PRIMARY AND DERIVED PARAMETERS OF COMMON RELEVANCE OF ASTRONOMY, GEODESY, AND GEODYNAMICS

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Abstract. Problem of selecting primary parameters has been discussed. Primaries should be defined uniquely, as well as, physically. Since no unique definition for semimajor axis exists, it should be replaced by the geoidal geopotential value W_0 or by the geopotential scale factor $R_0 = GM/W_0$, geocentric gravitational constant GM be also primary parameter. Current best estimates of some parameters are given numerically.

1. Introduction

Defining the system of primary constants is the open problem being discussed recently within Special Commission Fundamental Constants (SC-3 IAG). There are different opinions, e.g., to keep traditionally adopted the Earth's semimajor axis as primary. Some individual contributions by the SC-3 members have been published, e.g., the inspiring contribution by Kinoshita (1994). The paper is the individual contribution to the topic by the SC-3 Chairman. It describes his personal point of view only. The reason for it is, to be able to summarize the published contributions to the topic and to reach a consensus to be presented to the IAG community.

The system of parameters of common relevance of astronomy, geodesy, and geodynamics should be based on the physically defined quantities. We shall call them "primary parameters". If so, the set of the primary parameters will be physically well defined and it can serve as a solid base for computing the "derived parameters". We shall select the system of primary parameters and give their current best estimates on the basis of the most recent geopotential models, Satellite and Lunar Laser Ranging (SLR, LLR), and satellite altimetry. Further, after adopting the primaries, some derived parameters will be computed. All the values are to be given in SI units.

2. Primary Parameters

Primary parameters and/or constants should be physically defined. These, and these only, should be part of the system of fundamental constants (Kovalevsky, 1994). We adopt this statement and suggest, the primary parameters be selected as follows: (**a**) Newtonian gravitational constant; (**b**) geocentric gravitational constant

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GM; (**c**) angular velocity of the Earth's rotation ω; (**d**) geopotential value of the geoid W_0; (**e**) the second zonal Stokes parameter (geopotential coefficient) $J_2(J_2 = -J_2^{(0)})$; (**f**), (**g**) the second-degree sectorial Stokes parameters $J_2^{(2)}$ and $S_2^{(2)}$ or $J_{2,2} = [(J_2^{(2)})^2 + (S_2^{(2)})^2]^{1/2}$ and $\Lambda_{2,2} = \frac{1}{2}\tan^{-1}(S_2^{(2)}/J_2^{(2)})$; (**h**) parameter H in the precession constant.

(**a**) The current best estimate of G is (Sir A. H. Cook, 1991)

$$G = (6\,672.59 \pm 0.30) \times 10^{-14}\,\mathrm{m}^3\,\mathrm{s}^{-2}\,\mathrm{kg}^{-1}. \tag{1}$$

(**b**) The geocentric gravitational constant as determined by SLR and LLR (Satellite and Lunar Laser Ranging) contains the mass of the atmosphere; its current best estimates is (Ries, 1992)

$$GM = (398\,600\,441.8 \pm 1.0) \times 10^6\,\mathrm{m}^3\,\mathrm{s}^{-2}. \tag{2}$$

Note that if expressed in TDB units (Fukushima, 1994), it comes out as

$$GM = (398\,600\,435.9) \times 10^6\,\mathrm{m}^3\,\mathrm{s}^{-2}.$$

(**c**) The angular velocity of the Earth's rotation (rounded value) is

$$\omega = 7.292\,115 \times 10^{-5}\,\mathrm{s}^{-1}. \tag{3}$$

(**d**) Geopotential value W_0 determined on the basis of the GEOSAT satellite altimetry is (Nesvorný, 1993), (Nesvorný and Šíma, 1994)

$$W_0 = (62\,636\,857.5 \pm 1.0)\,\mathrm{m}^2\,\mathrm{s}^{-2}. \tag{4}$$

It is independent of the tidal distortions due to the Moon and the Sun, but it depends on GM, ω, as well as, on the volume defined by surface $W = W_0$. It should be redetermined recently on the basis of the ERS-1 altimeter data.

