ON THE MOMENTS OF INERTIA DIFFERENCES OF THE MOON

JAFAR ARKANI-HAMED
Dept. of Physics, Arya-Mehr University of Technology, Tehran, Iran
and
MEHDI B. KERMANI
Computer Center, Arya-Mehr University of Technology, Tehran, Iran

(Received 4 July, 1977)

Abstract. In this paper it is shown that the differences of the moments of inertia of the Moon are, most likely, due to the surface irregularities, the over-all front side mare fillings and the backside topography.

1. Introduction

The evidence for the different moments of inertia of the Moon has been deduced from lunar physical libration (Koziel, 1967) and it has been confirmed later by the analysis of the lunar orbiter data (Lorell and Sjogren, 1968; Tolson and Gapcynski, 1968; Kaula, 1969; Lorell, 1969; Michael *et al.*, 1969; Liu and Laing, 1971; Michael and Blackshear, 1972; Williams *et al.*, 1975). For the formation and the duration of the moments of inertia differences, the following theories have been proposed:

(1) – Heterogeneous accretion: Urey and MacDonald (1971) related the formation of the different moments of inertia to the lateral variations of density produced through the non-uniform accretion of the Moon. They suggested that the lunar interior has been strong enough to support differences of about 15–20 bars near its center throughout the lunar history. The strong radial dependence of the required lateral density variations led Kopal (1972, 1977) to suggest that the variations are mainly in the upper 200 km. The main difficulty with this theory is the geochemical evidence for a global differentiation of the upper several hundred km of the Moon within 0.1 b.y. of the lunar history (Fouad *et al.*, 1973). This implies the partially molten state of the upper region of the Moon and, thus, the diminishing of the lateral variation of density which might be produced by the accretion process.

(2) – Dynamical process: Jeffreys (1959) postulated that the moments of inertia differences are due to the tidal bulge produced during the capture of the Moon by the Earth. According to the capture theory, the close approach took place about 2.8–3.5 b.y. ago (Alfvén and Arrhenius, 1972; Singer, 1972). If the thermal energy released during the capture was large enough to soften the lunar interior and to provide a suitable condition for the creation of a strong tidal bulge, the condition would remain suitable for diminishing the bulge long after the time of the close approach, because of the very slow thermal processes inside the Moon. Therefore, it is implausible to assume that the present lunar bulge is due to the tidal effect of the capture event. Runcorn (1967) proposed that pos-

The Moon 17 (1977) 167–176.

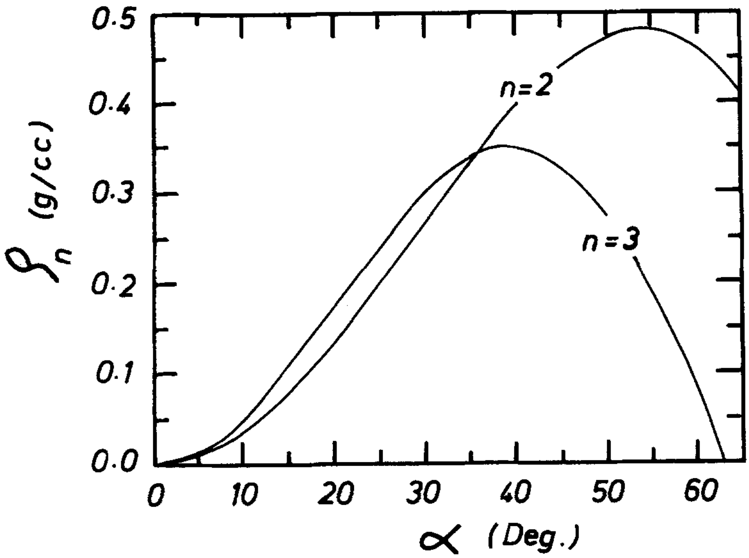

Fig. 1. The second and third harmonic coefficients of the spherical harmonic presentation of the volume density given by Equation (2). $\rho_0 = 1\ \mathrm{g\,cm^{-3}}$ and the units are in ($\mathrm{g\,cm^{-3}}$).

sible convection currents in the lunar interior produce the moments of inertia differences. Such a convection current, however, requires a viscosity of less than 10^{24} poise for the lunar interior (Turcotte and Oxburg, 1969), which is much lower than the value required (within the upper 600 km) for the prolonged existence of the mascons (Baldwin, 1971; Arkani-Hamed, 1973; Kunze, 1974; Meissner, 1975). On the other hand, the strong criteria required for a proper Rayleigh number makes it implausible to confine the convection cells below 600 km depth.

