# ON THE INTERACTION BETWEEN A STRONG STELLAR WIND AND A SURROUNDING DISK NEBULA*

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**Abstract.** The interaction between a strong stellar wind carrying no intrinsic angular momentum and a surrounding disk nebula is investigated. We analyze the shape and stability of the wind-nebula interface, the strength and direction of the ensuing mass motions and the time scales for nebular disruption. The resultant time scale is given by Equation (44). The dominant physical process is one of nebular accretion onto the central star due to turbulent viscosity in the disk. The turbulence will be driven in the upper layers of the disk by the wind. We note that if the accretion supplies mass for the wind (after the absorption of stellar energy), then the particle fluxes may undergo a runaway increase until the energy or momentum flux in the wind is limited by the total stellar luminosity. This may explain the origin of strong, pre-Main-Sequence winds.

## 1. Introduction

The primitive solar nebula is believed to have been dispersed by a combination of several different physical processes. Mass loss during the early stage is probably dominated by the mechanism of Lynden-Bell and Pringle (1974), in which viscous dissipation of orbital energy in the nebula results in an inward drift of mass and an outward migration of angular momentum. Recent evolutionary models for a massive ($\sim 1\ M_\odot$) protoplanetary nebula by Cameron (1978) differ from this in the inclusion of a coronal-type wind that originates all along the surface of the disk. The mass migration described by Lynden-Bell and Pringle (1974) presumably occurs as long as mass infall from the surrounding cloud provides a significant ram pressure to choke off the wind. After this accretion subsides, wind-driven mass loss from the disk could remove most of the remaining nebula in a few tenths of a million years.

Lynden-Bell and Pringle (1974) and Cameron (1978) emphasize that the viscous dissipation of orbital energy or the generation of a wind will be significant only when the disk is highly turbulent. This will occur throughout the early stages of evolution when the disk is massive and optically thick to cooling radiation. Under these circumstances, turbulence may be driven by meridional circulation currents in the disk (Cameron and Pine, 1973) or by convection and dynamical instabilities (Cameron, 1978). Mass infall from the surrounding cloud may also be an important source of turbulence during these early stages.

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After the infall stops and the disk becomes too small to provide an internal source of turbulence, then a new phase in the disruption of the nebula may begin. Here we investigate a suggestion by Cameron (1973) that the radially directed wind from a central star can remove a surrounding disk nebula. Although such a wind may be present even during the early stages of nebular dissipation, it may not play an important role in controlling nebular evolution until the large-scale turbulence in the midplane of the disk subsides.

Handbury and Williams (1976) stated that the pressure from a radial wind could not remove a surrounding disk nebula. They suggested that it would only push the nebula out to some finite distance determined by force balance and the conservation of angular momentum. However, they assumed that the wind acts on the nebula as a whole and that the total nebular mass could resist the wind's pressure by moving from one mean radius to another, larger mean radius. We show here that the wind induces a small scale outward mass motion in the top surface layer of the nebula. This mass motion in itself is enough to create a differential torque in the disk, but it also acts as a source of turbulence and mixing which lead to additional torques. The result of these torques, as in the Lynden-Bell and Pringle investigation (1974), will be the steady drift of nebular matter along the topmost layers of the disk. We show that the wind-induced mass outflow is exactly compensated by the wind's contribution to an inward drift. Thus our results are essentially in agreement with Handbury and Williams (1976): it is only the additional drift due to turbulence which leads to permanent changes in the mass distribution of the disk.

## 2. Two Surfaces of Pressure Balance

The interaction between a radially directed wind and a surrounding disk nebula will be similar in some respects to the interaction between the present day solar wind and a comet (for a discussion of this process, see Biermann *et al.*, 1967; or Wallis, 1973). A difference arises because the mean free path from collisions between energetic ions and neutrals in the wind and the high density nebular gas will be extremely small. We shall be considering examples of disk nebulae with masses of around $10^{-2}\ M_{\odot}$. Charge exchange reactions between a $\sim 100\,\mathrm{km\,s^{-1}}$ wind and such a nebula will occur within a few meters of the nebular surface, and energetic neutrals will be absorbed within hundreds of meters. The collective plasma effects thought to be responsible for the acceleration of type-I comet tails (Chernikov, 1974) will be inoperative. The wind will shock at the nebular interface as it does ahead of a comet, and wind particles will thermalize and collide with nebular particles behind the shock.

Evidently, the momentum carried by the wind will be rapidly transferred to the exposed parts of the nebula. This implies that another shock will exist in the nebular gas (in addition to a shock in the wind itself) wherever the ram pressure from the wind, $P_{\mathrm{W}}$, greatly exceeds the thermal pressure in the nebula, $P_{\mathrm{N}}$. The relatively low density gas at great distances above the midplane of the nebula will be shocked by the wind and removed from this position on a time scale of years; this shocked nebular material will either immediately acquire escape velocity and leave the system altogether, or it may accumulate more material on the way out and eventually fall back to the plane of the nebula as a result of Rayleigh-Taylor instabilities. On the other hand, higher density gas closer to the midplane of the nebula will not be significantly affected if its thermal pressure is greater than the ram pressure of the wind. Between these two regimes, there will be a surface at which the *normal* component of the wind's momentum flux balances the thermal pressure of the nebular gas. Gas which drifts above this surface will be pushed back onto the nebula by the wind. For a steady, isotropic wind, this surface of normal pressure balance will be the physical boundary of the nebula. For the more likely case of a variable wind, the surface will be the nebular boundary only in a time or position averaged sense.

### 2.1. A SURFACE OF NORMAL PRESSURE BALANCE

The surface of normal pressure balance (SNPB) is where

$$P_W(x, z) \sin \alpha = P_N(x, z) \tag{1}$$

for cylindrical coordinates $x$, measured along the plane, and $z$ perpendicular to the plane, and for $\alpha$ equal to the angle between the radius vector to a point $(x, z)$ and the local tangent to the surface at $(x, z)$, measured in the $x$-$z$ plane: i.e.,

$$\alpha = \beta - \theta, \tag{2}$$

where

$$\tan \beta = \mathrm{d}z/\mathrm{d}x, \text{ and } \tan \theta = z/x. \tag{3}$$

Using Equations (2)–(3), we may rewrite Equation (1) as a differential equation for the points $z(x)$ on the surface of revolution using $R \equiv (x^2 + z^2)^{1/2}$:

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{z}{x}\left[1 + \frac{P_N}{P_W}\frac{R^2}{xz}\left(1 - \frac{P_N^2}{P_W^2}\right)^{1/2}\right]\left[1 - \frac{P_N^2}{P_W^2}\frac{R^2}{x^2}\right]^{-1}. \tag{4}$$

The shape of this surface, $z(x)$, depends on the boundary conditions for $x, z$, and $\mathrm{d}z/\mathrm{d}x$ near the central star, and on the variation of wind and nebular pressures with position.