(**e**) However, the second zonal Stokes parameter, the function of the principal moments $C > B > A$ of the Earth's inertia

$$J_2^{(0)} = \frac{\frac{1}{2}(A+B) - C}{Ma_0^2}, \tag{5}$$

is sensitive to the permanent tide effect due to the zero frequency term $\delta V_2^{(0)}$ in the zonal tidal potential $V_2^{(0)}$. The term $\delta V_2^{(0)}$ was derived and investigated by Zadro and Marussi (1973). After solving the first (Dirichlet's) boundary-value problem, the effect $\delta J_2^{(0)}$ on $J_2^{(0)}$ due to $\delta V_2^{(0)}$ can be derived. In the terms used in Zadro and Marussi (1973) it reads as follows:

$$\delta J_2^{(0)} = \frac{3}{4} k_s \left\{ \frac{GM_{☾}}{GM} \left(\frac{R_0}{a_0}\right)^2 \left[\frac{R_0}{a_{☾} (1 - e_{☾}^2)}\right]^3 \times \right.$$
$$\times \left[(\sin^2 \epsilon + \sin^2 i_{☾}) \left(1 + \frac{3}{2} e_{☾}^2\right) - \frac{2}{3} - e_{☾}^2 \right] +$$
$$+ \frac{GM_{\odot}}{GM} \left(\frac{R_0}{a_0}\right)^2 \left[\frac{R_0}{AU(1 - e_{\odot}^2)}\right]^3 \times$$
$$\left. \times \left[\sin^2 \epsilon \left(1 + \frac{3}{2} e_{\odot}^2\right) - \frac{2}{3} - e_{\odot}^2 \right] \right\} \times$$
$$\times \left[1 + \frac{25}{21} \left(\frac{R_0}{a_0}\right)^3 q + \frac{10}{7} \left(\frac{a_0}{R_0}\right)^2 J_2^{(0)} \right]; \tag{6}$$

$$q = \frac{\omega^2 a_0^3}{GM}, \tag{7}$$

$$R_0 = \frac{GM}{W_0}. \tag{8}$$

Notations: k_s is the secular Love number; $GM_{☾}$ and $GM_{\odot}$ the selenocentric and heliocentric gravitational constant, respectively; $a_0 = 6\ 378\ 137$ m is the length factor used in Equation (4) rendering $J_2^{(0)}$ to be dimensionless; $a_{☾}$ is the semi-major axis of the Moon's orbit; $e_{☾} = 0.0549$ and $e_{\odot} = 0.0167$ the eccentricities; ϵ the obliquity of the ecliptic; $i_{☾} = 5°08'$ the inclination of the Moon orbital plane to the ecliptic. Numerically

$$\delta J_2^{(0)} = -(3.08 \times 10^{-8}) k_s. \tag{9}$$

The values of $J_2^{(0)}$, Equation (5), in the recent geopotential models as listed in Table I, are tide-free. It means, they do not contain the zero-frequency tidal effect given by Equation (6). However, the tide-free values of $J_2^{(0)}$ were derived from the values observed in the zero-frequency tide system. The tidal effect given by Equation (6) was subtracted, with $k_2 = 0.3$ (Pavlis, personal communication, 1994). However, the question is, whether $k_2 = 0.3$ is the appropriate Love number for the permanent tidal Earth's mass deformation. That is why we prefer to deal primarily with $J_2^{(0)}$ values in the zero-frequency tide system. Value $J_2^{(0)}$ in the zero-frequency tide system is the primary parameter and that tide-free is the derived parameter.

(f), **(g)** The second sectorial Stokes parameters $J_2^{(2)}$ and $S_2^{(2)}$ or $J_{2,2}$ and $\Lambda_{2,2}$ are free of any tides. The numerical values in the recent geopotential models are given in Table II.

