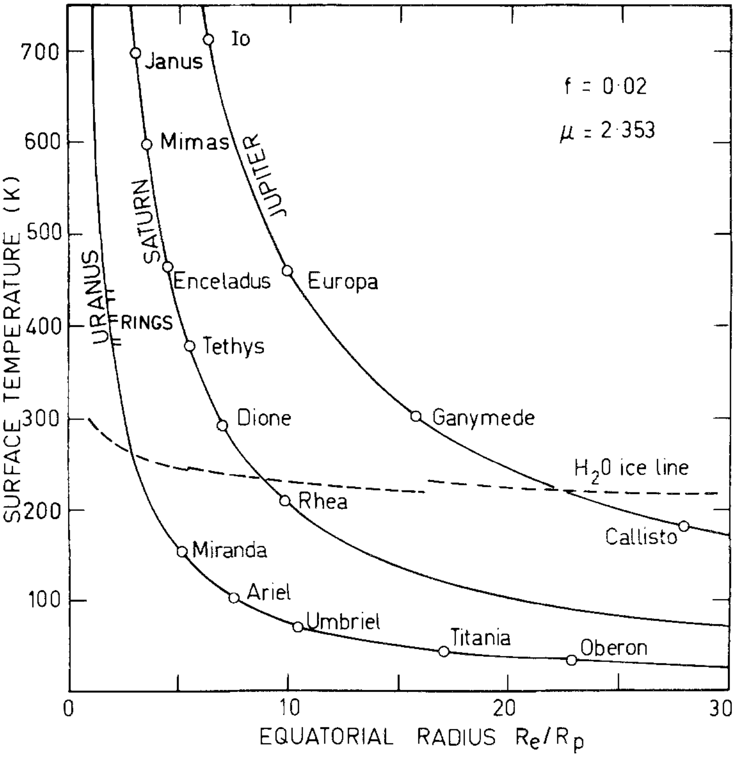

Fig. 1. Surface temperatures of turbulent polytropic models of the primitive gravitationally contracting gaseous envelopes of Jupiter, Saturn and Uranus, plotted against equatorial radius R_e, which is measured in units of the present planetary polar radius R_p. The envelopes are assumed to be in a state of supersonic convective equilibrium characterized by a turbulence parameter β, shown in Table I, which is adjusted to yield a moment-of-inertia coefficient $f = 0.02$. The envelopes are also supposed to be fully rotating, with gravitational force at the equator just balancing the centrifugal force, and to be chemically uniform with molecular weight $\mu = 2.353$ corresponding to a roughly solar composition with H: He = 0.7: 0.3. The condensation temperature at the position of each of the regular satellites and the Uranian rings is also shown, along with the temperature at which H_2O ice condenses for each envelope.

If we next consider Saturn, the temperatures derived from the model of the contracting protosaturnian envelope suggests that each of its regular satellites is made of rock. The observed densities of these satellites are only poorly known. A recent calculation by McLaughlin and Talbot (1977) of the mass of Saturn's rings, however, yielded a density for Rhea of $\sim 2.4\,\mathrm{g\,cm^{-3}}$. This is consistent with a composition of hydrous mineral rock. If this is so, Figure 1 tells us that the satellites interior to Rhea are increasingly rocky (less hydrous) moving towards the planet; that is, we expect Janus and Mimas to be similar in composition to Io. The fact that the observed masses of the inner satellites of Saturn are much smaller than the predicted mass of rock and decrease steadily towards the planet also points to a rocky composition. As we shall point out in the next section, the mass of a satellite depends on how much of the condensing material can segregate onto the central

Keplerian orbit of the gaseous Laplacian ring before the ring evaporates, or is dispersed. The segregation time depends on the size of the condensing particles, being longest for the small dusty particles. Most of the rock condensate may therefore have remained suspended in the gas. It is also likely that there may have been considerable stirring of the inner gaseous rings due to the intense heat bath of the contracting planet, which would have frustrated the settling out of the finer grains (*cf.* also Cameron, 1978; p. 69).

Turning now to Uranus we see that according to Figure 1 and Table I all of its satellites should consist of dirty ice and have a total mean mass of about 2×10^{24} g. Direct condensation of CH_4 occurs at the orbits of Titania and Oberon, thus possibly accounting for the higher mass of these two satellites. We note that the mass of Miranda is appreciably smaller than the expected mass of ice, suggesting perhaps that it may have formed from slowly segregating rock, as in the case of Saturn's satellites. From the temperature point of view, condensation of rock alone at the orbit of Miranda is possible provided that the envelope moment-of-inertia coefficient f exceeds 0.035. Computationally we find that the surface temperature T_e of the envelope varies in nearly linear proportion with f (i.e. $T_e \propto f$). Increasing the degree of turbulence in the cloud lowers both f as well as T_e. Thus when β is lowered from 0.084 to 0.06 in the Uranian envelope we find f increases from 0.02 to 0.035 and T_e from 150 K to 260 K, at Miranda's orbit, whilst the mass m of each gaseous ring ($m \propto f$ for constant R_n/R_{n+1}) increases from 1.2×10^{26} g to 2.1×10^{26} g. In order to account for the mass distribution of the Uranian system, however, we need $f \simeq 0.02$.