### 2.2. THE STABILITY OF THE SNPB

The SNPB will be unstable to small perturbations if ripples on the surface grow. The tendency for this to occur is always present, since the wind will intersect the nebula at an angle greater than $\alpha$ at the site of any such disturbance, and the total pressure there will be $P_W$, much larger than $P_N$ at the SNPB. If this is the case, then the wind will penetrate below the SNPB via the propagation of a shock. We may determine whether such instabilities will be significant by considering a channel that is created in the SNPB after the propagation of a shock. This channel will be continuously filled in by sonic motions and by an upward diffusion of the underlying gas. The formation of a surface instability will be important wherever the shock-clearing rate is large compared to the rate of sonic fill in. This condition may be written in terms of the shock velocity $v_s$, the sound speed $c$ and density $\rho_N$ at the SNPB if we take $\rho_N c/4$ to be the upward diffusion flux of nebular particles. Then the instability will occur to a significant degree wherever

$$\sin\alpha \gg \frac{c}{4v_s}. \tag{5}$$

Wind-induced shocks in the nebula will have a velocity of around $(P_W/\rho_N)^{1/2}$, which is $c/(\sin\alpha)^{1/2}$ from Equation (1). Condition (5) then simplifies to $\sin\alpha \gg 1/16$ or $\alpha \gg 3\overset{\circ}{.}6$. Wherever the solution to Equations (1) and (4) gives an $\alpha$ which is greater than $\sim 3\overset{\circ}{.}6$, the SNPB will be unstable against local and temporary penetration by the wind.

### 2.3. A SURFACE OF MAXIMUM PRESSURE BALANCE

The maximum depth below the SNPB which can be reached by either wind-induced mass motions or by the wind itself is the depth at which the total pressure from the wind balances the thermal pressure in the nebula. The surface of revolution where this balance occurs will be referred to as the surface of *maximum* pressure balance, or the SMPB. The equation governing its shape is simply

$$P_W(x, z) = P_N(x, z). \tag{6}$$

We shall refer to the layer between the SNPB and the SMPB as the pressure balance layer, or the PBL.

## 3. Wind-Induced Mass Motions and Torques

Matter near the SNPB will absorb the wind's momentum and move outward. The matter will in general have some small drift velocity, $u$, before it absorbs the wind's momentum, and then it will rapidly accelerate to a relative terminal velocity $v_0$ in the radial direction. Here we determine the outbound flux of nebular matter and angular momentum that results from this motion.

The rate $\delta\dot{M}$ (in grams per second) at which nebular material leaves a comoving annular ring with radius $x$ and width $\delta x$ is determined by the conservation of momentum in the wind:

$$\delta\dot{M}\, v_0 = P_W \cos\alpha\, 4\pi R^2\, \delta\Omega \tag{7}$$

for solid angle

$$\delta\Omega = \frac{x\,\delta x \sin\alpha}{2R^2 \cos\beta} \tag{8}$$

subtended by the annulus, and for $R = (x^2 + z^2)^{1/2}$. The outflow rate per unit length along the flow is thus

$$\frac{\delta\dot{M}}{\delta x} = \frac{2\pi x \sin\alpha \cos\alpha\, P_W}{v_0 \cos\beta}. \tag{9}$$

If the outflow mixes with underlying material after traversing some distance $\lambda_0 \ll x$ measured relative to the drift, then the total outbound flux relative to the drift is

$$F_{\text{wind}} = \frac{\lambda_0}{2\pi x}\frac{\delta \dot{M}}{\delta x}. \tag{10}$$

The values of $\lambda_0$ and $v_0$ depend on the stability of the SNPB and on the amount of mixing. If the SNPB is unstable to local shocking, then the high density post shock gas which may accumulate at the SNPB will continuously fall down to the SMPB, where it stops due to pressure balance. It is not supported in the $z$ direction at the SNPB so it falls because of its higher density. The fall time is roughly $x/v_c$ for orbital velocity $v_c = (GM/x)^{1/2}$ using $G$ and $M$ for the gravitational constant and stellar mass. The mixing length of each shocked element will then be $\sim v_s x/v_c$, and the outflow velocity will be $v_s$. The average values of $\lambda_0$ and $v_0$ will be less than these if the SNPB is only slightly unstable. It is interesting to note that $v_s x/v_c$ is comparable to the depth of the PBL (cf., Equation [15]).

If the SNPB is stable against local shocking, then material at the top of the nebula will gradually move outward (at subsonic velocities), at first conserving angular momentum. This will result in an ever increasing difference between the tangential component of the orbital velocity of the outflow and the circular velocity of the underlying material. Viscous friction acts to equalize these velocities, and the specific angular momentum of the outflowing matter increases while the underlying material experiences a decelerating torque. If mixing is not severe, then the maximum radial excursion of this material will be determined by the balance between centrifugal, gravitational and wind-derived forces at the SNPB. For turbulent viscosity $\nu$ and for density, column density and depth of flow $\rho_N$, $\sigma_f$, and $\Delta z$, the viscous damping length for motion along the outflow becomes $\Lambda = \sigma_f \Delta z (v_0 + u)(\rho_N \nu)^{-1}$. If viscosity is relatively low, e.g., $\Lambda \gtrsim x$, then the maximum radial excursion determined by force balance is $\lambda_{max} = x v_0/v_c$. For highly viscous flow, e.g. $\Lambda \ll x$, the outflow will continuously pick up angular momentum through viscous interactions with the underlying material; force balance will then occur after a much larger radial excursion, equal to $\lambda_{max}^2/\Lambda$. We find that for a $10^{-2}\,M_\odot$ nebula surrounding a solar mass star and with $\nu \sim v_0 \Delta z$, $\lambda_{max}$ is typically much larger than $\Lambda$ and $\lambda_{max}^2/\Lambda$ will be the same order of magnitude as $x$.

