THE MISSING PLANET*

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Abstract. Ovenden's hypothesis suggesting former existence of a planet of 90 Earth masses which supposedly filled the Titius–Bode gap in the asteroid belt and then suddenly disappeared 16 million years ago, is critically examined by the morphological method. It is shown that an explosive removal, however improbable, could have led to the formation of the asteroids from a non-explosive core (the nuclear charge being placed outside of it), but that life on Earth would have been completely destroyed by three successive blasts – one from the direct impact of the ejecta of the planet, another from the increased radiation suddenly emitted by the Sun when hit by the ejecta, and a third one (arriving, however, first) from the radiation emitted by the nuclear explosion. The geological record of the continuity of Life on Earth for the past 10^9 years definitely excludes the possibility of such an explosion in the late Tertiary.

The other mode of removal of the planet – in a gravitational encounter with an intruder either from interstellar space or from the unexplored outskirts of the solar system, under the condition of not having disturbed the existing regularity of planetary orbits – is not only extremely improbable, to be expected once during 100 million times the age of the solar system; but it would leave no asteroids behind, all of the previously existing primaeval asteroids having been rapidly eliminated in encounters with the hypothetical planet.

Whatever the merits of Ovenden's long-range calculations of the secular perturbations of coplanar 'circularized' planetary orbits, the hypothesis of a massive planet to have existed in the asteroidal region and then recently to have suddenly disappeared, belongs to the realm of the impossible. After such a hypothetical event, either we would not be here on Earth, or there would be no asteroids in their present place between Jupiter and Mars.

1. The Gap

The well-known Titius–Bode formula,

$$a = 0.4 + 0.3 \times 2^n, \tag{1.1}$$

approximates the mean heliocentric distances of the planets in astronomical units when n is a succession of integers (except the first), as shown in Table I. The agreement of the distances with the formula is excellent except for Neptune. This gives justification for the formula to be raised to the rank of a 'Law' – Bode's law of planetary distances which, in principle, applies also to the satellite systems and thus appears to be of universal validity. However, in the last column of Table I, while each of the entries except one refers to a planet of respectable size, at $n = 3$ there is no single planet while the total mass of the asteroids, quoted as 0.001 m_e (Earth masses), is so insignificant that the coincidence with the formula looks rather artificial

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TABLE I

Planetary distances (AU) and masses (Earth units)

n	a (Equation (1.1))	Planet	a (actual)	Mass
$-\infty$	0.4	Mercury	0.387	0.056
0	0.7	Venus	0.723	0.82
1	1.0	Earth	1.000	1.00
2	1.6	Mars	1.524	0.108
3	2.8	Asteroids	(2.8)	0.001
4	5.2	Jupiter	5.20	318
5	10.0	Saturn	9.54	95
6	19.6	Uranus	19.2	14.6
7	38.8	Neptune	30.1	17.3

and makes the identification at $n = 3$ doubtful, pointing instead to a virtual gap at this place.

While the widespread application to different systems of formulae similar to (1.1) has suggested that a universal cosmogonic law – Bode's law – determining hetegonic processes (in Alfvén's terminology) or the mode of formation of satellite systems is at its basis, opinions as to the nature of the 'law' have been divergent. Perhaps it reflects the spacing of the primordial rings as depending on turbulent friction in the flattened gaseous rotating nebular aggregate, e.g. the solar nebula as visualized by von Weizsäcker (1952). A sceptical note by Lecar (1973) suggests that the 'law' simply expresses the fact that neighbouring planets cannot be too close to each other without interfering with their existence, and that a sequence of random numbers in which the dangerously close distances are eliminated could lead to a similar orderly spacing. This, however, may be an oversceptical viewpoint. If mutual gravitational interference were the explanation, the massive giant planets should closely obey the 'law', while the small members could be spaced more at random. The figures of Table I convey, if anything, the opposite: the only important exception to Bode's spacing is the one for Neptune, a giant planet, while the small planets 'toe the line'. Mass seems to be unimportant for Bode's law, as it would be for von Weizsäcker's turbulent subdivisions in the solar nebula, and the asteroidal gap would not necessarily call for a specific explanation.