Of course, we should not be too surprised that it is impossible to fit all satellites exactly into our model with just one adjustable parameter. Rather, it is surprising that we succeed so very nearly.

4. Accumulation of the Satellite Material

In the previous section, we observed that the ice-like members of the regular satellite systems of the major planets by and large contained their full share of icy material. In contrast, the masses of the rocky ones in general fall short of the expected value. This same phenomenon is also observed in the distribution of planetary masses. There we find that Uranus and Neptune each contain about 10–15 $M_\oplus$ of ices, consistent with full condensation from gaseous rings each of mass 1000 $M_\oplus$ shed by the contracting protosun, whilst the terrestrial planets contain only a fraction of the available mass $\sim 4M_\oplus$ of rocky condensate that was available in their respective rings (Prentice, 1978a).

The physical reason for the short fall in the expected mass of the rocky satellites lies we feel in differences in the rate of segregation of the condensate material onto the central circular orbit R_n of each gaseous ring compared with the life-time of the gaseous rings. The gaseous rings play a vital role both in focussing the condensing grains onto the central orbit R_n as well as in damping out the excess kinetic energy of the larger aggregating satellitesimals which form on these orbits – as is demonstrated, for instance, by Hourigan's work (Hourigan, 1977). If the gaseous ring were to evaporate or disperse, or even to be vigorously stirred by the passage of thermal energy from the central protoplanet, so that the distribution of angular velocity and density given by Equation (2) were destroyed, then all segregation and aggregation would cease.

It is clear from the above discussion that if the segregation time t_{seg} exceeds the physical life-time t_f of the gaseous ring then only a fraction of the available condensate material is able safely to migrate onto the central orbit R_n for accumulation into the satellite mass. The remaining fraction remains suspended in the gas and is swept away from the region of that orbit when the gas ring disperses. Thus for a satellite to contain the full share of available condensate we shall require that

$$t_{\text{seg}} < t_f. \tag{4}$$

In addition, for the material which does successfully settle onto the orbit R_n to aggregate together in time we shall require that

$$t_{\text{agg}} < t_f, \tag{5}$$

where t_{agg} is the aggregation time. If this condition is not met the circular stream of satellitesimals will be left strewn around the mean orbit R_n, as one of us has suggested was the case for the asteroids (Prentice, 1978a). Let us therefore compute t_{seg}, t_f and t_{agg}. In the present section we shall ignore the effects of thermal stirring and turbulent dissipation and suppose instead that the time t_f is given simply by the thermal evaporation time t_{evap} appropriate to a quiscent gaseous ring.

4.1 Timescales Governing the Occumulation Process

(a) *Segregation time, t_{seg}*

A condensed particle migrates onto the circular Keplerian orbit R_n of the nth gaseous ring under the action of the unbalanced component of the protoplanet's gravitational attraction and the orbital centrifugal acceleration, $-\omega_n^2 \underset{\sim}{\xi}$, where $\underset{\sim}{\xi}$ is the meridional displacement of the particle off the mean orbit R_n (Prentice, 1978a), $\omega_n = \sqrt{GM/R_n^3}$ is the Kepler frequency appropriate to the radius R_n and M is the protoplanetary mass. As the particle sinks towards the circular axis R_n it experiences a drag force whose value depends on the size of the particle relative to the mean molecular path length λ_g of the gas, as well as the gas kinetic temperature T_n at a position R_n. For a gas of solar composition (H mass fraction $X \simeq 0.7$, He mass fraction $Y \simeq 0.3$) we find from the data of Williams and Crampin (1971) that

$$\lambda_g \simeq 2.0 \times 10^{-9}/\rho_g \,\text{cm}, \tag{6}$$

where ρ_g (g cm^{-3}) is the gas density.

Table II shows the values of T_n, the gas density ρ_n, and λ_g on the central orbit of each of the gaseous rings shed by the contracting envelopes of Jupiter, Saturn and Uranus at the present orbital positions of the regular satellites. We observe that in every instance λ_g is very much smaller than the mean particle radius $a = 10^{-3}$–1 cm which we assume to be

Table II
Physical characteristics of the Gaseous Rings

Satellite orbit	R_n/R_p	T_n(K)	ρ_n(g cm^{-3})	λ_g(cm)	α_n
		JUPITER			
Io	6.25	712	4.9(–4)†	4.1(–6)	119
Europa	9.95	460	1.1(–4)	1.8(–5)	116
Ganymede	15.78	301	2.5(–5)	7.9(–5)	111
Callisto	27.89	182	4.1(–6)	4.8(–4)	105
		SATURN			
Janus	2.94	697	9.6(–4)	2.1(–6)	97
Mimas	3.43	597	6.1(–4)	3.3(–6)	97
Enceladus	4.40	466	2.9(–4)	7.0(–6)	97
Tethys	5.44	377	1.5(–4)	1.3(–5)	97
Dione	6.98	294	7.2(–5)	2.8(–5)	97
Rhea	9.74	210	2.6(–5)	7.6(–5)	97
		URANUS			
Miranda	5.1	151	3.6(–4)	5.6(–6)	84
Ariel	7.5	103	1.1(–4)	1.8(–5)	84
Umbriel	10.4	74	4.2(–5)	4.8(–5)	84
Titania	17.0	45	9.6(–6)	2.1(–4)	84
Oberon	22.8	34	4.0(–6)	5.0(–4)	84