TABLE I

The Stokes second-degree zonal parameter; marked with a bar: fully normalized; $k = 0.3$ adopted for the tide-free system

Geopotential model	Zero frequency tide system $-\bar{J}_2^{(0)}$ (10^{-6})	$-J_2^{(0)}$ (10^{-6})	Tide-free $-\bar{J}_2^{(0)}$ (10^{-6})	$-J_2^{(0)}$ (10^{-6})
GEM-T1	484.16909	1082.6350	484.16491	1082.6258
GEM-T2	484.16958	1082.6361	484.16547	1082.6269
GEM-T3	484.16922	1082.6353	484.16510	1082.6261
JGM-1	484.16958	1082.6361	484.16549	1082.6269
JGM-2	484.16958	1082.6361	484.16548	1082.6269
JGM-3	484.16951	1082.6359	484.16537	1082.6267

TABLE II

The Stokes second-degree sectorial parameters; marked with a bar: fully normalized

Geopotential model	$\bar{J}_2^{(2)}$ (10^{-6})	$\bar{S}_2^{(2)}$ (10^{-6})	$J_2^{(2)}$ (10^{-6})	$S_2^{(2)}$ (10^{-6})	$J_{2,2}$ (10^{-6})	$\Lambda_{2,2}$ (deg)
GEM-T1	2.43893	−1.39984	1.57432	−0.90359	1.81520	165.073; 345.073
GEM-T2	2.43900	−1.40010	1.57440	−0.90380	1.81538	165.072; 345.072
GEM-T3	2.43907	−1.40009	1.57441	−0.90375	1.81536	165.072; 345.072
JGM-1	2.43907	−1.40005	1.57441	−0.90373	1.81535	165.072; 345.072
JGM-2	2.43908	−1.40011	1.57442	−0.90377	1.81538	165.071; 345.071
JGM-3	2.43926	−1.40027	1.57454	−0.90387	1.81553	165.071; 345.071

(**h**) Parameter H in the precession constant

$$H = \frac{C - \frac{1}{2}(A + B)}{C} = -J_2^{(0)} \left(\frac{C}{Ma_0^2}\right)^{-1}, \tag{10}$$

is the only parameter which enables to derive the principal moment of inertia C. The current best estimate (in the zero-frequency tide system) is as

$$H = (3\,273\,763 \pm 20) \times 10^{-9}, \tag{11}$$

(Williams, 1994).

3. Derived Parameters

3.1. Parameters of the best-fitting tri-axial Earth's ellipsoid

The Earth's ellipsoid should represent the geoid surface

$$W = W_0, \tag{12}$$

best. The best-fitting ellipsoid is the level ellipsoid E the potential on the boundary surface of which is just equal W_0. If tri-axial, however, a priori geocentric and a priori given direction of its polar axis, it is defined by four parameters as follows: a the longest semi-axis, α the flattening of the meridional section by the plane containing semi-axis a; α_1 the equatorial flattening; Λ_a the longitude of the meridian above. Six primary parameters define the surface

$$E = E(a, \alpha, \alpha_1, \Lambda_a), \tag{13}$$

as follows: GM, W_0, ω, $J_2^{(0)}$, J_{22}, Λ_a. Instead of W_0 the geopotential scale factor, defined by Equation (8)

$$R_0 = (6\,363\,672.4 \pm 0.1)\ \mathrm{m}, \tag{14}$$

can be used, and instead of ω the dimensionless parameter in the potential of centrifugal forces, defined by Equation (7)

$$q = (3\,461\,390 \pm 2) \times 10^{-9}. \tag{15}$$

However, the surface defined by Equation (12), to be represented by the ellipsoid, should be specified as regards the permanent tidal distortion. It seems to be reasonable, the indirect, as well as, the direct permanent tidal distortion be included. In that case, the primary Stokes parameter directly observed can be used in the solution, and there is no problem as regards the appropriate Love number. The basic Equation (12) defines the s.c. mean geoid and the parameters in Equation (13) define the mean tri-axial ellipsoid.