2. Ovenden's Hypothesis

Despite this, and with Bode's law in view, an interesting proposition by Ovenden (1972) purports to explain the asteroidal gap by placing there from the beginning a giant planet of 90 m_e (Earth masses) which suddenly vanished 16 million years ago. Ovenden's figures are linked to a streamlined model of secular perturbations of the mean heliocentric distances (a) of the planets, calculated numerically by integrations of 'circularized' orbits or by assuming zero eccentricities and inclinations. This artificial assumption enables the calculation to be extended over the entire age of the solar system – a task otherwise impossible even with modern calculating machines. On this model, the variation with time of the mean distances is found to be non-uniform. For a given set of initial conditions, there occur epochs when the a-values of all the planets vary slowly, so that a high probability exists that the system is observed at these epochs of 'Least Interaction Action' (Ovenden, 1975) or epochs of minimum change. Ours is not such an epoch; for the present initial condition this would arrive about 300 million years from now, when Neptune would settle at 37.3 AU from the Sun, in better agreement with Bode's law (see Table I), while Mars (at 1.24 AU) and Mercury (at 0.46 AU) would disagree. In view of this, Ovenden's contention that his principle of Least Interaction could account for the Titius–Bode regular spacing of planetary orbits must be subject to doubt, even when his approximate method of treating the divergent long-time many-body problem of secular perturbations is not objected to. The method is probably fit to describe qualitatively the trends of change, but fails in detail over long intervals of time as also do the more exact methods on a long run. This is well illustrated by Lecar (1968), when eleven numerical integrations of a 25-body problem made from identical initial conditions on electronic computers of different astronomical institutions, led ultimately to differences of up to 100% in the computed parameters.

Ovenden's work is concerned with the existence of the gap in the asteroidal belt. However, he quotes a total mass of 0.1 m_e for the Belt, about equal to that of Mars. The question may arise: 'Why is this a "gap" while the locations of Mars or Mercury are not identified as gaps?' However, the value of 0.1 m_e based on problematic analyses of perturbations (Leverrier, 0.1; Harzer, 0.17; Osten, 0.08 m_e; see Öpik, 1953) is grossly exaggerated, illustrating also the unreliability of the method of perturbations when the perturbing body cannot be definitely located. As shown in Section 3, a value smaller by a factor of 100 is definitely a close estimate of the total amount of matter in the asteroidal belt, and this is the value entered in Table I. It makes the belt look like a real gap, also justifying Ovenden's search for an explanation.

As mentioned, the present configuration (distribution of heliocentric mean distances) of the planets is not one of Ovenden's minimum action states (characterized by the sum of mutual potential energies of the planets) and change is relatively rapid. He finds that, working backwards, the state would become a minimum when a mass of 90 m_e is inserted 16 million years ago into the asteroid belt at $a = 2.794$ AU. This 'Planet A', missing at present but having stayed there from the very beginning, could have stabilized the system for all the 4500 million years since its inception and, from the criterion of Bode's law, would have restored the 'respectability' of the asteroidal region. However, grave physical problems are raised by the proposed sudden disappearance of a planet of this size in so recent time, as also recognized by

Ovenden himself. In critical reviews, Napier and Dodd (1973, 1974) conclude that '. . . Ovenden's theory . . . is physically improbable: one must look elsewhere for an explanation of Bode's law and the existence of the asteroids'. In the following it is shown, by additionally taking into consideration biological and paleontological facts, that not only is the hypothesis improbable, but that the event of sudden removal of a massive planet must not only be of extreme improbability among the planetary systems in our Universe but that it never could have occurred in our solar system except in the very beginning.