† Numbers in brackets refer to powers of 10.

of physical interest. The actual grain size depends on the number of nucleation sites available during the vapour condensation phase. This latter quantity is unknown and is an uncertainty which plagues all detailed theories of accretion. Both Goldreich and Ward (1973) and McCrae (1972) estimate the maximum particle size to be of order a few centimetres. For this reason, we consider a spread of grain sizes ranging up to this size. The magnitude of the drag force which a moving grain experiences equals the Stokes value $2.16\pi\rho a\lambda_g(8\mathscr{R}T/\pi\mu)^{1/2}v$ at low particle speeds v and the dynamic pressure value $\pi\rho a^2v^2$ at high speeds. Introducing a dimensionless parameter A to distinguish between these two velocity regimes, given by

$$A = \frac{a}{2.16\lambda_g}\left(\frac{\pi\,\rho_s a\sqrt{\alpha_n}}{6\mathscr{R}\rho_n R_n}\right)^{1/2}, \qquad \alpha_n = \frac{\mu GM}{\mathscr{R}T_nR_n}, \tag{7}$$

where ρ_s is the intrinsic particle mass density, $\mu \simeq 2.353$ is the gas mean molecular weight and $\mathscr{R}$ is the universal gas constant, we have integrated the particle equation of motion to obtain the result

$$t_{\text{seg}} = \omega_n^{-1}\begin{cases}(3\rho_nR_n/\rho_s a\sqrt{\alpha_n})^{1/2} & \text{if} \quad A > 1,\\ \left(\dfrac{2}{\pi}\right)^{1/2}\dfrac{2.16\lambda_g}{a}\,(3\rho_nR_n/\rho_s a\sqrt{\alpha_n}) & \text{if} \quad A \leqslant 1.\end{cases} \tag{8}$$

We have computed the segregation time t_{seg} for four cases of physical interest, namely particulate rock ($\rho_s = 3\,g\,cm^{-3}$ with $a = 0.001,\ 0.01$ cm) and icy flakes ($\rho_s = 1\,g\,cm^{-3}$ with $a = 0.1, 1$ cm). The results are shown in the 2nd column of Table III.

(b) *Evaporation time,* t_{evap}

Hoyle (1960) has shown how highly energetic molecules at the periphery of the solar nebula are able to escape from the gravitational field of the sun into outer space, thereby leading to a steady depletion of the mass of the nebula. In a detailed calculation (Prentice 1979) one of us has extended Hoyle's idea to the case of a system of gaseous rings where a similar result emerges. Any given ring centered on an orbital radius R_n evaporates from its edge on a *e*-folding time scale given by

$$t_{evap} = 2.73\ \omega_n^{-1}\sqrt{\alpha_n}\exp(5\alpha_n/64). \tag{9}$$

The factor $\exp(5\alpha_n/64)$ in this equation, in place of Hoyle's factor $\exp[\frac{1}{2}(\sqrt{2}-1)^2\alpha_n]$ arises from the different choice of angular velocity distribution in the gas, specified by Equation (2); the difference, though, is very small: $5/64 = 0.078$, $\frac{1}{2}(\sqrt{2}-1)^2 = 0.086$.

(c) *Aggregation time,* t_{agg}

The time-scale for the aggregation of a self-gravitating stream of satellitesimals of total mass m_{seg} spread around the mean circular orbit of a gaseous ring is given by

$$t_{agg} = \psi\sqrt{R_n^3/Gm_{seg}\lambda} = \omega_n^{-1}\left(\frac{M}{\lambda m_{seg}}\right)^{1/2}, \tag{10}$$

where $\lambda = \langle|\Delta\rho_{seg}|/\rho_{seg}\rangle$ is the initial mean fluctuation ratio in the line density distribution of satellitesimal mass around the ring and ψ is a number of order unity which is a function of the dimensionless drag coefficient $\rho_n R_n/\rho_s a$ (Prentice, 1978).

The calculated values of t_{seg}, t_{evap} and t_{agg} for each of the satellite orbits are shown in Table III. In computing t_{agg} we have set $\lambda = 0.01$, $\psi = 1$ and m_{seg} equal to the observed satellite mass. It seems likely that this may lead to slight overestimate of t_{agg}.

4.2 Discussion of the Results

Jupiter: The fact that the 4 Galilean satellites appear to contain the full share of condensates available in each gaseous ring seems to be in accord with values t_{seg}, t_{evap}, t_{agg} in Table III. Typically in the case of the icy condensate the segregation time is two orders of magnitude smaller than the evaporation time so that Ganymede and Callisto would have swept their respective gaseous rings completely clean. For the rocky condensate we also have $t_{seg} < t_{evap}$ provided we assume a grain size $a \sim 0.01$ cm. In both cases, the aggregation time is much shorter than the ring evaporation time, showing that the aggregation phase is only a brief event in the formation process.