Numerical values of the ellipsoidal parameters based on different Geopotential Models, are given in Table III. The corresponding parameters for the mean Earth's rotational ellipsoid are in Table IV. They are closed to values derived by Rapp *et al.* (1994).

If necessary, the parameters defining the ellipsoid representing the tide-free geoid can be computed as

$$a(\text{tide-free}) = a(\text{mean}) + \frac{1}{2} R_0 (1 + k_s) \frac{\delta J_2^{(0)}}{k_s}, \tag{16}$$

TABLE III

Parameters of the mean tri-axial Earth's ellipsoid

Geopotential model	a (m)	$1/\alpha$	$1/\alpha$	Λ_a (deg)
GEM-T1	6378 171.55	297.7663	91043	14.9270 W
GEM-T2	6378 171.55	297.7661	91034	14.9281 W
GEM-T3	6378 171.55	297.7662	91035	14.9285 W
JGM-1	6378 171.55	297.7661	91035	14.9281 W
JGM-2	6378 171.55	297.7661	91034	14.9286 W
JGM-3	6378 171.55	297.7661	91026	14.9291 W

TABLE IV

Parameters of the mean Earth's rotational ellipsoid

Geopotential model	$\bar{a}$ (m)	$1/\bar{\alpha}$
GEM-T1	6378 136.52	298.2524
GEM-T2	6378 136.52	298.2523
GEM-T3	6378 136.52	298.2524
JGM-1	6378 136.52	298.2523
JGM-2	6378 136.52	298.2523
JGM-3	6378 136.52	298.2523

$$\alpha(\text{tide-free}) = \alpha(\text{mean}) + \frac{3}{2}(1 + k_s)\frac{\delta J_2^{(0)}}{k_s},$$

The value of $\delta J_2^{(0)}$ is negative; it is given by Equation (9). In Table V the numerical values are given for the tide-free system defined by the Love number k_2. Analogously, in the zero-frequency tide system

$$a(\text{zero-frequency}) = a(\text{mean}) + \frac{1}{2}R_0\frac{\delta J_2^{(0)}}{k_s},$$
$$\alpha(\text{zero-frequency}) = \alpha(\text{mean}) + \frac{3}{2}\frac{\delta J_2^{(0)}}{k_s}. \tag{17}$$

The same relations hold for $\bar{a}$ and $\bar{\alpha}$, however, the tide-free solution needs the numerical value of the Love number responsible for the permanent tidal distortion.

TABLE V

Parameters of the rotational Earth's ellipsoid in the tide-free system

Geopotential model	$\bar{a}$ (m)	$1/\bar{\alpha}$
GEM-T1	6378 136.39	298.2578
GEM-T2	6378 136.39	298.2576
GEM-T3	6378 136.39	298.2577
JGM-1	6378 136.39	298.2576
JGM-2	6378 136.39	298.2576
JGM-3	6378 136.39	298.2576

Only the solution in the zero-frequency tide system and in the mean system are free of the problem above. However, the primary $J_2^{(0)}$ should be used as observed, it means, including the zero-frequency tidal distortion.

In the level ellipsoid system the mean equatorial gravity g_e can be computed as function

$$g_e = g_e(GM, \bar{a}, J_2^{(0)}, q). \tag{18}$$

Omitting terms of order $(J_2^{(0)})^4$, q^4 of magnitude and smaller, it is in the tide-free system

$$\begin{aligned} g_e &= \frac{GM}{a^2}\left[1 - \frac{3}{2}J_2^{(0)} - q - \frac{9}{56}q^2 - \frac{9}{14}J_2^{(0)}q + \frac{27}{8}(J_2^{(0)})^2 - \right. \\ &\quad \left. -\frac{1}{588}q^3 + \frac{83}{784}J_2^{(0)}q^2 - \frac{135}{16}(J_2^{(0)})^3 + \frac{123}{49}(J_2^{(0)})^2 q\right] \\ &= 978\,032.716 \times 10^{-5}\ \mathrm{m\,s^{-2}}. \end{aligned} \tag{19}$$