Contrasting with this morphologically derived conclusion, Ovenden's hypothesis of a catastrophical removal of a planet has purportedly found support in the crowding of the aphelion directions of 60 long-period comets around the same common point on the 'heliocentric celestial sphere' (van Flandern, 1976), implying that these comets originated in the explosion of Planet A. The argument is unphysical for two reasons: ejecta of a totally exploding planet would spread out in all directions, not just in one; and the force of the explosion would inevitably pulverize or atomize the material, even when it would all be as hard as nickel-iron. Indeed, the hydrostatic pressure of the explosion, of the order of $\frac{1}{2}\varrho v^2$ (ϱ = density, v = velocity), would amount to a multiple of 10^{13} dyne cm^{-2}, about 18 million atmospheres at least (when $\varrho = 1$ g cm^{-3}, $v = 60$ km s^{-1}) or 600 times the crushing strength of meteoritic iron. From the tidal disruption of Sun-grazing comets, the crushing strength of cometary nuclei has been found equal to from 2×10^4 to 2×10^6 dyne cm^{-2}; 10^9 to 10^7 times less than the estimated pressure in the explosion. How could such brittle objects survive in the super-explosion? Besides, the same argument applies to some suggestions about comets being ejected from Jupiter and other giant planets by volcanic explosions (cf. Vsekhsvyatsky, 1977 and earlier). Van Flandern's argument is, therefore, irrelevant to the problem of the missing planet, while the crowding of cometary aphelia can be easily explained by fission and break-up of the nuclei, an observed but only partly explained fact (in the case of tidal break-up) (Štefánik, 1966; Whipple and Štefánik, 1966; Öpik, 1966). Fission 'could be caused by rotational instability and increased pressure of the imprisoned gases, especially hydrogen' (Öpik, 1971).

3. The Total Mass in the Asteroidal Belt

In the still continuing quest of a history of the solar system – its origin and evolution – one of the conditions for a credible solution is that it should cover all the members. Peculiar cases which may be difficult to include in a general picture must not be left out, but must be morphologically included, so that the ultimate model, adapted to *all* cases without exception, will acquire a high probability of reality as corresponding to a coincidence of all the essential facts or criteria. At present we are far from having a unified cosmogonical description of theory. Although there may be some agreement about the general outlines, controversial propositions exist, especially in the quantitative aspects or because of one-sided non-morphological approach and neglect of peculiar cases or facts which do not suit the theory. If not now, we may hope that at some time a unified history of the solar system, however approximate and sketchy, will emerge. At present, work is going on more on the details which are not yet complete enough to be put together in an unquestionable manner.

The asteroidal belt, as one of the outstanding peculiar cases, has rightfully attracted special attention of cosmogonic theories. The literature is very large, and only a few examples could suffice. Thus, Alfvén (1964) considers the asteroids as a critical case for models of the formation of celestial bodies, and finds support for a process of condensation, although the majority of opinion would prefer collisional break-up in an encounter or encounters of two or more primitive larger bodies in the beginning of the solar system. Except for Ovenden, until now there has been agreement in one point: that the asteroidal belt is very old, of an age comparable to or equal to the entire age of the solar system. The literature dealing with the structure, origin, orbital and collisional evolution of the asteroids is too extensive to be quoted here, but statistics from another publication (*Irish Astron. J.* **13**, 22 (1977), in press) for a still incomplete list of references for the period after 1960 would suffice; the names of the authors are followed by the number of publications as quoted (in brackets) in the list: Alfvén (2); Anders (3); Arrhenius and Alfvén (1); C. R. Chapman (3); Dermott and Lenham (1); Dohnanyi (4); Gehrels (2); Hanner *et al.* (1); Hartmann (4); J. G. Hills (1); Kaula (2); Kotsakis (1); Kresak (1); G. P. Kuiper (1); Lecar and Franklin (1); Marsden (1); Morrison (1); Napier and Dodd (2); Öpik (5); Ovenden (3); Schubart (1); Sobermann *et al.* (1); Weidenschilling (1); Wetherill (6); and J. G. Williams (1). The list conveys an idea of the enormous extent of research dedicated to the study of the cosmic role of the asteroids.