Saturn: Our model of the chemical condensation sequence in the system of gaseous rings shed by proto-Saturn suggests that each of its satellites is made of rock. This prediction is reflection in the results shown in Table III. For if each of the regular satellites were

Table III

Time-scales of the Accumulation Process

Satellite orbit	Segregation time, t_{seg}(yr) $a = 10^{-3}$ cm	$a = 10^{-2}$ cm	$a = 10^{-1}$ cm	$a = 1$ cm	t_{evap} (yr)	t_{agg} (yr)
			JUPITER			
Io	1.1(4)S†	1.1(2)S	6.0	1.9	250	1
Europa	3.4(4)S	3.4(2)S	1.0(1)S	2.3	390	3
Ganymede	1.1(5)S	1.1(3)S	3.4(1)S	2.7	520	4
Callisto	4.7(5)S	4.7(3)S	1.4(2)S	3.5	720	10
			SATURN			
Janus	1.9(3)S	1.9(1)S	2.2	0.7	17	25
Mimas	2.8(3)S	2.8(1)S	2.4	0.8	21	16
Enceladus	5.1(3)S	5.1(1)S	2.8	0.9	31	16
Tethys	8.7(3)S	8.7(1)S	3.0	1.0	42	8
Dione	1.6(4)S	1.6(2)S	4.9 S	1.1	61	8
Rhea	3.7(4)S	3.7(2)S	1.1(1)S	1.3	100	9
			URANUS			
Miranda	3.1(3)S	3.1(1)S	2.4	0.8	11	6
Ariel	8.2(3)S	8.2(1)S	2.9	0.9	19	3
Umbriel	1.9(4)S	1.9(2)S	5.6 S	1.1	31	8
Titania	6.3(4)S	6.3(2)S	1.9(1)S	1.4	65	5
Oberon	1.3(5)S	1.3(3)S	4.0(1)S	1.6	100	12

† Letter S means Stokes law drag ($A < 1$).

made of ice, with characteristic particle size 0.1 and 1 cm and where $t_{seg,ice} \ll t_{evap}$, we should expect to find satellite masses each roughly the same and equal to about 1.6×10^{21} g. Instead the observed masses fall very much short of this value, particularly those of the inner ones, implying $t_{seg} \gg t_{evap}$, and hence a dusty composition, assuming that the rocky condensate is finely divided with typical particle size $a \leqslant 0.01$ cm. The segregation time is very sensitive to particle size in the Stokes regime ($A \lesssim 1$) varying as $t_{seg} \propto a^{-2}$, as we see from Equation (8).

Uranus: Both conditions given by Equations (4) and (5) are easily satisfied by the 5 Uranian satellites, assuming an icy composition. Miranda has a mass which is appreciably smaller than the expected mass of ice at this orbit, perhaps suggesting a rocky composition. If the temperature of Miranda's gaseous ring were raised above the condensation point for water ice, however, by increasing the moment-of-inertia factor f of the envelope to 0.035, t_{evap} would drop to 1/10th of t_{agg}, namely 0.5 yr, thus rendering this idea inconsistent. It seems more likely that the onset of vigorous stirring in the gaseous ring, due to the increasing temperature of the contracting proto-Uranian envelope, prevented the efficient settling out of its ice particles. We discuss this phenomenon in more detail in the next section.

4.3 Rate of Contraction of the Protoplanetary Envelopes

Finally, we should mention that the various times $t_{\rm seg}$, $t_{\rm agg}$, $t_{\rm evap}$ we have calculated in this section are each very much shorter than the time-scale which governs the contraction of the protoplanetary envelopes. Figure 2 shows the locus of equatorial radius R_e versus elapsed time, in units of the present polar radii R_p, computed on the basis that the rate of collapse is determined by the rate at which the envelope radiates away from its surface the excess gravitational energy released as heat throughout the interior (Prentice, 1978b). In all it takes some 3×10^6yr for Jupiter, 6×10^6yr for Saturn and 1×10^8yr for Uranus to contract through the dimensions of their regular satellite systems, commencing from an initial radius of $30R_p$. For Uranus, the time is reduced to 2×10^7yr if the envelope moment-of-inertia coefficient f is 0.03 in place of 0.02. In any event it is clear that the formation of any given satellite is a very brief affair in the lifetime of the protoplanet and takes place immediately after the gaseous ring is detached. This latter process also occurs very rapidly once the envelope reaches the fully rotating configuration (when the centrifugal force at the equator matches the gravitational force there). During ring detachment the equatorial regions of the envelope withdraw cataclysmically over a time scale of order $\omega_n^{-1} = \sqrt{R_n^3/GM}$, which is of the order of only a day.