The Kelvin-Helmholtz instability will occur over a much smaller length than $\lambda_{max}$, however. This instability will grow at the interface between the outflow and the underlying nebula as a result of the relative motion of these two fluids. It will become important after the outflow moves a distance comparable to its own thickness, which is $\Lambda \sim \Delta z \sim \sigma_f/\rho_N \ll \lambda_{max}$. Thus the outflow will continuously mix with underlying material, making $\lambda_0 \sim \Delta z$. The terminal velocity, $v_0$, will be regulated by viscous friction and by the production of sound waves. The outflow will be severely damped if $v_0$ is close to the local sound speed, so we might expect $v_0 \lesssim c$. If in fact $\lambda_0$ is comparable to the depth of the outflow, $\lambda_0 \sim F_{\text{wind}}/(v_0 \rho_N)$, then Equations (1), (9) and (10) give $v_0 \approx c$ for typically small angles $\alpha$ and $\beta$.

With continuous mixing on a small scale $\lambda_0$, the outbound flux of angular momentum at the top of the nebula is

$$\Phi_{\text{wind}} = F_{\text{wind}} h_c \tag{11}$$

for specific angular momentum $h_c = (GMx)^{1/2}$. The torque per unit length in the radial direction that is exerted on the underlying material by the outflow is then $-\partial(2\pi x \Phi_{\text{wind}})/\partial x$.

## 4. Turbulence in the Pressure Balance Layer

Although the entire layer between the SNPB and the SMPB is susceptible to penetration by the wind, only the topmost part of this layer will take part in the forced outflow if the SNPB is stable. The depth of the outflow may be estimated from the fraction $f$ (which is a function of $x$), equal to the ratio of the column density involved in the flow, $\sigma_f \sim F_{\text{wind}}/v_0$, to the total column density in the PBL, denoted here by $\sigma_\epsilon$:

$$\sigma_\epsilon = \int_{z(\text{SMPB})}^{z(\text{SNPB})} \rho_{\text{N}}(z)\, \text{d}z. \tag{12}$$

Since

$$P_{\text{W}}(1 - \sin\alpha) \cong \frac{GMz}{x^3}\sigma_\epsilon \tag{13}$$

in hydrostatic equilibrium with $z \ll x$, we have for typically small drift velocities $u \ll v_0$,

$$f \cong \lambda_0 \frac{v_c^2}{v_0^2}\frac{z}{x^2}\sin\alpha. \tag{14}$$

Here we have set $\cos\alpha \simeq \cos\beta \simeq 1$ and $\sin\alpha \ll 1$ for the small angles involved. From our model calculations of Section 6, we obtain $\lambda_0/z \sim 0.3$, $z/x \sim 0.2$, $v_c/v_0 \sim 30$, $\sin\alpha \sim 0.02$ at $x \sim 1$ AU, so $f \gtrsim 0.2$.

The rest of the PBL will be turbulent with characteristic length equal to the depth of the PBL, which is

$$\lambda_t \cong z(\text{SNPB}) - z(\text{SMPB}). \tag{15}$$

The wind will not induce turbulence below the SMPB because the gas density there is too large to allow any significant motion after it absorbs the wind's momentum. The turbulent velocity, $v_t$, will be determined by the local sound speed; a value of $v_t \sim c/3$ (Cameron, 1978) will be used in Section 6.

Turbulence in the PBL will be driven in part by Kelvin-Helmholtz instabilities, and in part by occasional wind-driven shocks at the nebular surface. The primary source of energy for this turbulence must be the wind. Most of the wind's energy will be radiated away at the shock in the wind's flow, but the rest will induce both the outflow and the turbulence. The power dissipated per unit length in the radial direction by the turbulent viscosity involved with the outflow is $\sim 2\pi x(v_c\lambda_0/2x)$ times the viscous stress, $\rho_{\text{N}}\nu \text{d}v/\text{d}z$, with $\text{d}v/\text{d}z \cong v_c\lambda_0(2x\Delta z)^{-1}$. The viscous forces supply energy to sound waves and to turbulence throughout the PBL. The rate at which energy is dissipated by the turbulence per unit length in the radial direction is approximately $2\pi x \rho_N v_t^3$. The ratio of the energy source to the energy dissipation rate is $(v_c \lambda_t)^2 (2v_t x)^{-2}$, obtained to order of magnitude by setting $\sigma_\epsilon/\rho_N \sim \Delta z \sim \lambda_t$ and $\nu \sim v_t \lambda_t$. This ratio is slightly dependent on the nebular model, but for a typical case studied in Section 6 it is greater than 1. Similarly, one can show that the energy dissipated by the turbulence anywhere in the PBL is the fraction $v_t$ divided by the wind speed, or $\sim 10^{-3}$ times the energy available in the wind.

The viscous stress due to turbulence will be $\rho_N \nu$ times the local rate of shear in the circular velocity (Lynden-Bell and Pringle, 1974). The outbound mass and angular momentum fluxes measured in comoving coordinates are, respectively,

$$F_{\text{vis}} = 0, \tag{16}$$

$$\Phi_{\text{vis}} = -x^2 \sigma_\epsilon \nu \frac{\partial}{\partial x}(v_c/x). \tag{17}$$

These are evaluated in the frame of reference that moves at some velocity $u$ along with the gradual drift of the material in the PBL. They also refer to each half of the disk, i.e., plus or minus $z$, so the total fluxes in the entire nebula would be twice as large. For simplicity of notation, we set

$$\nu = \zeta \lambda_t v_t, \tag{18}$$

where $\zeta$ absorbs all of the physics involved with the viscous momentum transport. For $v_t \sim c/3$, $\zeta$ might range between 0.3 (Schwarzschild, 1959; Cameron, 1978) and $\sim 10^{-3}$ (Lynden-Bell and Pringle, 1974).

## 5. Torques, Drifts and Nebular Evolution

The negative gradients of $2\pi x$ times the above angular momentum fluxes are equal to the torques on the material in the top layer of the nebula. These torques lead to a slow drift of matter in the PBL and to a permanent change in the distribution of surface density along the entire disk. Here we determine the drift velocity and the equation of evolution.