The present paper is concerned with the question that the moments of inertia differences are due to the irregularities of the lunar surface, formed by volcanic activity which occurred between 3.7 and 3.2 b.y. ago and by earlier processes. In the first section, the spherical harmonic analysis of the gravitational field of a spherical cap is determined and its effect on the moments of inertia differences of the Moon is discussed. In the second section, the surface irregularities are approximated by several spherical caps and the moments of inertia differences of the Moon subject to these surface irregularities are determined. The third section is devoted to the concluding remarks.

2. Gravitational Potential of a Spherical Cap

Let a thick and uniform spherical cap be limited by two spherical surfaces of radii R_1 and R_2 and by a conic surface with an angle α. Let its axis of symmetry, z', be specified by the spherical angles θ_0 and ϕ_0 and its volume density by ρ_0. The purpose is to find the spherical harmonic coefficients of the gravitational field of such a mass distribution. The

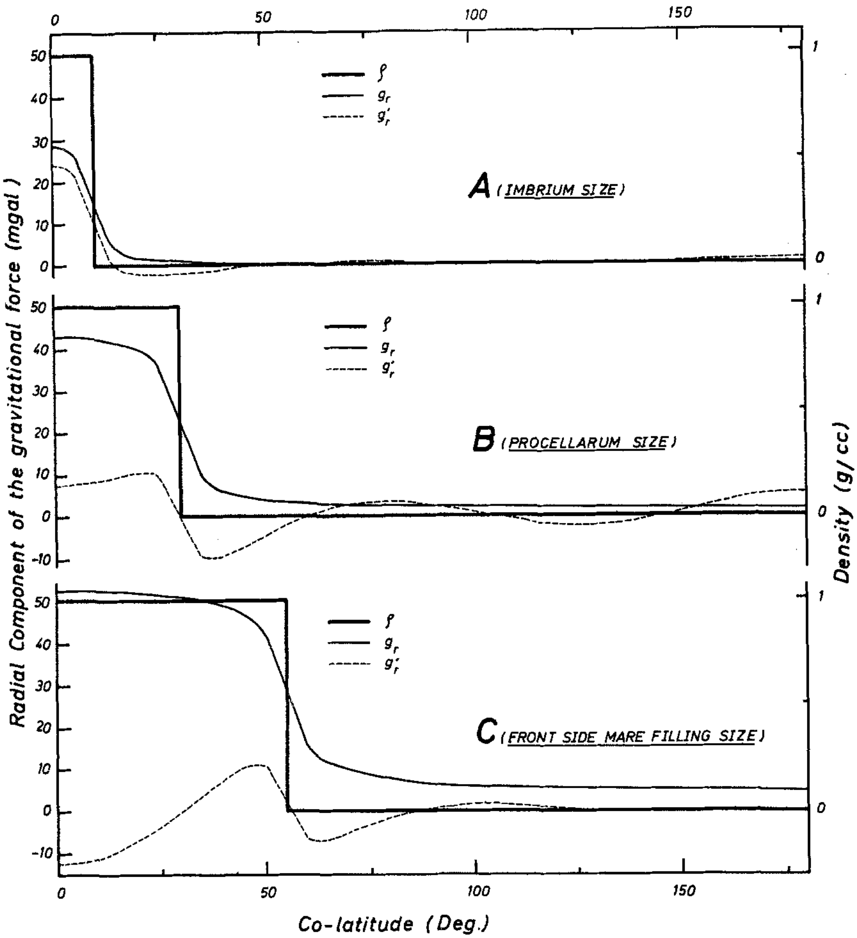

Fig. 2. Spherical caps and the radial components of their gravitational force at 100 km altitude as functions of colatitude with respect to the axis of symmetry of the caps. g_r is the total value and g'_r is the reduced value. The parameters of the caps are $\theta_0 = \phi_0 = 0$, $R_1 = 1737$ km, $R_2 = 1738$ km, $\rho_0 = 3.3\ \mathrm{g\,cm^{-3}}$. The values of α are given in the figures.

procedure is to expand the mass distribution in spherical harmonics and then determine the corresponding harmonic coefficients of the potential.