In this section we are concerned chiefly with the total mass present in the asteroidal belt. Instead of the uncertain method of perturbations when applied to marginal effects of unknown origin (see Section 2), direct statistics of photometric data can give a definite realistic answer, without much uncertainty involved. We note that the decision must be made on estimates differing by two orders of magnitude (0.1 m_e versus 0.001 m_e), and that therefore approximate calculations are sufficient, irrespective of such niceties as the *exact* albedo of the asteroids or the precise scattering power of a gas cloud.

From a statistical study by Putilin (1952), as reviewed by Öpik (1953), it turns out that the total mass of observable asteroids or those exceeding 1 km in diameter is close to one-thousandth of the Earth's mass, or 0.001 m_e as quoted in Table I. Allen (1973, p. 151) gives even a lower figure of 0.0004 m_e, as based on statistics by Kiang. With the well-established frequency law of asteroid diameters, most of the asteroidal mass must be locked in the larger objects – Ceres alone amounting to 0.0002 m_e – and very little can be expected from the unobserved range below 1 km down to the Poynting–Robertson limit of elimination of 10 cm or less (depending on the assumed age). Besides, if quite unnaturally it were assumed that a hidden mass of 0.1 m_e, equal to that of Mars, could exist below the 1 km diameter limit, and that all this mass is concentrated at the optically least efficient upper limit of 1.3 km (reflecting area or total brightness being inversely proportional to particle diameter for a given total mass), the total power reflected from this complex would equal 1500 times that reflected from the surface of Mars (6000 × $\frac{1}{4}$, for one-quarter illumination as appropriate to its greater distance from the Sun). It would equal about five times the integrated power of starlight, concentrated in the asteroidal belt and appearing as another Milky Way. Smaller particles for the same total mass would reflect still more light, but even the unnatural assumption of the unobserved mass being concentrated just below the observational limit does not work. Certainly, the hypothetical mass cannot be hidden in solid bodies, asteroids or dust.

The other possibility, of the hidden mass being in the form of gas, is even less favourable. If distributed between 2.2 and 3.2 AU (4.5×10^{41} cm^3), with a moderate concentration toward the median plane, a mass of 0.1 m_e, or 6×10^{26} g, would correspond to an average density *at present* of 1.3×10^{-15} g cm^{-3}. An asteroid of average eccentricity and inclination, moving with a velocity of 4 km s^{-1} relative to the rotating gas, would have swept about 7×10^7 g of the gas in 4.5×10^9 years per cm^2 of cross-section, thus accreting to a minimum diameter of 350 km and, besides, losing completely its peculiar motion. This, of course, contradicts the facts. In 16 million years the accretion would result in a minimum diameter of 1.25 km, thus not so much in contradiction with the factual data, however improbable this would appear. However, the photometric criterion also destroys this possibility. If the scattering power of the gas is assumed equal to that of the pure terrestrial atmosphere (hydrogen would be much more efficient), the mass of 0.1 m_e spread uniformly over a sphere of 2.7 AU radius would produce a surface load of 0.04 g cm^{-2}, or 1/25 000 of the terrestrial atmosphere. With the known scattering power of the latter (effective albedo 0.10), and one-half of the light scattered inwards from the belt, the total scattered power would be equivalent to a star of -10.3 mag or 20 times the combined starlight. It would be concentrated toward the ecliptic and especially in the direction of 90° from the Sun where its optical thickness would be greatest, of the order of 5×10^4 10th-mag. stars per square degree (zodiacal light equals about 100 in these units). When observed from inside the Belt the surface brightness in 90° from the Sun should be even greater, yet Pioneer 10 recorded less than 10 units in this case (Hanner *et al.*, 1974), 5000 times less than expected. Certainly, there is no hidden mass of any substantial amount in the asteroidal belt. Its mass is close to 0.001 m_e and the 100 times larger value suggested from unidentified perturbations is not real. On the other hand, the smallness of the total mass gives some justification to calling the locus at 2.8 AU a real gap from the standpoint of Bode's law.