Consider again a differential annulus $(x, \delta x)$ in the PBL. The mass and angular momentum contained in $\delta x$ are, respectively,

$$\delta M = 2\pi x \sigma_\epsilon \delta x, \tag{19}$$

$$\delta L = 2\pi x \sigma_\epsilon h_c \delta x. \tag{20}$$

Their comoving time derivatives are

$$\frac{D}{Dt}\delta M = -\delta x \frac{\partial}{\partial x} 2\pi x F_{\text{wind}}, \tag{21}$$

$$\frac{D}{Dt}\delta L = -\delta x \frac{\partial}{\partial x} 2\pi x(\Phi_{\text{wind}} + \Phi_{\text{vis}}). \tag{22}$$

The rate at which matter is deposited by the wind itself is less than $D\delta M/Dt$ by the factor $v_0/v_w \ll 1$, so we ignore it here. The angular momentum carried by the wind is also assumed to be negligible.

The drift velocity $u$ may be determined by differentiating $\delta L = \delta M h_c$ by parts, by using the fact that $h_c$ is time invariant for each $x$ and that $D/Dt = \partial/\partial t + u\partial/\partial x$. We find after substitution from Equations (19)–(22) that

$$u = -\frac{F_{\text{wind}}}{\sigma_\epsilon} - \frac{3\zeta}{x\sigma_\epsilon}\left(\frac{y}{2} + x\frac{\partial y}{\partial x}\right), \tag{23}$$

where

$$y = \lambda_t \sigma_\epsilon v_t. \tag{24}$$

This drift takes place only in the PBL.

The total radially directed mass flux relative to the star, $F_{\text{tot}}$, is the sum of the mass flux from the drift, $\sigma_\epsilon u$, and the mass flux relative to the drift, $F_{\text{wind}}$. For each half of the disk, we obtain

$$F_{\text{tot}} = -3\zeta\left(\frac{y}{2x} + \frac{\partial y}{\partial x}\right). \tag{25}$$

The time rate of change of mass column density at a fixed position $x$ is the negative divergence in cylindrical coordinates of the absolute flux

$$\frac{\partial \sigma_\epsilon}{\partial t} = 3\zeta\left(\frac{3}{2x}\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}\right). \tag{26}$$

If there is no independent process of evolution for the part of the disk below the SMPB, then $\partial\sigma/\partial t = 2\partial\sigma_\epsilon/\partial t$, and Equation (26) describes the evolution of the entire disk above or below $z = 0$. We note that there is no contribution to $\partial\sigma/\partial t$ from the wind so that $\lambda_0$ and $v_0$ are insignificant as long as they are relatively small.

## 6. Results of Model Calculations

### 6.1. THE MODEL

We apply the above results to highly simplified model nebulae in order to estimate the time scales for their evolution. We consider a disk with a mass $M_{\text{N}}$ of around $10^{-2}\ M_\odot$. The local gas temperature is taken to be (cf. Hills, 1973)

$$T(R) = \left(\frac{L}{4\pi R^2 \sigma_{\text{SB}}}\right)^{1/4} \tag{27}$$

for stellar luminosity $L$, and Stefan-Boltzmann constant $\sigma_{SB}$. The isothermal sound speed in molecular hydrogen is then $c(R) = (kT/\mu)^{1/2}$ for $\mu = 2.36\, m_H$.

The equation of hydrostatic equilibrium in the $z$ direction requires that

$$\frac{dP_N}{dz} = -\frac{GMz}{R^3}\rho_N. \tag{28}$$

Since $\rho_N = P_N/c^2$, this admits of the solution

$$P_N(x,z) = P_{N0}(x)\exp\left\{-\frac{2v_c^2}{c_0(x)^2}\left[1-\left(\frac{x}{R}\right)^{1/2}\right]\right\}, \tag{29}$$

where $P_{N0}$ and $c_0$ are the local pressure and sound speed in the midplane of the disk. The pressure in the midplane is determined by the total column density, since

$$\sigma(x) = 2\int_0^{z(\mathrm{SNPB})} \frac{P_N(x,z)}{c^2(R)}\,dz. \tag{30}$$

In the appropriate limit of $z \ll x$, we have to a good approximation

$$P_N(x,z) = P_{N0}(x)\exp\left(-\frac{v_c^2}{2c_0^2}\frac{z^2}{x^2}\right) \tag{31}$$

and

$$P_{N0}(x) = c_0(x)v_c(x)\sigma(x)\left[(2\pi)^{1/2}x\,\mathrm{erf}\left(\frac{v_c z(\mathrm{SNPB})}{2^{1/2}c_0 x}\right)\right]^{-1}, \tag{32}$$

where

$$\mathrm{erf}\,(q) = \frac{2}{\pi^{1/2}}\int_0^q e^{-p^2}\,dp. \tag{33}$$

The total mass in the nebula, $M_N$, the stellar luminosity, $L$, and the column density $\sigma(x)$ define the nebular model. The maximum nebular extent in the initial state is $x_{max}$, where

$$M_N = \int_0^{x_{max}} 2\pi x\sigma(x)\,dx. \tag{34}$$

The pressure from the wind is taken to be

$$P_W(R) = \frac{\dot{M}_W v_W}{4\pi R^2} \tag{35}$$

for stellar mass loss rate $\dot{M}_W$, and for a wind velocity $v_W$ that is constant, as it is to a good approximation in the case of the present day solar wind for $x \lesssim 1$ AU.

The surface of normal pressure balance may now be found from Equation (4) for any set of boundary conditions near the star. We assume arbitrarily that the nebula starts at some $x = x_{min} \ll x_{max}$, and solve for $z$ at $x_{min}$ to be such that $P_W = P_N$. The initial slope of the interface at this point $(x, z)$ is then $-x/z$ from Equation (4). Other values of $z(x)$ along the SNPB may be found by a predictor-corrector method. The slope turns positive within the fractional distance $\sim 10^{-7}$ of $x_{min}$, and we are not likely to be seriously affected by the highly unrealistic conditions at the inner nebular boundary. Model calculations with $x_{min} = 5$, 10, and 20 $R_\odot$ differ in evolutionary time scale by less than 1%. Typical solutions for the SNPB and the SMPB are shown in Figure 1. Figure 1a compares a high-pressure wind with a low-pressure wind, and Figure 1b shows solutions $z(x)$ for nebulae with different masses, as discussed below.