At a given radius $r (R_1 \leqslant r \leqslant R_2)$ the spherical harmonic representation of ρ is

$$\rho(\theta') = \sum_{n=0}^{\infty} \rho_n P_n^0(\theta'), \tag{1}$$

where

$$\rho_n = \tfrac{1}{2}\rho_0 [P_{n-1}^0(\alpha) - P_{n+1}^0(\alpha)]; \tag{2}$$

P_n^0 being an associated Legendre function of degree n and order zero, and θ' is the latitude

TABLE I

Gravitational Potential of the Moon

The low degree spherical harmonic coefficients of the lunar gravitational potential presented by different authors. JPL1=Lorell and Sjogren (1968); JPL2 and JPL3=Lorell (1969); MB1=Michael *et al.* (1969); MB2=Michael and Blackshear (1972); ODP=Orbit determination program used for the reduction of the S-band transponder data.

Coefficients $\times 10^4$	JPL-1	JPL-2	JPL-3	MB1	MB2	ODP
C_{20}	-0.9062	-0.8582	-0.8749	-0.9260	-0.9114	-0.9262
S_{21}	0.0116	0.0171	0.0999	-0.0354	0.1008	0
C_{21}	-0.0680	0.0333	-0.0990	-0.0034	0.0856	0
S_{22}	0.2030	-0.0923	0.1571	0.0033	-0.00015	0
C_{22}	0.3394	0.1739	0.2458	0.3437	0.3850	0.3209
C_{30}	-0.0840	-0.0560	-0.0491	-0.0238	0.1075	0.0794
S_{31}	0.0685	0.0959	0.0877	0.0213	0.1927	0
C_{31}	0.3366	0.3193	0.3233	0.2256	0.2236	0.3418
S_{32}	-0.0585	0.0810	0.0829	0.0594	0.0665	0
C_{32}	-0.0753	0.0008	0.0221	0.1470	0.2234	0
S_{33}	-0.3558	-0.2209	-0.3205	0.0488	-0.0222	0
C_{33}	-0.1899	0.2533	0.2040	0.1190	0.1011	0.1852

n	MB2	ODP
	degree power $\times 10^{-8}$	
2	0.9965	0.9605
3	0.1637	0.1397
	degree correlation	
2		0.989
3		0.646

with respect to z'. A very strong non-linear dependence of ρ_n on α is displayed in Figure 1 for n values of 2 and 3 and $\rho_0 = 1$. Notice that the maximum value of ρ_2 corresponds to $\alpha \approx 55°$; this means that for $\alpha > 55°$ the ρ_2 value is less despite the larger mass of the cap. Similar characteristics are true in the case of ρ_3 and others.

Using the addition theorem, the gravitational potential at a given point (r, θ, ϕ) is

$$\Phi(r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{1}{r^{n+1}} [\Phi_n^{m,e} \cos m\phi + \Phi_n^{m,0} \sin m\phi] P_n^m(\theta), \tag{3}$$

where

$$\begin{bmatrix} \Phi_n^{m,e} \\ \Phi_n^{m,0} \end{bmatrix} = \frac{4\pi G(R_2^{n+3} - R_1^{n+3})\rho_n}{(2n+1)(n+3)} \begin{bmatrix} \cos m\phi_0 \\ \sin m\phi_0 \end{bmatrix} P_n^m(\theta_0). \tag{4}$$

Bearing in mind the linear relationship of Φ_n^m and ρ_n, figure 1 implies that the small spherical caps, such as those proposed for the lunar mascons, contribute very little to the second-degree harmonics, while a broad cap has significant second-degree coefficient, even if it is relatively thin.