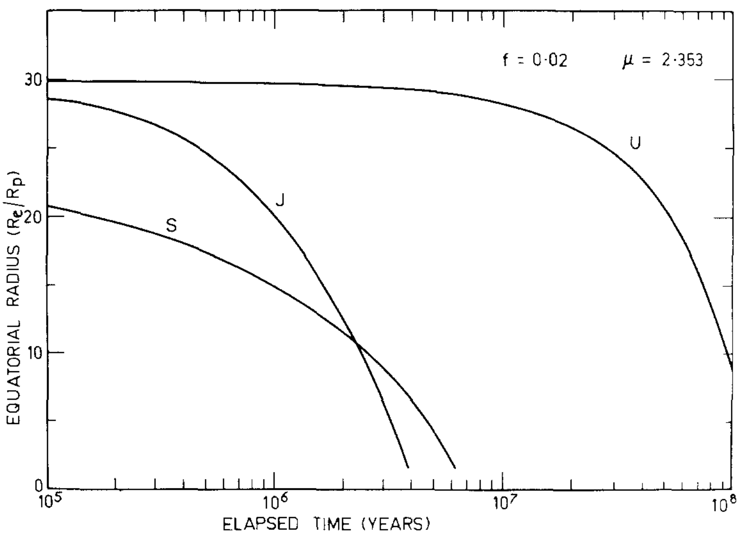

Fig. 2. Equatorial radius of the turbulent protoplanetary envelopes, described in Figure 1, plotted against time elapsed from initial radius $30\,R_p$. The rate of contraction is governed by the radiation loss at the surface, according to Kelvin–Helmholtz equilibrium.

5. The Inner Boundary of the Regular Satellite Systems

According to our theory, the contracting envelopes of Jupiter, Saturn and Uranus also shed several rings of gas between the orbits of the innermost regular satellites and their present planetary radii. Why then do there appear to be no satellites formed at these inner

Table IV
Characteristics of the Innermost Gaseous Rings

R_n/R_p	R_n (cm)	ρ_n ($g\,cm^{-3}$)	T_n (K)	Broad chemical nature of the condensate
		JUPITER		
Io	4.2(10)	4.9(−4)	712	Rock
4	2.7(10)	1.9(−3)	1100	Rock
$2\frac{1}{2}$	1.7(10)	8.0(−3)	1800	Rock
		SATURN		
Janus	1.6(10)	9.6(−4)	697	Rock
$2\frac{1}{2}$	1.4(10)	1.6(−3)	820	Rock
2	1.1(10)	3.1(−3)	1000	Rock
		URANUS		
Miranda	1.3(10)	3.6(−4)	151	Ice + rock
$3\frac{1}{2}$	9.0(9)	1.1(−3)	220	Ice? + rock
$2\frac{1}{2}$	6.4(9)	3.0(−3)	310	Rock
1.8	4.6(9)	8.1(−3)	430	Rock

positions? And how do we account for the rings of Saturn and Uranus within the framework of this model?

In Table IV, we have listed the broad physical properties of the gaseous rings shed at the next two or so positions in the geometric sequence of reducing satellite orbital radii R_n, commencing at the orbits of Io, Janus and Miranda, respectively. The broad chemical nature to be expected of the condensate is shown in the last column. We implicitly assume that the contraction of the planetary envelopes proceeds homologously, maintaining the same turbulence parameter β and moment-of-inertia coefficient $f \simeq 0.02$. That is, the surface temperature and density behave as $T_e \propto R_e^{-1}$, $\rho_e \propto R_e^{-3}$ where R_e is the equatorial radius of the envelope. For Saturn, the position of the first ring at radius $\sim 2\frac{1}{2} R_p$ (R_p = present polar radius) coincides with the outer edge of the A-ring. For Uranus we can fit two gas rings between Miranda and the recently discovered rings which have a mean radius of $1.8 R_p$ (i.e.4.6×10^{10} cm).

Onset of thermal stirring and the termination of grain settling

The most striking feature which we notice in Table IV is that the lower cut-off point in the observed sequence of satellites appears to be associated with that point in the contraction of the envelope where the gas density exceeds about $10^{-3}\,g\,cm^{-3}$. Physically, therefore, this suggests that at this density, which corresponds to a pressure of about one atmosphere, the ring of gas is so compressed that it becomes vigorously stirred by the intense heat bath of the contracting envelope. Efficient segregation and aggregation of the condensate material can take place only as long as the gaseous ring remains quiescent, with angular velocity distribution given by equation (2). If the gas in the ring is strongly stirred then only a small fraction of the condensing particles may be able to settle down onto the circular Keplerian orbit R_n. Cameron (1978) has also drawn attention to this phenomenon in the context of his most recent nebula theory. In fact if the turbulence is sufficiently strong, so that the orbital angular velocity $\omega(s, z)$ of the gas is rendered uniform through the action of turbulent viscosity, the density profile of the gas ring is destroyed and the gases dispersed away from the mean orbit. As the gases are dispersed they carry away the fine particulate material. Hence as the temperature at the inner ring positions does suggest a mostly rocky condensate composition, we can perhaps link the absence of any significant inner satellites with the onset of large-scale thermal stirring of the rings.

We note that the density at the orbit of Janus lies only just below this apparently critical gas density $\rho_{crit} = 10^{-3}\,\mathrm{g\,cm^{-3}}$. It is not surprising, therefore, that Janus is the most insignificant member of the Saturnian family, and indeed of all the satellite families.