4. Explosive Removal

A sphere of 90 m_e, at a low density of 1 g cm^{-3} as for a giant planet, would possess an average gravitational energy of 4.8×10^{12} erg g^{-1}, corresponding to an *average* escape velocity of the fragments of 31 km s^{-1} in break-up. With a velocity of 18 km s^{-1} in its nearly circular orbit, and an escape velocity from the solar system of 24.4 km s^{-1}, complete removal of the fragments would require a minimum velocity in the retrograde direction of $18 + 24.4 = 42.4$ km s^{-1}, after overcoming the gravitational potential of the planet (we call it 'A') itself. Yet there is no retrograde orbiting material left in the belt and, considering that some dispersion in the velocities of ejection must have existed, a minimum average velocity after break-up of the escaped fragments (or gaseous material) when out in free space can be set equal to 60 km s^{-1} relative to the former center of mass of A. In the inertial frame of the Sun, this would correspond to an average forward ejection velocity of $60 + 18 = 78$ km s^{-1}, and to a retrograde velocity of $60 - 18 = 42$ km s^{-1}, with values in between in other directions. After overcoming also the gravitational potential of A, the minimum energy input would thus amount to about 150 times that per unit mass released in the explosion of an optimum mixture of hydrogen and oxygen. Chemical explosives are therefore entirely out of question, and only a nuclear explosion would be adequate. Leaving aside the near-impossibility of a nuclear explosion occurring in so small a body after 4500 million years of its settled existence, let us consider the other aspects or consequences of such an event.

Through gravitational encounters (Öpik, 1976 and elsewhere) any primordial asteroids in the belt would have been eliminated by a planet A of 90 m_e on a time-scale of less than one million years. Therefore, if we believe in an explosive removal of A, new asteroids must have been created and placed in the belt. With a centrally located explosive, the debris (dust and gas) ejected in all directions from the orbiting planet would have the best chances to remain in the solar system when ejected backwards. Bodies condensed from these ejecta could have formed, predominantly, asteroids moving in retrograde orbits, of which those of small eccentricity would have survived the danger of encounters with Jupiter. Such retrograde asteroids are definitely absent, so that a central nuclear charge cannot be postulated.

However, if a solid non-explosive central core in A of about 0.001 m_e is assumed, the nuclear explosive being placed concentrically adjacent and outside the core – either as a shell, or throughout the bulk of A – the explosion, while ejecting and pulverizing the outward main mass of the planet, would compress the core, causing at first pressure ionization. When relieved of the pressure, the core would rebound, breaking up into solid fragments ejected in all directions at the expense of the energy of recombination. On a model of atomic collisions, verified by experiment and adapted to the theory of meteor phenomena, the potential energy of atomic interaction versus compressibility of a solid can be derived as depending on the radii and energies of the Bohr orbitals (Öpik, 1933; also Bates, 1972).

On this basis it can be calculated that a shock pressure of 18 million atmospheres (as estimated in Section 2) applied to solid iron would cause compression to a density of 33 g cm^{-3}. At a compression energy of 25 eV intermediate between second and third ionization of iron, the kinetic energy of subsequent expansion (4.3×10^{11}

erg g^{-1}) would correspond to a velocity of 9.3 km s^{-1}, about one-half of the circular orbital velocity, so that no retrograde orbits would have been created. Subsequent encounters with Jupiter would eliminate all objects with aphelia exceeding 4.6 AU (near Jupiter's perihelion) whatever their inclinations (up to 25–30°) as conditioned by the secular motions of node and apsides (Öpik, 1976 and elsewhere). Inward-bound Apollo type objects, of an encounter lifetime of the order of 100 million years, would mostly survive for 16 million years, so that a ready-made asteroid belt could indeed be produced in the explosion. Unfortunately, from what follows it appears that after such an explosion *we* could not be here to discuss its consequences.