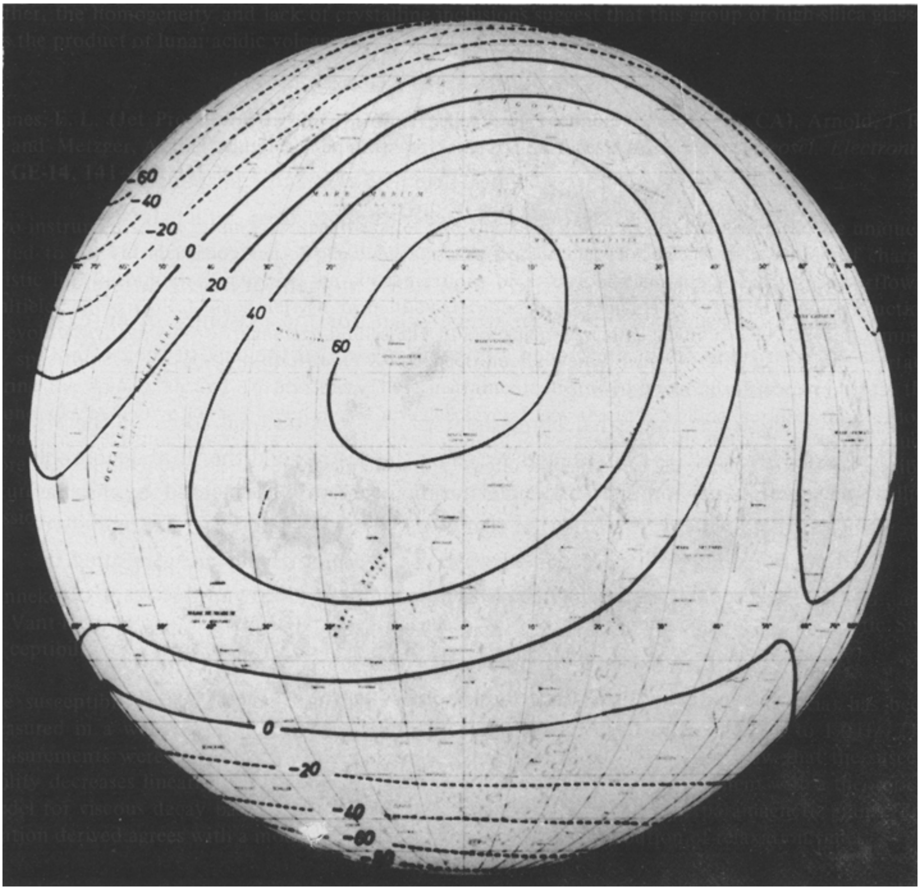

Fig. 3. Radial component of the gravitational force of the Moon at 100 km height produced by the second and third degree harmonic. Units are in (m gal).

Figures 2a–c illustrate the spherical caps and the spherical harmonic representations of the radial components of their gravitational forces, $g_r = -\partial/\partial r\, \Phi$, at 100 km altitude. The expansions include harmonics up to $n = 50$.

3. Moments of Inertia Differences of the Moon

The moments of inertia differences of the Moon are related to the coefficients of the second degree spherical harmonics of its gravitational potential, C_2^0 and C_2^2, through the relationships

$$C - A = \sqrt{5}Ma^2(3^{-1/2}C_2^2 - C_2^0) \tag{5}$$

and

$$C - B = -\sqrt{5}Ma^2(3^{-1/2}C_2^2 + C_2^0), \tag{6}$$

where C, A and B are moments of inertia about the polar axis, the Earth–Moon line, and the third principal axis, respectively. M is the mass and a is the radius of the Moon. C_2^0 and C_2^2 are the fully normalized spherical harmonic coefficients. Table I gives their values which are obtained by different authors through independent procedures, and it implies a good agreement among the values of C_2^0 and some differences among the values of C_2^2.