The parameters which specify the wind were chosen to make the energy flux in the wind equal to a fraction of the stellar luminosity, $L$. Three cases studied were

$$\dot{M}_W = (1.0, 0.1, 0.01) \times \frac{L}{0.5\, v_W^2}. \tag{36}$$

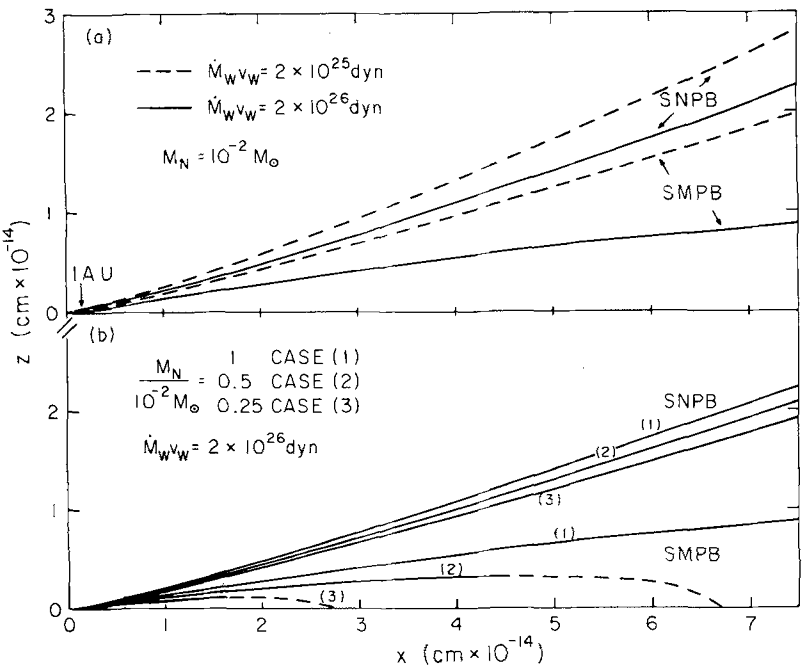


Fig. 1. Wind-nebula interfaces showing height $z$ of the nebulae as a function of radial distance $x$, measured along the plane. The Surfaces of Normal Pressure Balance (SNPB) and the Surfaces of Maximum Pressure Balance (SMPB) are indicated. A surface density distribution of the form $\sigma(x) \propto x^{-1}$ has been used. Figure (a) shows the interfaces for two values of the wind pressure; the solid line corresponds to the case where the energy flux in a 400 km s$^{-1}$ wind equals 1 $L_\odot$; the dashed line has a wind pressure that is one-tenth as large. In both cases, the nebular mass was taken to be $10^{-2} M_\odot$ within a radius of 50 AU (7.5 × $10^{14}$ cm). Figure (b) shows the interfaces for nebulae with different masses in the same maximum radius of 50 AU. The strong value of the wind pressure was used in each case.

The highest value of $\dot{M}_W$ corresponds to the case where the particle luminosity equals the photon luminosity. Apparently, such a high particle luminosity is observed for Lick H$\alpha$120 (Herbig, 1962), for which $L \gg L_\odot$. With $L = L_\odot$ and $v_W = 400 \text{ km s}^{-1}$, the present day wind speed, the value of $\dot{M}_W$ corresponding to this maximum loss rate is $5 \times 10^{18} \text{ gm s}^{-1}$, or $0.079 M_\odot$ in $10^6$ yr.

Although $\dot{M}_W$ and $v_W$ independently specify the wind parameters, only the combination $\dot{M}_W v_W$ that occurs in the expression for $P_W$ affects the structure of the wind-nebula interface and the nebular evolution. If $\dot{M}_W$ has its maximum value as given above, and $v_W = 400 \text{ km s}^{-1}$, then $\dot{M}_W v_W = 2 \times 10^{26}$ dyn. Figure 1a shows two cases, one where $\dot{M}_W v_W$ equals this maximum value, and another where $\dot{M}_W v_W$ is one-tenth as large. We note that $z$ for a given $x$ increases and that $\lambda_t$ decreases with decreasing $\dot{M}_W v_W$.

The distribution of column density in the model nebulae were taken to be either of the form

$$\sigma(x) = \sigma_1 (x/1 \text{ AU})^{-\gamma} \tag{37}$$

or

$$\sigma(x) = \sigma_G \exp\left(-\frac{x^2}{2x_G^2}\right). \tag{38}$$

The case we discuss in detail will have $\gamma = 1, x_{max} = 50$ AU. We shall also consider the three nebular masses

$$M_N = (1.0, 0.5, 0.25) \times 10^{-2} M_\odot . \tag{39}$$

for which $\sigma_1 = 260$, 130, and $65 \text{ gm cm}^{-2}$, respectively, if $\gamma = 1$. Figure 1b shows the SNPB and SMPB for these nebular parameters, and for $\dot{M}_W v_W = 2 \times 10^{26}$ dyn. For the two low-mass nebulae, the wind pressure exceeds the nebular pressure even in the mid-plane for sufficiently large $x$. We draw the portions of the SMPB which have negative slope as dashed lines. In these outer regions where $P_W(x) > P_{N0}(x)$, we expect that the turbulence driven in the upper layers by the wind will penetrate all the way to the mid-plane. However, in what follows, we shall not give any evolutionary time scales or other results for the region where $P_W(x) > P_{N0}(x)$.

The shape of the wind-nebula interfaces for a Gaussian distribution of surface density, or for $\gamma \neq 1$ are not shown here, but they are similar to those in Figure 1. In the case of a Gaussian distribution, the values of $x$ where $P_W(x) > P_{N0}(x)$ is slightly larger than $x_G$, which equals $1.1 \times 10^{14}$ cm for $M_N = 10^{-2} M_\odot$ and $\sigma(1 \text{ AU}) = 260 \text{ gm cm}^{-2}$, because the surface density drops rapidly after this point.