Consider now the aggregation of material within a gaseous ring where very little condensate has settled onto the central orbit of the ring. A circular stream of satellitesimals is able to aggregate in time into a single body only if it lies outside the Roche radius of the parent planet (see below) and provided its mass exceeds a minimum amount m_{min}, determined from the condition $t_{agg} < t_{evap}$. Using Equations (9) and (10) we find that

$$m_{min} \simeq (0.13\psi^2/\alpha\lambda)M\exp(-5\alpha/32) = \begin{cases} 2\times 10^{21}\ [\text{Jupiter}] \\ 2\times 10^{22}\ [\text{Saturn}] \\ 3\times 10^{22}\ [\text{Uranus}] \end{cases} \mathrm{g}, \qquad (11)$$

where we have used typical values $\psi = 1$, $\lambda = 0.01$. This lower mass bound is satisfied by all of the regular satellites of Jupiter, Saturn and Uranus, including Janus if its radius exceeds 130 km. We therefore suggest that Uranus's belt formed from a gaseous ring where the total segregated mass of rock was too small to meet this minimum mass criterion. This conclusion is supported by preliminary estimates of the mass and composition of the belt (Smith, 1977; Matthews, Neugebauer and Nicholson, 1978). Once the gas has evaporated, any stream of unaggregated material is first broadened under the action of collisions then later, perhaps, re-structured by orbital resonances with the outer satellites. According to our theory there should exist two further satellite belts of Uranus having orbital radii about 65,000 and 90,000 km which correspond to the remains of two gaseous rings shed by the contracting proto-Uranian envelope at those positions. The suspected faint stream of satellites of Saturn reported by Fountain and Larson (1978) to exist between Janus and the A-ring and having semimajor axis 151,300 km, may also have formed in this manner. If Jupiter does possess a rocky satellite belt at orbital radius 2.7×10^{10} cm, its mass would not exceed 2×10^{21} g.

6. Formation of the Rings of Saturn

6.1. THE FINAL STAGES OF THE PROTOPLANETARY CONTRACTION AND THE SHEDDING OF A GASEOUS DISC

The present effective surface temperatures of Jupiter, Saturn and Uranus are about 125 K, 97 K and 57 K, respectively. These values are maintained partly through the absorption of solar radiation and partly through heat released by the gravitational contraction of the planet. Nonetheless, comparing these present temperatures with those indicative of the very early stages of the protoplanetary contraction [Figure 1] it is clear that at some point in the contraction the homologous phase ($T_e \propto R_e^{-1}$) must have terminated and the surface temperature T_e and the luminosity of the envelope must have undergone a massive downturn. Since a decline in T_e is due to a slowing down in the rate of release of gravitational energy in the envelope, we should also expect to find an accompanying decline in the degree of supersonic convective turbulence in the cloud, measured by the turbulence parameter β. A decline in β, however, leads to a closer spacing between the gaseous rings shed in the equatorial plane. As soon as β falls below 0.05, the contracting envelope is unable to shed a discrete system of rings, but instead leaves behind a continuous disc of gas (Prentice, 1978b). The orbital angular velocity at each point in this disc is closely Keplerian. We therefore suggest that the process of discrete ring shedding ceased at some point during the final stage of the contraction and that the envelopes of Jupiter, Saturn and Uranus got rid of the last vestige of their excess angular momentum through the shedding of a gaseous Keplerian disc.

6.1 CONDENSATION OF THE GASEOUS DISC

Consider now the condensation of material within the gaseous disc. Any material condensing out of the gas tends to settle towards the mid-plane ($z = 0$) to form a disc-like sheet. Nevertheless, as the gas is very dense ($\rho > 10^{-3}\,\mathrm{g\,cm^{-3}}$) and likely to be strongly stirred, any fine rock-like condensate will be unsuccessful in settling out, as described earlier. If the temperature, however, is low enough for H_2O ice to condense out so that largish grains can form. segregation of material onto the central plane will be far more efficient.

We therefore suggest that the outer edge of the Saturnian system of rings delineates that point in the contraction of the envelope of this planet where the surface temperature T_e first dropped below the ice-point. This suggestion is supported by the observation that the rings of Saturn most probably consist predominantly of water ice (Pollack, 1975).

Once the water-ice grains have settled onto the mid-plane of the gaseous disc there is no further focussing of the material into discrete orbital radii as in the case of gaseous rings. Satellitesimals are therefore able to grow only by direct collisions with fresh grain material segregating at the same orbital distance. The so-called Goldreich–Ward instability mechanism (Goldreich and Ward, 1973) for the self-aggregation of material from neighbouring radii fails to operate here since the majority of the condensate disc lies interior to the Roche limit of the planet, defined by

$$D/R_P = 2.44(\rho_p/\rho_s)^{1/3}. \quad (12)$$

In this equation, $\rho_p = 3M/4\pi R_p^3$ is the mean density of the planet and ρ_s that of the accreting satellitesimals. Taking $\rho_s = 1\ \mathrm{g\,cm^{-3}}$, corresponding to H_2O ice, we find $D/R_P = 2.31$ for Saturn, 2.62 for Uranus and 2.78 for Jupiter where R_P, as before, denotes the present polar radius of the planet. Interior to a radius D, the tidal stresses induced by the planet prevent any aggregation of satellitesimals under the action of their own self-gravitation.