It can be computed directy from Geopotential Models, adopting value (4). E.g., from JGM-2 it came out as

$$g_e = 978\,032.759 \times 10^{-5}\ \mathrm{m\,s^{-2}}.$$

However, g_e is different for the tide-free, the zero-frequency and the mean systems. The correction δg_e due to the zero-frequency tidal distortion (only) is, in the linear approximation, putting $R_0/a = 1$,

$$\begin{aligned} \delta g_e &= -\frac{GM}{a^2}\left(\frac{3}{2}\delta J_2^{(0)} + 2\frac{\delta a}{a}\right) \\ &= -\frac{1}{2}\frac{GM}{a^2}\delta J_2^{(0)} = (1.508 \times 10^{-7}\ \mathrm{m\,s^{-2}})k_s. \end{aligned} \tag{20}$$

Note that the direct zero-frequency tidal variation in g_e is opposite in sign. It amounts

$$\frac{GM}{a^2}\frac{\delta J_2^{(0)}}{k_s} = -3.016 \times 10^{-7}\ \mathrm{m\ s^{-2}}. \tag{21}$$

3.2. Principal Moments of Inertia

The primaries $J_2^{(0)}$ and $J_{2,2}$ make it possible to derive the relative differences of the Earth's principal moments of inertia $(C > B > A)$. In the zero-frequency tide system, with

$$J_2 = -J_2^{(0)} = (1082.6362 \pm 0.0006) \times 10^{-6}, \tag{22}$$

$$J_{2,2} = (1.8154 \pm 0.0009) \times 10^{-6}, \tag{23}$$

they come out as follows:

$$\frac{C-A}{M\bar{a}^2} = J_2 + 2J_{2,2} = (1086.267 \pm 0.001) \times 10^{-6},$$

$$\frac{C-B}{M\bar{a}^2} = J_2 + 2J_{2,2} = (1079.005 \pm 0.001) \times 10^{-6},$$

$$\frac{B-A}{M\bar{a}^2} = 4J_{2,2} = (7.262 \pm 0.004) \times 10^{-6}. \tag{24}$$

Adopting

$$\bar{a} = 6\,378\,136.4\ \mathrm{m},$$

$$M\bar{a}^2 = \frac{GM}{G}\bar{a}^2 = (243.012 \pm 0.00005) \times 10^{38}\ \mathrm{kg\ m^2},$$

the differences between the principal moments are as follows:

$$C - A = (2.6398 \pm 0.0001) \times 10^{35}\ \mathrm{kg\ m^2},$$

$$C - B = (2.6221 \pm 0.0001) \times 10^{35}\ \mathrm{kg\ m^2},$$

$$B - A = (1.765 \pm 0.0001) \times 10^{33}\ \mathrm{kg\ m^2}. \tag{25}$$

Adopting the coefficient H in the precession constant, Equation (11), the relative values of the principal moments of inertia are numerically

$$\frac{C}{M\bar{a}^2} = \frac{J_2}{H} = (330\,701 \pm 2) \times 10^{-6},$$

$$\frac{A}{M\bar{a}^2} = (329\,615 \pm 2) \times 10^{-6},$$

$$\frac{B}{M\bar{a}^2} = (329\,622 \pm 2) \times 10^{-6}. \tag{26}$$

The coefficients in the Euler's dynamical equations come out as

$$\alpha = \frac{C - B}{A} = (327\,353 \pm 6) \times 10^{-8},$$

$$\beta = \frac{C - A}{B} = (329\,549 \pm 6) \times 10^{-8},$$

$$\gamma = \frac{B - A}{C} = (2\,196 \pm 6) \times 10^{-8}. \tag{27}$$

The principal moments of inertia themselves are as follows

$$A = 8.0101 \pm 0.0002 \times 10^{37}\ \text{kg m}^2,$$

$$B = 8.0103 \pm 0.0002 \times 10^{37}\ \text{kg m}^2,$$

$$C = 8.0365 \pm 0.0002 \times 10^{37}\ \text{kg m}^2. \tag{28}$$

4. Discussion

The system of primaries suggested is formed by quantities which are physically defined. However, instead of W_0 the mean equatorial gravity g_e could be used as primary for defining the dimension of the body, as suggested by Rapp (1967). He considered the four primary parameters as follows: g_e, GM, $J_2^{(0)}$, ω. The equatorial gravity is a physical quantity and could serve also well as primary geodetic parameters. The only reason for preferring W_0 is, the accuracy required as present, i.e., 10^{-8}–10^{-9} order of magnitude.