Unlike the high frequency variations of the surface irregularities, such as mascons, mountains and the unfilled craters, which affect the S-band transponder data strongly, the low frequency variations show little effect on these data, which lead to a general agreement that the extensive irregular mare type fillings of the front side are isostatically compensated and thus have little contribution to the moment of inertia differences of the Moon. Here some evidences are given which argue for the existence of an extra mass associated with these fillings and then a possible reason for the lack of their effect on the S-band transponder is pointed out.

Figure 3 illustrates the radial component of the gravitational force of the Moon at 100 km altitude, which is constructed by using the second and the third degree spherical harmonic coefficients of the lunar gravitational potential presented by Michael and Blackshear (1972). It is clear from the figure that there is a good correlation between the front side mare filling and the broad positive gravity anomaly. Bearing in mind that this broad positive anomaly on the front side is not due to the known mascons, figure 3 indicates the existence of a broad excess mass distribution there. For further clarification, note from figure 2 that if a part of the radial component of the gravitational force of a spherical cap is constructed by using the second- and the third-degree coefficients of its spherical harmonic expansion, a maximum positive gravity would occur above the cap and its amplitude would decay at greater distances. This indicates that it is implausible to think that the strong and broad positive gravity of the front side is due to a mass distributed anywhere other than the front side.

The Apollo laser altimetry data show an overall depression of about 1 km of the front side besides the strong depressions of the surfaces of the circular maria (Kaula *et al.*, 1972, Brown *et al.*, 1974). The latter depressions have been interpreted as the manifestation of the an-elastic deformation of the lunar interior (Arkani-Hamed, 1973). Such depressions, however, cannot produce the broad depression of the front side since their ranges of influence are limited. Therefore, bearing in mind the interpretation proposed for the mascons, it is plausible to assume that the present-day positive gravity anomaly over the front side (figure 3) is the remainder of the larger anomaly which existed at the termination of the filling process. The relaxation time of any surface load as large as the overall front side filling does not differ from that of a mascon-size load by a factor of more than about 2 for the case of a uniform viscous lunar interior (Arkani-Hamed, 1973), or a layered lunar model with a lithosphere overlying a soft asthenosphere (Shimazu, 1966). This is to say, any significant departure from isostatic compensation at the termination of the filling process would be noticeable at the present time. Using a relaxation time of about 3 b.y. (Arkani-Hamed, 1973) and Figure 3 as the present-day gravity anomaly, the present-day surface depression of the front side, due to the viscous deformation of the lunar interior since the mare filling, is found to be about 1 km which is in agreement with the Apollo laser altimetry results.

The lack of any prominent positive gravity anomaly associated with the front side mare filling in the S-band transponder results is most likely due to the reduction of the data on the basis of a non-spherical lunar model, through using the JPL orbit determination program (ODP). In the case of Apollo data, the non-sphericity was due to the inclusion of perturbations, to the otherwise spherically symmetric lunar model, which were specified by the second-degree spherical harmonics (Muller and Sjogren, 1968; Sjogren, 1971; Sjogren *et al.*, 1971, 1972, 1974a, 1974b; Muller *et al.*, 1974), while in the case of Apollos 15 and 16 subsatellite data, the perturbations were specified by the second and the third-degree harmonics (Sjogren and Wollenhaupt, 1973; Sjogren *et al.*, 1974c). The last column of Table I displays the coefficients of the latter perturbations. Included in the table are, also, the degree powers and the degree correlations of these coefficients and the observed values of Michael and Blackshear (1972). The close agreement of the corresponding degree powers and the high degree correlation coefficients imply that the triaxial lunar model contains a significant part of the second- and the third-degree harmonics of the lateral variations of the actual lunar gravity. Therefore, there remains a minute portion of the variations to be detected by the S-band transponder method, which cannot, however, be resolved due to local topographic variations. To clarify this point, the radial component of the gravitational forces excluding the harmonics up to the third degrees is displayed in figure 2. It is clear that such an exclusion does not affect the gravity of small caps significantly, while it reduces the gravity of a larger cap considerably, and it even changes the feature of the gravity from an outstanding positive to an oscillatory one with a small amplitude. Figure 2 indicates that a small local variation of the thickness of filling, for example, due to the topography of the surface of the sites before the filling, would make the reduced gravity of the front side indistinguishable.