## 6.2. Evolutionary Time Scales

A large uncertainty in the values of $F_{tot}$ and $\partial\sigma/\partial t$ arise because of the unknown value of $\zeta$. For greatest simplicity and practicality of application, we give the derived fluxes and evolutionary time scales in terms of $\zeta$. Figure 2 shows the quantity

$$-\frac{4\pi x F_{tot}}{3\zeta} = 4\pi\left(0.5\,y + x\frac{\partial y}{\partial x}\right), \tag{40}$$

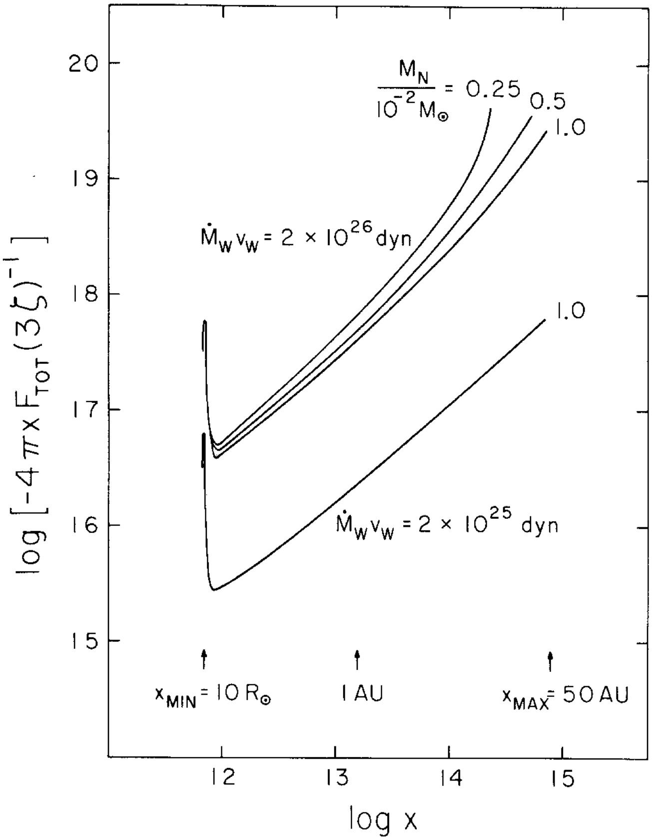


Fig. 2. The total rate of mass motion in gm s$^{-1}$ along both Pressure Balance Layers (at plus and minus $z$) is shown relative to a coefficient of turbulent viscosity, $\zeta$. The radial distance, $x$, measured along the plane, is in cm. A surface density distribution of the form $\sigma(x) \propto x^{-1}$ has been used. The influence of the pressure in the wind or the nebular mass (within 50 AU) is shown. The anomaly at $x_{\min} = 10\,R_\odot$ is an artifact of our simplified boundary condition near the central star.

which is proportional to the total mass moving through a cylinder of radius $x$ per unit time. Figure 3 shows the time scale

$$\tau = \frac{1.5\,\zeta\sigma}{\partial\sigma_\epsilon/\partial t} = 0.5\,\sigma\left(\frac{3}{2x}\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}\right)^{-1}. \tag{41}$$

The factor 0.5 arises because $\partial\sigma_\epsilon/\partial t$ in Equation (41) represents the evolutionary rate in each half of the disk, while the total column density, $\sigma$, has been measured for the whole disk.

We see from Figure 2 that the rate of mass motion is relatively insensitive to $M_N$, but that it depends strongly on $\dot{M}_W v_W$. We also obtain the interesting result that $F_{\text{tot}}$ is approximately constant with $x$ for $\gamma = 1$. This means that $\partial\sigma/\partial t$ is proportional to $x^{-1}$

and relatively independent of $M_N$, and that $\tau$ is almost independent of $x$ for $x \gg x_{min}$ and $P_W(x) \ll P_{N0}(x)$, as shown in Figure 3a. Thus $\sigma$ increases at a rate that is approximately constant with time, and the form of the mass distribution for an initial $\gamma = 1$ power law will be invariant during this evolution. In this simplified analysis, the nebula will evolve by becoming thicker and by shrinking so that $x_{max} \propto (1 + t/\tau)^{-1}$. As in Lynden-Bell and Pringle (1974) the angular momentum in the disk will end up in a small amount of mass at a very large distance from the central star.