6.3 Masses of the Condensate Ring Systems

The maximum mass of the icy discs can be estimated from Equation (3) taking R_n/R_{n+1} equal to the observed ratio of outer and inner radii of the annular disc and proceeding the same manner as in Table I.

Saturn: Choosing $f = 0.02$, as before, the maximum mass of dirty-ice condensate available in the gaseous disc spanning the interval of the A and B rings of Saturn is found to be

$$m_{\max} \simeq 2 \times 10^{25}\,\mathrm{g}. \quad (13)$$

This upper bound is consistent with the best empirical mass $(3.5 \pm 1.4) \times 10^{24}$ g obtained by McLaughlin and Talbot (1977). This suggests that only 1/5th or so of the available condensate in the gaseous disc was successful in settling onto the mid-plane before the disc was dispersed through thermal evaporation and turbulent dissipation. The fact that rings are substantially more massive than the regular inner satellites (e.g. $m_{\mathrm{Rings}}/m_{\mathrm{Mimas}} \sim 100$) also supports the view that they consist primarily of water-ice and not finely segregated rock.

Lastly, we should point out that our explanation of the formation of Saturn's rings and the inner satellites differs markedly from that of Pollack *et al.* (1976). They propose that all of the inner satellites are made of ice and that the variation in the masses of the satellites and rings can be attributed to differences in the period of elapsed time before the initially hot ($T \simeq 600$ K) gaseous disc cools sufficiently for the condensation of H_2O ice to occur. Since we have shown that the gas evaporates away over a time-scale of at most 10^2 yr, whilst the nebula takes a time $\gtrsim 10^6$ yr to cool sufficiently for the condensation of ice, it seems very difficult to see how this latter hypothesis can have time to operate.

We should mention that after the gaseous components of the discs have dispersed, orbital resonances of the outer satellites with the material of the condensate disc will lead to a re-structuring of the radial distribution of this material. The inner edge of the C-ring at 1,33 R_p, for example, lies well inside the radius $1\frac{1}{2} R_p$ which marks the inner edge of the Keplerian gaseous disc which could be shed by a fully rotating proto-Saturn envelope having the present polar radius. Thus, satellite resonances and other tidal phenomena are probably needed to account for the present detailed structure of the rings (Pollack, 1975).

Uranus: For Uranus it is possible that the radius at which the envelope surface temperature T_e began to downturn and disc shedding commenced did not occur until the final Keplerian radius $R_e = 1.5\ R_p$ had been passed. We note from Figure 1, that for $R_e = 1.5\ R_p$ the temperature $T_e \sim 500$ K is well below that (~ 700 K) enjoyed by the last forming satellites of Jupiter and Saturn, namely Io and Janus. Thus, if proto-Uranian envelope did not shed a gaseous Keplerian disc, we can account for the absence of any icy disc surrounding this planet. Of course, any material condensing within a sub-Keplerian disc will spiral in onto the planet.

6.4 ABSENCE OF JOVIAN AND NEPTUNIAN RING SYSTEMS

Jupiter: Considering Jupiter, we concur with Pollack *et al.* (1976) that no icy ring system is expected for this planet simply on the basis that the surface temperature T_e of the protojovian envelope probably never fell low enough for the condensation of ice in the shedded gaseous disc of this planet. At equatorial radius $R_e = 2R_p$, corresponding to polar radius 1.33 R_p for the fully rotating envelope, T_e exceeds 600 K, judging from the calculations of Graboske *et al.* (1975).

Further evidence that the temperature in the inner protojovian disc never become low enough for the condensation of water ice before being turbulently dispersed comes from the tiny satellite Amalthea which lies interior to the Roche limit of this planet ($D = 1{,}88 \times 10^{10}$ cm for $\rho_s = 1$ g cm^{-3}). Had this satellite somehow formed from the protojovian gaseous disc then it certainly could not have accreted from icy condensates because of the tidal stresses. In view of the absence of any other satellite interior to Io, and the vigorous stirring of the gases of the disc which would have prevented the settling out of any rocky condensate, it seems likely the Amalthea is a captured satellite. The dark reddish spectral features of this body, observed recently by Millis (1978), suggests a composition similar to the Carbonaceous chondrite material found in C-type asteroids. This raises the strong possibility that Amalthea was a stray asteroid captured by Jupiter at some time after its formation.

Neptune: Nepture is not expected to possess a condensed ring system for the same reason that it does not possess a regular satellite system, namely that it did not acquire any significant primitive hydrogenic envelope.