The foregoing evidence implies that there is an extra mass distribution on the front side of the Moon associated with the extensive mare filling.

In the present section it is shown that the surface irregularities produce a gravitational potential whose second degree harmonics are similar to those given in Table I. A very simple model is adopted for the surface irregularities. Table II is a list of the spherical caps which are considered to present the main surface irregularities of the Moon. The first six caps represent the associated mascons. The thickness of these caps is assumed to be 3 km and the radii of the caps are determined so that the total mass of each of them is about the value obtained from the S-band transponder data (Wong *et al.*, 1971). The next three caps represent the overall mare filling of the front side, which includes Oceanus Procellarum, Mare Tranquillitatis and Mare Fecunditatis.* The actual thickness of the filling is not known, though the analysis of the seismic data indicates a thickness of about 25 km for the mare material at the northeastern part of Oceanus Procellarum (Toksöz *et al.*, 1972, 1973). The excess filling is assumed to be 1 km. The last three spherical caps shown in Table II are associated with the back-side topography. Bearing in mind that, unlike the front side, the back-side topography has a positive correlation with the undu-

* Note that the second and third caps are located on the highlands, and therefore their density is assumed negative so as to exclude these areas from the first cap.

TABLE II

The parameters of the spherical caps and some of the spherical harmonic coefficients of their gravitational potentials.

		R_1	R_2	α	ρ_0	θ_0	ϕ_0	C_2^0	C_2^2	C_3^1
								$\times 10^4$		
MASCONS	Imbrium	1735	1738	10	3.3	54	344	0.003	0.081	0.048
	Serenitatis	1735	1738	9	3.3	62	19	−0.023	0.073	0.006
	Crisium	1735	1738	6	3.3	73	58	−0.023	−0.021	−0.009
	Nectaris	1735	1738	6	3.3	109	33	−0.021	0.019	−0.012
	Humorum	1735	1738	5	3.3	114	322	−0.011	0.008	−0.027
	Smythii	1736	1738	4	3.3	89	86	−0.009	−0.016	−0.001
OVER ALL FRONT SIDE FLOODING		1737	1738	55	3.3	85	355	−0.357	0.618	−0.101
		1737	1738	10	−3.3	100	10	0.026	−0.044	0.023
		1737	1738	15	−3.3	125	10	0.001	−0.067	−0.031
BACK SIDE TOPOGRAPHY	Back side depression	1736	1738	20	−2.8	115	180	0.082	−0.252	−0.016
	Back side High land I	1738	1740	30	2.8	95	225	−0.342	0	0.195
	Back side High land II	1738	1739	20	2.8	105	120	−0.071	−0.072	0.027
	Total	–	–	–	–	–	–	−0.745	0.327	0.102

lations of the lunar gravity (Ferrari, 1975), the parameters of these caps are determined such that the caps represent a good approximation of the back-side topography, obtained by the Apollo laser altimetry data.

Table II includes the coefficients C_2^0, C_2^2, and C_3^1 of the spherical harmonics of the gravitational potentials associated with the spherical caps. It is clear that mare Imbrium has a small and negative effect on the equatorial bulge of the lunar potential, while mare Crisium, due to its location, has a positive and relatively larger contribution, despite its smaller excess mass. The total contribution of the front side mascons to the moments of inertia differences of the Moon is about an order of magnitude smaller than the observed differences, the conclusion which has already been reached by different authors (Booker *et al.*, 1970). The main contributions to the moments of inertia differences, however, are the back-side topography and the overall front-side mare filling.

Included in Table II are the total values of the coefficients. Bearing in mind that, for the sake of simplicity, the thickness of the caps are assumed as round values, the close approximation of the coefficients to those of the observed ones is very striking. By a slight change of thickness, one can obtain the observed values of the coefficients. Therefore, it is concluded that the moments of inertia differences of the Moon are, most likely, due to surface irregularities.