The pulverized or atomized material of the explosion would travel outwards in all directions, initially as a spherical wave centered on its orbiting origin. When reaching the Earth at, say, a distance of 3 AU from the origin, the mass load of the wave would equal 23.6 g cm^{-2}, to be deposited per cm^2 of the Earth's cross-section. Neglecting the condensing and accelerating action of the solar and terrestrial fields, at an impact velocity of 60 km s^{-1} and at one-half of the impact heat being lost to space, the heat input of this super-meteoric display would equal 2.12×10^{14} erg cm^{-2}. The input of solar radiation – 40% being reflected to space – equals 8.4×10^5 erg s^{-1} per cm^2 of the cross-section. The blast thus would suddenly deliver an amount of heat equal to that of 8 years of sunshine; this could boil a water layer of 21 m all round the globe, creating a mass of steam twice the mass of our atmosphere. The duration of the blast would be from 80 s (time of covering the radius of A at 60 km s^{-1}) up to 10 days (at a velocity dispersion of 10% over the entire path), the material expanding into space as a gigantic thinning-out bubble. Even if only one hemisphere were hit by a blast of short duration, the wave of hot steam would rush around the globe, reaching the antipodal point in a few hours. Life could not have survived such a blast, except perhaps in the abyssal depths of the oceans. The continuity of the biological–geological record shows that – not to mention such a recent epoch as the late Tertiary 16 million years ago – no such catastrophe has ever befallen the Earth since almost its very beginning.

An explosive removal of A, especially in so recent times, is not only improbable: it is impossible.

This conclusion is strengthened (if it requires strengthening) by considering that the direct blast of the expanding shell is not the only one. The ejecta when reaching the Sun would cause a devastating radiative blast from the solar surface. The kinetic energy per gram of the accelerated infalling material, at about 600 km s^{-1} or at ten times the original velocity, would equal 100 times that received at the Earth's orbit; and conservation of angular momentum in the solar gravitational field would require a tenfold reduction in the impact parameter or a hundredfold concentration of the infalling material jet. Against this is to be set the dilution of solar radiation reaching the Earth according to the inverse-square law. Thus, as compared with the direct material blast, the Earth would receive a radiative blast of relative intensity $100 \times 100/40\,000$ or one-quarter. This would boil an equivalent layer of water equal to one-half the mass of the atmosphere: a lethal catastrophe in its own right. In addition, the initial radiation from the nuclear explosion, of an energy comparable to the material blast, would reach the Earth in advance, as soon as the expanding and *superhot* shell had thinned out enough to transmit the trapped radiation. There is thus a triple guarantee that no life could have survived on the surface of our planet after the hypothetical explosive removal of A.

5. Removal by a Gravitational Encounter

As an alternative, a 'painless' removal of Planet A could have taken place through the gravitational action of a foreign body passing close to it. It can be shown that, with the known frequency function of stellar masses in our galactic surroundings, action of a definite strength by large masses (encounter cross-section proportional to m^2) is more probable than that by small masses despite the greater number of the latter. This follows from the limitation of the frequency function of stellar masses, $m^{-n}\,\mathrm{d}m$, requiring $n < 1$ (otherwise the sum total of small masses would tend to infinity). The 'frequency of action', $m^2 \cdot m^{-n}\,\mathrm{d}n = m^{2-n}\,\mathrm{d}m$, with $n < 1$, is thus an increasing function of m and the most probable effect would correspond to the largest admissible mass.