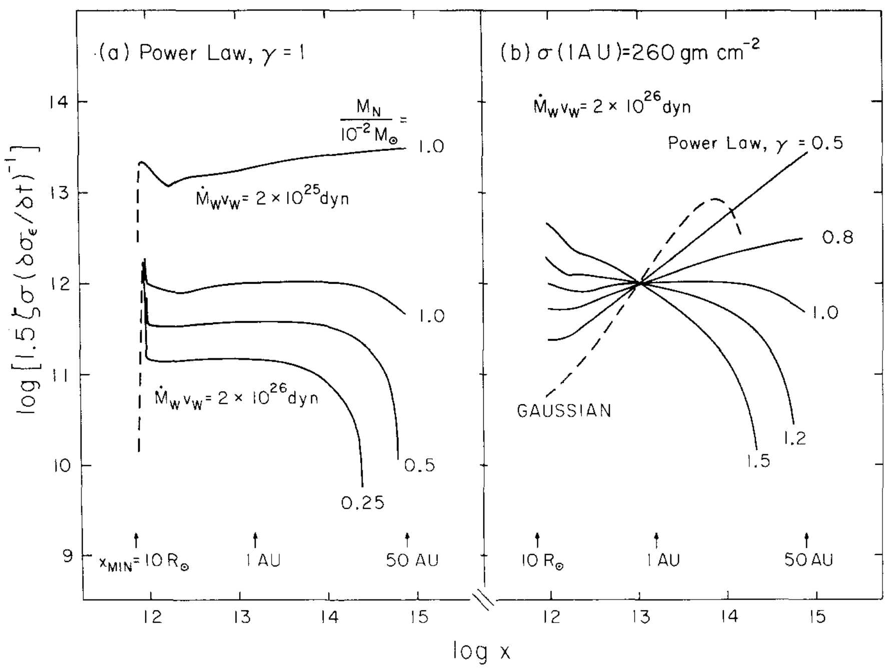


Fig. 3. The evolutionary time scale, in seconds, is shown relative to the coefficient of turbulent viscosity, $\zeta$. The radial distance $x$ along the plane is in cm. Figure (a) compares the time scales for nebulae with different masses and for two different wind pressures. A surface density distribution of the form $\sigma(x) \propto x^{-1}$ has been used, for which the time scale is relatively independent of $x$. Figure (b) compares the time scales for nebulae with different distributions of surface density but with the same value of $\sigma$ at 1 AU and with the same wind pressure. Power laws of the form $\sigma(x) \propto x^{-\gamma}$ were used with the indicated values of $\gamma$, as was a Gaussian distribution with a standard deviation of $1.1 \times 10^{14}$ cm, thereby giving the nebula a mass of $10^{-2} M_\odot$. From this we determine that the evolutionary rate, $\partial\sigma_\epsilon/\partial t$, is proportional to $x^{-1}$, as explained in the text. The anomaly at $x_{min} = 10 R_\odot$ is an artifact of our simplified boundary condition near the star; it has been suppressed in Figure (b).

Figure 3b shows $\tau(x)$ for cases where $\gamma \neq 1$ and for a Gaussian distribution. We find that $\tau(x)$ is approximately proportional to $x^{\gamma-1}$, so that

$$\frac{\partial\sigma}{\partial t} \mathrel{\widetilde{\propto}} x^{-1} \tag{42}$$

for a wide range of $\gamma$ in a power law distribution and for the approximation $\gamma \cong 0$ for $x \ll x_G$ in the case of a Gaussian. This may be understood from simple arguments since (a) $\sigma_\epsilon$ is roughly independent of $x$ from Equation (13) with $z/x$ constant as in Figure 1, and (b) the ratio $\sigma_\epsilon/\sigma$ is given by

$$\begin{aligned} \frac{2\sigma_\epsilon}{\sigma} &= 1 - \frac{\text{erf}\,|az\,(\text{SMPB})\,|}{\text{erf}\,|az\,(\text{SNPB})\,|} \cong \lambda_t\, a \exp\left(-a^2 z^2\right) \\ &\cong \lambda_t\, P_\text{W} \sin\alpha\, \pi^{1/2}\, \text{erf}\,(az)\, c_0^{-2}\, \sigma^{-1} \end{aligned} \tag{43}$$

for $a = v_c(2^{1/2}xc_0)^{-1}$, where we have used a Taylor series expansion of the error function and Equations (31) and (1). Typically $az \gtrsim 1$ so erf $\cong 1$ and $\lambda_t$ scales with $c_0^2$ $(P_\text{W} \sin\alpha)^{-1}$; then $y \propto x^{5/4}$ $(\sin\alpha)^{-1}$. We find that $\sin\alpha$ varies roughly as $x^{0.2}$ near 1 AU, so $y$ is approximately proportional to the first power of $x$. Equation (42) follows immediately from Equation (26).

It is convenient to parameterize $\tau$ in terms of powers of the independent variables $\dot{M}_\text{W}$, $v_\text{W}$, $M_\text{N}$, $L$, $x_\text{min}$ and $x_\text{max}$. The lack of significant dependence of the results on $x_\text{min}$ has already been mentioned, as well as the fact that $\dot{M}_\text{W}$ and $v_\text{W}$ enter only as the product $\dot{M}_\text{W} v_\text{W}$. The extent of the nebula does not influence the results at $x \ll x_\text{max}$, but the value of $\sigma(x)$ does, so a dependence of $\tau$ on $M_\text{N}/x_\text{max}$ arises.

We have calculated $\tau$ at 1 AU for a $\gamma = 1$ power law, for the three mass loss rates given in Equation (36), for the three nebular masses in Equation (39), for $v_\text{W} = 400$ and $100\,\text{km}\,\text{s}^{-1}$, and for $L = L_\odot$. To a good approximation, the evolutionary time scale at 1 AU may be written in all cases as

$$\begin{aligned} \frac{0.5\sigma}{\partial\sigma_\epsilon/\partial t} &= \frac{\tau(1\,\text{AU})}{3\zeta} = \\ &= \frac{1.0 \times 10^{12}\,\text{s}}{3\zeta} \times \left(\frac{M_\text{N}/10^{-2}\,M_\odot}{x_\text{max}/50\,\text{AU}}\right)^{1.37 \pm 0.21} \left(\frac{\dot{M}_\text{W} v_\text{W}}{2 \times 10^{26}\,\text{dyn}}\right)^{-1.33 \pm 0.15} \end{aligned} \tag{44}$$

We have also calculated values of $\tau(1\,\text{AU})/3\zeta$ for two stellar luminosities, $L_\odot$ and $10L_\odot$, with all other parameters the same. This allows for different nebular temperatures. We find that although $v_t$ scales with $L^{1/8}$, $\sin\alpha$ scales with a similar power of $L$, and $\tau$ is nearly independent of $L$.

The dependencies of $\tau$ on $M_\text{N}/x_\text{max}$ and $\dot{M}_\text{W} v_\text{W}$ follow from its variation with $\sigma x^2$ $(\lambda_t \sigma_\epsilon v_t)^{-1}$ from Equations (24) and (26). From Equations (34) and (13) we have that at 1 AU, $\sigma \propto M_\text{N}/x_\text{max}$ and $\sigma_\epsilon \propto \dot{M}_\text{W} v_\text{W}$, respectively, while the results of our calculations give a $\lambda_t$ proportional to $(M_\text{N}/x_\text{max})^{-0.4}(\dot{M}_\text{W} v_\text{W})^{0.3}$ at 1 AU.