4. Conclusion

The foregoing analysis leads to the following conclusions:

(1) The gravitational field of a spherical cap indicates that the contribution of an excess mass to the moments of inertia differences of the Moon depends strongly on the location of the mass.

(2) The overall mare filling of the front side of the Moon is associated with a broad mass excess, and

(3) The moments of inertia differences of the Moon are, most likely, due to the surface irregularities, mainly the overall mare filling of the front side and the topography of the back side. The known mascons have little effect on the differences. If this is the case, one can deduce that a significant part of the moments of inertia differences have been created much later than the accretion of the Moon, i.e., during the mare formation and successive floodings.

Acknowledgement

This research is partially supported by the Ministry of Science and Higher Education of Iran, grant no. 650-5-35/1.

References

Alfvén, H. and Arrhenius, G.: 1972, 'Origin and the Evolution of the Earth–Moon System', *The Moon* **5**, 210–230.

Arkani-Hamed, J.: 1973, 'Viscosity of the Moon, I: After Mare Formation', *The Moon* **6**, 100–111.

Baldwin, R. B.: 1971, 'The Question of Isostasy on the Moon', *Phys. Earth–Planet. Interior* **4**, 167–179.

Booker, J. R., Kovach, R. L., and Lu, L.: 1970, 'Mascons and Lunar Gravity', *J. Geophys. Res.* **75**, 6558–6564.

Brown, W., Adams, G., Eggleton, R., Jackson, P., Jordan, R., Kobrick, M., Peeples, W., Phillips, R., Porcello, L., Schaber, G., Sill, W., Thompson, T., Ward, S., and Zelenka, J.: 1974, 'Elevation Profiles of the Moon', *Proc. Fifth Lunar Sci. Conf.* **3**, 3037–3048.

Ferrari, A. J.: 1975, 'Lunar Gravity: The First Farside Map', *Science* **188**, 1297–3000.

Fouad, T., Papanastassiu, D. A., and Wasserburg, G. J.: 1973, *Proc. Lunar Science Conference* **4**, pp. 723–725.

Jeffreys, H.: 1959, *The Earth*, 4th ed., The University Press.

Kaula, W. M.: 1969, 'The Gravitational Field of the Moon', *Science* **166**, 1581–1588.

Kaula, W. M., Schubert, G., Lingenfelter, R. E., Sjogren, W. L., and Wollenhaupt, W. R.: 1972, 'Analysis and Interpretation of Lunar Laser Altimetry', *Proc. Third Lunar Sci. Conf.* **3**, 2189–2204.

Kopal, Z.: 1971, 'Moment of Inertia of the Lunar Globe, and Their Bearing on Chemical Differentiation of Its Outer Layers', *The Moon* **4**, 28–34.

Kopal, Z.: 1977, 'Dynamical Arguments which Concern Melting of the Moon', *Phil. Trans. Roy. Soc. London A.* **285**, 561–568.

Koziel, K.: 1967, 'Differences in the Moon's Moments of Inertia', *Proc. Roy. Soc. Ser. A.* **296**, 248–253.

Kunze, A. W. G.: 1974, 'Creep Response of the Lunar Crust in Mare Regions from an Analysis of Crater Deformation', *Phys. Earth–Planetary Interior* **8**, 375–387.

Liu, A. S. and Laing, P. A.: 1971, 'Lunar Gravity Analysis from Long-Term Effects', *Science* **173**, 1017–1020.

Lorell, J.: 1969, 'Lunar Orbiter Gravity Analysis', *JPL Tech. Rep. 32-1387.*

Lorell, J. and Sjogren, W. L.: 1968, 'Lunar Gravity: Preliminary Estimates from Lunar Orbiter', *Science* **159**, 625–627.

Meissner, R.: 1975, 'Lunar Viscosity as Obtained from the Selenotherms', *The Moon* **12**, 179–191.

Michael, W. H., Blackshear, W. T., and Gapcynski, J. P.: 1969, 'Results on the Mass and the Gravitational Field of the Moon as Determined from Dynamics of Lunar Satellites', presented at the 12th planetary meeting of COSPAR held in Prague.