The primary parameter W_0 enables to define well the length scale of the body, no a priori conditions are needed. Moreover, it does not depend on the zero-frequency tidal term. It can be proved on the basis of its expression in terms of the ellipsoidal parameters

$$W_0 = \frac{GM}{\bar{a}} \left(1 + \frac{1}{3}\bar{\alpha} + \frac{1}{3}q + \text{smaller terms}\right). \tag{29}$$

Let $\bar{a}$, $\bar{\alpha}$ be tide-free values. The hypothetic tidal variation δW_0 in W_0 due to the indirect tidal variations $\delta\bar{a}$, $\delta\bar{\alpha}$ in $\bar{a}$ and $\bar{\alpha}$ respectively,

$$\delta\bar{a} = -\frac{1}{2}R_0\delta J_2^{(0)},$$

$$\delta\bar{\alpha} = -\frac{3}{2}\delta J_2^{(0)}, \tag{30}$$

is as follows (again at the linear approximation)

$$\begin{aligned}\delta W_0 &= -\frac{GM}{a^2}\delta\bar{a} + \frac{1}{3}\frac{GM}{a}\delta\bar{\alpha} \\ &= \frac{1}{2}\frac{GM}{a^2}R_0\delta J_2^{(0)} - \frac{1}{2}\frac{GM}{a}\delta J_2^{(0)} = 0.\end{aligned} \tag{31}$$

Primary parameter W_0 does not depend on the long-term variation in $J_2^{(0)}$ (Eanes, 1991)

$$\frac{\mathrm{d}J_2^{(0)}}{\mathrm{d}t} = -3.6 \times 10^{-9}\ \mathrm{cy}^{-1}. \tag{32}$$

The phenomenon given by Equation (32) gives rise to variation in semimajor axis as

$$\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} = -\frac{1}{2}R_0\frac{\mathrm{d}J_2^{(0)}}{\mathrm{d}t} = -0.009\ \mathrm{m\ cy}^{-1}, \tag{33}$$

and the impact on W_0 is zero:

$$\frac{\mathrm{d}W_0}{\mathrm{d}t} = -\frac{GM}{a^2}\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} - \frac{1}{2}\frac{GM}{a}\frac{\mathrm{d}J_2^{(0)}}{\mathrm{d}t} = 0. \tag{34}$$

In linear approximation $R_0/a = 1$.

The facts above can be proved on the basis of Pizzetti's theory (1913) from which the expression for W_0 is as follows (Heiskanen and Moritz, 1967)

$$W_0 = \frac{GM}{\bar{a}\bar{e}}\mathrm{atan}\frac{\bar{e}}{1-\bar{\alpha}} + \frac{1}{3}\omega^2\bar{a}^2; \tag{35}$$

$\bar{e}^2 = 2\bar{\alpha} - \bar{\alpha}^2$. Then

$$\begin{aligned}\delta W_0 &= -\frac{GM}{\bar{a}}\left[\left(\frac{1}{\bar{e}}\mathrm{atan}\frac{\bar{e}}{1-\bar{\alpha}} - \frac{2}{3}\frac{\omega^2\bar{a}^3}{GM}\right)\frac{\delta\bar{a}}{\bar{a}} + \right. \\ &\quad \left. + \left(\frac{1}{\bar{e}}\mathrm{atan}\frac{\bar{e}}{1-\bar{\alpha}} - \frac{1}{1-\bar{\alpha}}\right)\frac{1-\bar{\alpha}}{\bar{e}^2}\delta\bar{\alpha}\right],\end{aligned} \tag{36}$$

numerically, with tide-free parameters $\bar{a}$, $\bar{\alpha}$

$$\delta W_0 = -\frac{GM}{\bar{a}}\left(0.998\,115\,09\frac{\delta\bar{a}}{\bar{a}} - 0.334\,229\,34\delta\bar{\alpha}\right).$$