This has a ceiling in that the intruder which ejected A must not have disarranged the regularity of the solar system, or must have spared the small eccentricities and inclinations of *all* the planets as they are now. While passing near A, it would disturb the Sun without much affecting distant planets whose elements are therefore practically changed according to the velocity vector Δv acquired by the Sun alone as the result of the passage. For a distant planet such as Neptune, not appreciably perturbed by the passage, the outcome would be close to imparting a vector $-\Delta v$ in the opposite direction relative to the Sun. An upper limit to the impulse can be set as $|\Delta v| = 1\ \mathrm{km\,s^{-1}}$ which could change Neptune's eccentricity by 0.2 or its inclination by 12°, or generally, with the present near-zero values, could put Neptune's orbit 'out of tune' into a non-conforming state limited by the condition

$$e^2 + \sin^2 i = 0.04. \qquad (5.1)$$

A close estimate of Δv in a distant passage from the Sun along an only moderately curved hyperbolic path is that corresponding to a rectilinear passage:

$$\Delta v = 2Gm/aW \qquad (5.2)$$

where m is the mass, W the velocity and a the closest distance from the Sun of the passing object; a is to be identified with the heliocentric distance of A, and G is the gravitational constant. With $a = 2.8$ AU $= 4.5 \times 10^{13}$ cm, $\Delta v = 10^5\ \mathrm{cm\,s^{-1}}$, $W = 3 \times 10^6\ \mathrm{cm\,s^{-1}}$, the upper limit of the mass of the intruder that could not perturb the regularity of the solar system becomes

$$m < m_0 = 1.0 \times 10^{32}\ \mathrm{g}\ \text{or}\ 0.05\, m_s\ \text{(solar mass)}.$$

Remembering that the encounter efficiency increases with mass, an upper limit to the probability of an efficient encounter is obtained by disregarding the variety of small masses and by assuming that the total interstellar mass of stars smaller than m_0 is made up of masses equal to the upper limit, m_0. Also, to achieve ejection, the gravitational potential of m_0 upon A must exceed the potential of the Sun upon A. This requires the distance of closest approach of m_0 to A, D_0, be limited to

$$D_0 < 2.8 \times 0.05 = 0.14\,\text{AU}. \tag{5.3}$$

Because of gravitational bending, for an interstellar velocity of about $W_0 = 20$ km s^{-1}, the target area for an efficient encounter ending in ejection can be assumed equal to twice πD_0^2 or 0.12(AU)^2, so that the volume swept by the target area at 20 km s^{-1} = 4.19 AU yr^{-1} becomes 0.5 (AU)^3 yr^{-1}. With the known distribution of interstellar masses (Allen, 1973, p. 246), and considering that ejection can be achieved only when the intruder overtakes A from behind (limiting thus the directional angle of approach to $\pm 30°$ or to a solid angle of $4\pi/15$), the *upper limit* for the probability of ejection becomes $1/(4.5 \times 10^{17})$ yr^{-1}, equivalent to 100 million times the age of the solar system – practically tantamount to impossibility.

Furthermore, even should such an almost impossible event have happened, there would be no asteroids, the space having been swept clean in gravitational encounters with A during its long existence. Not even Trojan-type asteroids could have been there in the 60° Lagrangian points: these are only the privilege of Jupiter, the most massive of the planets, while Jupiter's perturbations render unstable would-be Trojans of the smaller planets (Öpik, 1970, p. 294). Also, no new asteroids could have been created in the encounter. The 'painless' removal of Planet A would thus have left the space of the belt empty. Thus it, too, could not have happened.

The same reasoning would apply also to a possible outermost member of the solar system, placed in Oort's sphere of comets and sent inwards by a stellar perturbation. The total mass of the cometary population of Oort's sphere is estimated to be only about 1.5 m_e (Öpik, 1975), and even if an exceptional body of 0.05 solar mass = 15 000 m_e could be there accommodated, unless ejected to space and as a self-luminous star of the 4th apparent magnitude, it could not have escaped detection when having returned to the sphere after the encounter.

The final conclusion is that the hypothesis of the former existence and subsequent removal of a massive planet in the present asteroidal belt cannot be supported with any degree of credibility.

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