Michael, W. H. and Blackshear, W. T.: 1972, 'Recent Results on the Mass, Gravitational Field and Moment of Inertia of the Moon', *The Moon* **3**, 388–402.

Muller, P. M. and Sjogren, W. L.: 1968, 'Mascons: Lunar Mass Concentrations', *Science* **161**, 680–684.

Muller, P. M., Sjogren, W. L., and Wollenhaupt, W. R.: 1974, 'Lunar Gravity: Apollo 15 Doppler Radio Tracking', *The Moon* **10**, 195–205.

Runcorn, S. K.: 1967, 'Convection in the Moon and the Existence of a Lunar Core', *Proc. Roy. Soc., Ser. A* **296**, 270–284.

Shimazu, Y.: 1966, 'Survival Time of Lunar Surface Irregularities and Viscosity Distribution within the Moon', *Icarus* **5**, 455–458.

Singer, S. F.: 1972, 'Origin of the Moon by Tidal Capture and some Geophysical Consequences', *The Moon* **5**, 206–209.

Sjogren, W. L.: 1971, 'Lunar Gravity Estimate: Independent Confirmation', *J. Geophys. Res.* **76**, 7021–7026.

Sjogren, W. L., Muller, P. M., and Wollenhaupt, W. R.: 1971, 'Apollo 15 Gravity Analysis from the S-band Transponder Experiment', *The Moon* **4**, 411–418.

Sjogren, W. L., Gottlieb, P., and Muller, P. M.: 1972, 'Lunar Gravity via Apollo 14 Doppler Radio Tracking', *Science* **175**, 165–168.

Sjogren, W. L. and Wollenhaupt, W. R.: 1973, 'Gravity: Mare Humorum', *The Moon* **8**, 25–32.

Sjogren, W. L., Wimberly, R. N., and Wollenhaupt, W. R.: 1974a, 'Lunar Gravity: Apollo 16', *The Moon* **11**, 35–40.

Sjogren, W. L., Wimberly, R. N., and Wollenhaupt, W. R.: 1974b, 'Lunar Gravity: Apollo 17', *The Moon* **11**, 41–52.

Sjogren, W. L., Wimberly, R. N., and Wollenhaupt, W. R.: 1974c, 'Lunar Gravity via the Apollo 15 and 16 Subsatellites', *The Moon* **9**, 115–128.

Toksöz, M. N., Press, F., Dainty, A., Anderson, K., Latham, G., Ewing, M., Durman, J., Lammlein, D., Sutton, G. and Duennebier, F.: 1972, 'Structure, Composition, and Properties of the Lunar Crust', *Proc. Third Lunar Sci. Conf.* **3**, 490–504.

Toksöz, M. N., Dainty, A. M., Solomon, S. C., and Anderson, K. R.: 1973, 'Velocity Structure and Evolution of the Moon', *Proc. Fourth Lunar Sci. Conf.* **3**, 2529–2547.

Tolson, R. H. and Gapcynski, J. P.: 1968, 'An Analysis of the Lunar Gravitational Field as Obtained from Lunar Orbiter Tracking Data', in *Moon and Planets*, Vol. II (ed. by A. Dollfus) North-Holland Publ. Co., pp. 178–186.

Turcotte, D. L. and Oxburgh, E. R.: 1969, 'Implications of Convection Within the Moon', *Nature* **223**, 250–251.

Urey, H. C. and MacDonald, G. J. F.: 1971, 'Origin and History of the Moon', in *Physics and Astronomy of The Moon*, 2nd ed. (ed. by Z. Kopal), Academic Press Inc., New York and London, pp. 213–289.

Williams, J. G., Slade, M. A., Eckhardt, D. H., and Kaula, W. M.: 1973, 'Lunar Physical Libration and Laser Ranging', *The Moon* **8**, 469–483.

Wong, L., Buechler, G., Downs, W., Sjogren, W., Muller, P., and Gottlieb, P.: 1971, 'A Surface-Layer Representation of the Lunar Gravitational Field', *J. Geophys. Res.* **76**, 6220–6236.