7. Origin of the Irregular Satellites of the Major Planets

We have asserted in this paper that the outermost members of the regular satellite systems of the major planets mark the point where the primitive contracting envelopes of these planets first became rotationally unstable. This being the case, all of the satellites lying beyond the orbits of Callisto, Rhea, and Oberon, respectively, must represent captured bodies, possibly residual icy planetesimals left over from the formation of the icy cores of Jupiter, Saturn and Uranus. That is, we propose that they originally accreted from the same icy condensate streams that segregated from the gaseous rings that were disposed by the contracting protosun at the orbits of the major planets, but which failed to aggregate with the main planetary cores of these planets, each of mass $\sim 10_{\oplus}$. Presumably there must have been many such stray planetisimals at the orbit of each major planets and of the terrestrial planets, judging from the copious distribution of impact craters on the Moon and Mars and Mercury. Most of these stray planetesimals would have been swallowed up by each of their parent planets during the first 10^8 yr or so of their evolution. A few large stray icy planetisimals on each orbit may, however, have been captured in highly irregular orbits. The fact that the outer satellites of Jupiter, Saturn and Uranus do have highly eccentric orbits, which are also highly inclined to the planets equatorial plane, does support this capture hypothesis. All of the satellites of Neptune, therefore, are captured bodies according to this theory.

8. Conclusions

We have shown that the formation of the regular satellite systems of the major planets can be accounted for in terms of a modern version of the original Laplacian hypothesis, namely, through condensation from a system of gaseous rings shed by the primitive turbulent contracting envelopes of each of these planets. If the contraction of the envelope proceeds nearly homologously, preserving its moment-of-inertia coefficient f and ratio m/M of ring-to-envelope mass, the orbital radii R_n of the rings, when numbered from the outermost one ($n = 0$) inwards, form a geometric progression of the form

$$R_n/R_{n+1} = [1 + m/Mf]^2.$$

If the supersonic turbulent convection is sufficiently strong, the masses m of the disposed rings are each about the same and are sufficiently small when $f \sim 0.02$ to account for the masses of the satellite systems.

We have examined the chemical condensation sequence in each gas ring and the rate of segregation of the condensing particles onto the central mean orbit of the ring, where the local orbital angular velocity equals the Kepler value. We have also considered the rate of accumulation of the segregating material into single satellite masses and computed the expected lifetime of each gaseous ring, on the basis of the thermal evaporation rate.

For Jupiter, we achieved a consistent picture for the formation of its regular satellite system in which each satellite accumulated from all of the available condensate in its respective gaseous ring, of mass 1.0×10^{28} g and solar conposition, and with the chemical composition of the condensing species determined from the temperature law $T_n \propto 1/R_n$, appropriate to a uniform contraction of the protojovian envelope.

For Saturn and Uranus, the observational chemical composition of the regular satellites is largely unknown. We have predicted that the moons of Saturn are made mostly of hydrous mineral rock whilst those of Uranus consist mostly of dirty water ice. The shortfall and steady decline in the mass of the Saturnian group, moving towards the planet, is consistent with a picture of condensation from finely-divided solid material, having particle size $a \lesssim 0.01$ cm, which remained suspended in the gas ring unable to settle onto the mean orbit R_n before the gaseous ring was dispersed, and where vigorous stirring of the inner rings due to the heat bath of the contracting protoplanet also frustrated the settling out of the dusty grains. For Uranus the sizes of the icy grains were apparently large enough ($a \gtrsim 0.1$ cm) to permit the full segregation of all the ice condensate in each gas ring, except at the orbit of Miranda where the onset of thermal stirring in the ring may have retarded the settling process. In all cases the time scales for the accumulation of the satellitesimal material and the thermal evaporation of the rings are very much shorter than the time scale governing the rate of contraction of the protoplanetary envelopes which is about 10^7–10^8 yr. This means that in the present scenario the satellites are formed one at a time and immediately following the catastrophic detachment of the gas ring from the turbulent envelope of the parent planet.

According to our theory, the outer member of each regular satellite group marks the point where the protoplanetary envelope first became rotationally unstable. The inner boundary of the group is probably governed by the point where the gas ring is so dense ($\rho > 10^{-3}\,\mathrm{g\,cm^{-3}}$) that vigorous thermal stirring due to the heat of the protoplanet almost completely prevents the settling of the condensing grains. Uranus's belt formed like the asteroidal belt from a gaseous ring where the total mass of segregated rock was too small to permit aggregation before the gaseous ring dispersed. Two further such belts may exist at orbital radius $2\frac{1}{2}\,R_{\mathrm{U}}$ and $3\frac{1}{2}\,R_{\mathrm{U}}$, respectively, whilst Jupiter may possess a rocky belt near 4 R_{J}. Amalthea is probably a captured asteroid.

We have suggested that during the final stages of the contraction of the protoplanetary envelope, when the rate of contraction and degree of turbulence drastically decline, the envelope disposes of its excess spin angular momentum through the shedding of a continuous gaseous Keplerian disc, rather than a discrete system of gaseous rings. We have suggested that the well-known Saturnian ring systems may have formed through condensation in such a disc. The outer edge of these systems is defined by the point in the contraction where the temperature at the equator of the envelope first became cool enough for the condensation of largish grains of water ice – the dust component in the stirred gaseous disc being too finely divided to settle out. The subsequent accretion of this icy condensate disc into large satellitesimals is prevented because it lies mostly within the Roche radius of the protoplanet. We concur with Pollack *et al.* (1976) that Jupiter does not posses an icy ring system simply because it never became cool enough for the formation of icy condensates in its inner gaseous disc. For Uranus, the disc formation phase is never reached. Lastly, we suggest that Neptune does not possess a regular satellite system or ring system because it never acquired any significant gaseous envelope.

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