We may also derive the total mass given off by the wind in a nebular disruption time. We obtain the useful ratio

$$\frac{\dot{M}_\text{W}\tau}{3\,\zeta M_\text{N}} \cong (12\,\zeta)^{-1} \left(\frac{v_\text{W}}{400\,\text{km}\,\text{s}^{-1}}\right)^{-1} \left(\frac{x_\text{max}}{50\,\text{AU}}\right)^{-1} \left(\frac{\tau}{10^{12}\,s}\right)^{0.25} \tag{45}$$

for $\tau$ at 1 AU evaluated from Equation (44). We see that for typical wind and nebular parameters and for $\zeta \sim 0.1$, the wind must radiate an amount of gas (into $4\pi$ steradians, as assumed) that is roughly equal to the mass in the nebula. Since the nebula subtends a solid angle equal to $4\pi \sin\theta$ (cf. Equation (3)), the total mass of the wind absorbed by the nebula in a disruption time will be $\sin\theta \, \dot{M}_W \tau$. From Figure 1, $\sin\theta \cong 0.2$ so the nebula will absorb more mass than its loses if

$$\zeta \gtrsim \frac{\dot{M}_W \tau}{15 M_N}. \tag{46}$$

If the wind is very weak or if the magnitude of the turbulent viscosity is small, then the nebula could evolve so slowly that it picks up more mass from the wind than it loses via inward drifts. For the wind and nebular parameters used here, e.g. $M_N \sim 10^{-2} M_\odot$, $x_{max} \sim 50$ AU, $v_W \sim 400 \text{ km s}^{-1}$ and $\dot{M}_W \sim 5 \times 10^{18} \text{ gm s}^{-1}$, $\zeta$ would have to be less than 1/60 for this to occur. For one-tenth the mass loss rate, $\zeta$ would have to be less than $\sim 1/30$.

### 6.3. Feasibility of Nebular Disruption

The insensitivity of $\dot{M}_W \tau (3\zeta M_N)^{-1}$ to $\tau$ in Equation (45) allows one to readily estimate the feasibility of a wind's disruption of a surrounding disk nebula in a case where the wind's pressure varies with time. For typical $v_W$ and $\zeta \sim 0.1$, the wind must give off a total amount of mass that is comparable to the mass in the nebula. For example, rapid mass loss during a short-lived T-Tauri phase will disperse a surrounding disk nebula to the same extent as will a weaker wind that sheds the same amount of gas over a proportionally longer time. Of course, this is subject to the constraint of inequality (46). It is reasonable to expect that a strong stellar wind that sheds some $0.1 M_\odot$ will contribute to the disruption of a comparable amount of mass in a disk nebula. Whether the wind alone leads to the final disruption of the nebula depends primarily on the total mass lost from the star after the time when other processes creating turbulence in the disk subside.

## 7. Speculations on the Origin of T-Tauri Winds

If inequality (46) is not satisfied, then the nebular mass will decrease with time as the nebula accretes onto the central star. In that case there arises the interesting possibility that a small fraction of the accreting nebular mass will pick up energy near the star and get blown out with the wind. One of the important results of the present work is that the accretion rate of the nebula onto the star, i.e., $\sim M_N/\tau$, is proportional to the wind's mass loss rate, varying approximately as $\dot{M}_W^{1.3}$ if $v_W$ is fixed. This dependence could be one-half of a system with positive feedback. If a small fraction of the accreting mass were continuously channeled back into the wind, then the accretion rate and the mass loss rate would iteratively increase to the point of saturation, when the maximum possible energy or momentum flux in the wind is attained. Thus the final destruction of a disk nebula and the creation of a very strong wind may be intimately related.

It is also evident that a young star could appear to be losing a large amount of mass in the form of a wind, whereas the material could actually be coming from a surrounding disk nebula. This alternative to extreme mass loss from the star itself could have important implications for pre-Main-Sequence evolution. The stellar mass could even increase during the disk accretion-wind phase.

## 8. Final Stages of Evolution

In the evolutionary process just described, much of the nebula will be accreted onto the central star, a small amount of mass will carry the disk's angular momentum out to large distances, and a small amount may leave the system altogether if it joins the wind. However, as soon as $P_W(x) > P_{N0}(x)$ for all $x$, then the interaction between the wind and the nebula will begin to differ from what we have described. Wind-driven shocks will occur throughout most of the nebula with a mixing length equal to either $\lambda_{max}$ or $v_s x/v_c$ as discussed in Section 3.1. The nebula will be completely turbulent and it will continue to evolve as described here (but with $z$(SMPB) $= 0$) until the mixing length becomes comparable to the nebular radius. Then the wind can push out the remaining gas all at once. Presumably, this will be the final stage in nebular disruption. From Equations (32) and (35), we see that $P_W(1\ \text{AU}) > P_{N0}(1\ \text{AU})$ if $\sigma(1\ \text{AU}) < 10\ \text{gm cm}^{-2}$ or if $M_N < 10^{-3}\ M_\odot$, assuming $\dot{M}_W v_W = 2 \times 10^{26}$ dyn. Furthermore, the mixing length will be the same size as the nebula if $v_s = v_c$, or if $\sigma(1\ \text{AU}) \sim 10^{-2}$ or $M_N \sim 10^{-6}\ M_\odot$.

## 9. Summary

The disruption of a disk nebula by a stellar wind occurs as a result of wind-driven turbulence in the top layers of the disk. The outward motion that is directly induced by pressure from the wind will be exactly compensated by the wind's contribution to an inward drift of the top layer. Turbulent viscosity also contributes to the drift of the top layer, but this contribution leads to a permanent change in the nebular surface density. The net result is an inward drift of most of the original mass in the nebula.

We have analyzed the structure and stability of the wind-nebula interface, the strength and direction of all mass motions, and the time scales for nebular evolution. An approximation for this time scale, showing its dependence on the nebular mass, the pressure in the wind and a coefficient, $\zeta$, for turbulent viscosity, is given by Equation (44). The results showed that the total amount of mass given off by a strong wind in the nebular disruption time will be comparable to the mass in the nebula. This simple relationship (Equation [45]) allows one to readily estimate whether a wind can disrupt a surrounding disk nebula.

We have also discussed the interesting possibility that some of the influx of nebular material (resulting from the viscosity-induced drift) may absorb energy near the star and turn around to join the wind. If this is possible, then there will be two important consequences: The rate of nebular accretion onto the star and the mass loss rate in the wind will be mutually dependent, creating a system with positive feedback. The particle fluxes may then undergo a runaway increase until the limit of energy or momentum flux in the wind is attained. Thus the destruction of a disk nebula and the creation of a strong wind may be related, and the energy flux in the wind may have a natural tendency to become saturated at a value controlled by the stellar luminosity.

The second consequence follows immediately, namely that the mass lost from a young stellar system during a T-Tauri phase may originate in a disk nebula and not in the star itself. Thus the total amount of material given off as a radially directed wind could be quite large, without seriously affecting the internal structure of the star.

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