TABLE VI

Geopotential value on the geoid computed in the tide-free, the zero frequency and the mean systems; adopted: GM = 398 600.4418 × 10^9 m^3 s^{-2}, ω = 7.292 115 × 10^{-5} rad s^{-1}, k_2 = 0.3

System	a (m)	$1/\alpha$	W_0 (m^2 s^{-2})	R_0 (m)
Tide-free	6 378 136.39	298.25765	62 636 857.5	6 363 672.40
Zero	6 378 136.42	298.25642	62 636 857.5	6 363 672.40
Mean	6 378 136.52	298.25231	62 636 857.5	6 363 672.40

After substituting the zero-frequency tidal corrections, see Equations (16), (17) and (9),

$$\frac{\delta \bar{a}}{\bar{a}} = -\frac{1}{2} k_s \delta J_2^{(0)}, \qquad \delta \bar{\alpha} = -\frac{3}{2} k_s \delta J_2^{(0)}, \tag{37}$$

one gets

$$\frac{1}{k_s} \delta W_0 = 0.003\ 7289\ \mathrm{m}^2\ \mathrm{s}^{-2},$$

$$\frac{1}{k_s} \frac{\delta W_0}{W_0} = 5.95 \times 10^{-11}; \tag{38}$$

it is zero practically, i.e., as regards the computational accuracy.

The fact, W_0 and/or R_0 is independent on the system (tide-free, zero, mean) is illustrated numerically in Table VI.

The facts above are of importance from the point of view of the realization of the reference system for the long-term geodynamic studies. Parameter W_0 is relatively very stable, it depends only on the volume of surface $W = W_0$, on geocentric gravitational constant GM and on angular velocity of the Earth's rotation. It does not depend on any perturbation which does not change the volume of surface $W = W_0$, as well as, the mass of the Earth and its angular velocity of rotation.

The determination of W_0 need not the global coverage by the input data. However, if accuracy on the dm-level is required, the determination of the semimajor axis meets the difficulties as regards the estimates of its actual accuracy. Theoretically, the global coverage by input data is necessary to reach accuracy on the dm-level. E.g., the area covered by satellite altimetry is limited and that is why, the solution for the semimajor axis based on it only, should be distorted by the geoidal features over the areas not covered. As regards W_0, it is free of this major disadvantage. Note that W_0 is needed for determining the difference between the geocentric coordinate time TCG and terrestrial time TT (Fukushima, 1994)

$$\mathrm{TCG} - \mathrm{TT} = \frac{W_0}{c^2}(t - t_0),$$

as well as for determining the difference between the solar system barycentric coordinate time TCB and the solar system barycentric dynamical time TDB (Fukushima, 1994).

For many reasons, Kinoshita (1994) suggested the Earth's semimajor axis be fixed as a defining constant like the Gaussian constant which defines the astronomical uint AU. In that case, a would not be involved into derived parameters, however, the basic system of primaries remains unchanged.

5. Conclusion

Primary parameters should be defined physically. Therefore, the semimajor axis a of Earth's ellipsoid is to be replaced by the geopotential geoidal value W_0 or by geopotential scale factor $R_0 = GM/W_0$. Geocentric gravitational constant GM be also primary parameter. Disadvantages of a are: (1) It is not defined uniquely, (2) global coverage by input data is needed for its derivation, (3) it depends on permanent tidal perturbation. However, redetermination of $W_0(R_0)$ using ERS-1 satellite altimeter data should be done for reestimating actual accuracy of W_0 (R_0).

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