POYNTING–ROBERTSON EFFECT AND ORBITAL MOTION

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Abstract. Time evolution of the meteoroid's orbit under the action of the solar electromagnetic radiation is discussed in terms of perihelion and aphelion distances. Perturbation equations for secular changes of orbital elements are written for the most simple case. Initial conditions are formulated for the obtained system of perturbation equations and simple example is presented.

1. Introduction

The Poynting–Robertson effect (P–R effect) is the most important nongravitational effect influencing the orbital evolution of small (less than $\approx$1 mm in diameter) interplanetary dust particles (IDPs). In spite of its importance, the P–R effect is not thoroughly understood. Many incorrect statements exist as for the physical nature of this effect and also as for the orbital evolution of meteoroids under the action of the solar electromagnetic radiation. The physical nature of the P–R effect is discussed in detail in Klačka (1992a) and in a more elementary manner by Klačka (1992b, 1992c). The orbital evolution of meteoroids under the action of the P–R effect is discussed in Klačka (1992d), Klačka and Kaufmannová (1992a, 1992b). The aim of this paper is to discuss the perturbation equations of celestial mechanics in terms of perihelion and aphelion distances instead of generally used orbital elements, semimajor axis and eccentricity.

2. Secular Changes of Orbital Elements

As was pointed out by Klačka (1992d), perturbation equations of celestial mechanics for secular changes of orbital elements can be used only for the case of elliptical orbit if $e \approx 0$ (e – eccentricity) does not hold. Time evolution of secular changes of orbital elements for the case $e \approx 0$ behaves in a qualitatively different manner and the whole process of its calculation must be done numerically (Klačka and Kaufmannová 1992a, 1992b). The case $e \approx 0$ does not hold, is considered in this section. We want to formulate the perturbation equations in terms of perihelion and aphelion distances, now.

Time derivatives for the perihelion (q) and aphelion (Q) distances in terms of semimajor axis (a) and eccentricity (e) follow from definitions of these orbital elements ($q = a(1 - e)$, $Q = a(1 + e)$)

$$\frac{dq}{dt} = \frac{da}{dt} - \left(a \frac{de}{dt} + e \frac{da}{dt} \right), \quad \frac{dQ}{dt} = \frac{da}{dt} + \left(a \frac{de}{dt} + e \frac{da}{dt} \right). \tag{1}$$

We have to substitute da/dt and de/dt by their expressions obtained from the P–R effect:

$$\frac{da}{dt} = -2\beta \frac{\mu}{c} \frac{1}{r^2} \frac{a}{1-e^2} (1 + 2e^2 + 2e \cos f - e^2 \cos^2 f),$$

$$\frac{de}{dt} = -\beta \frac{\mu}{c} \frac{1}{r^2} (2e + e \sin^2 f + 2 \cos f) \tag{2}$$

(see Equations (20) and (22) in Klačka, 1992d). We are interested in secular changes of perihelion and aphelion distances. We have to substitute Equation (2) into Equation (1) and make time averaging according to formula

$$\langle g \rangle = \frac{1}{a^2 \sqrt{1-e^2}} \frac{1}{2\pi} \int_0^{2\pi} g(f) r^2 \, df \tag{3}$$

(see Equation (105) in Klačka, 1992e): the result is

$$\frac{dq}{dt} = -\beta \frac{\mu}{c} \frac{1}{a(1-e^2)^{3/2}} \left\{ 2 + 3e^2 - \left(\frac{9}{2} - e^2 \right) e \right\},$$

$$\frac{dQ}{dt} = -\beta \frac{\mu}{c} \frac{1}{a(1-e^2)^{3/2}} \left\{ 2 + 3e^2 + \left(\frac{9}{2} - e^2 \right) e \right\}; \tag{4}$$

and we have to substitute equations

$$a = \frac{Q+q}{2}, \quad e = \frac{Q-q}{Q+q} \tag{5}$$

into equations (4) to be this system consistent.

Equations (4) lead to the conclusion that the secular decrease for the perihelion distance is smaller than that for the aphelion distance. Although this result is generally known (Wyatt and Whipple, 1950), we have shown this statement in the analytical way without numerical solution of a system of differential equations.

3. Initial Conditions

We must define initial conditions for q and Q to be the system (4)–(5) complete. In the case of sudden releasing of the IDP from a large parent body (e.g., comet) we have

$$q_{in} = a_{in}(1 - e_{in}), \quad Q_{in} = a_{in}(1 + e_{in}), \tag{6}$$

where

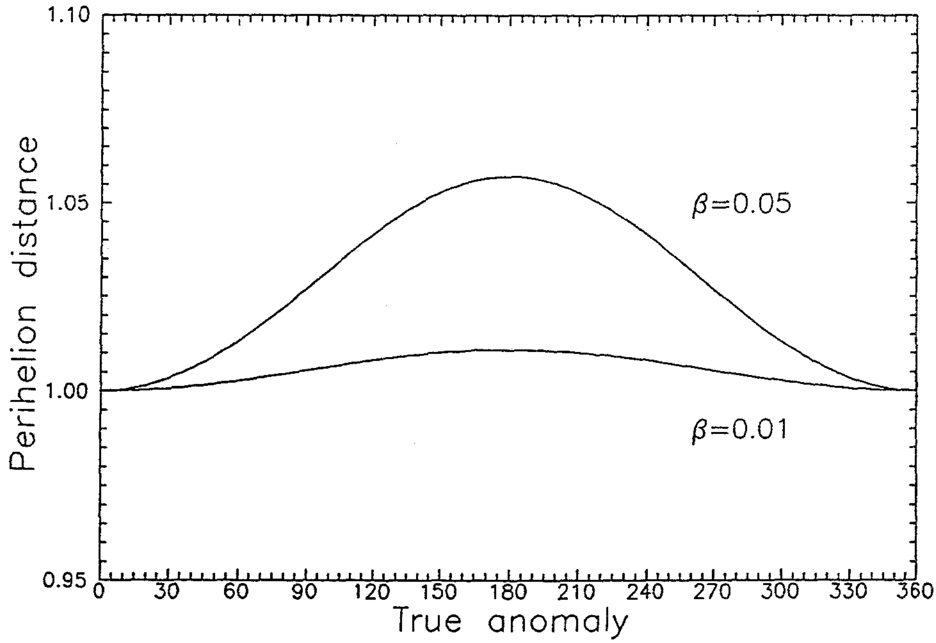

Fig. 1. Initial perihelion distance (q_{in}/q_0) for dust particle released from a large parent body (perihelion distance q_0) as a function of positions on the orbit of the parent body. Figure depicts situation for $e_0 = 0.85$.

$$a_{in} = a_0(1-\beta)\left[1 - 2\beta\frac{1+e_0\cos f_0}{1-e_0^2}\right]^{-1},$$

$$e_{in} = \sqrt{1 - \frac{1-e_0^2-2\beta(1+e_0\cos f_0)}{(1-\beta)^2}}. \tag{7}$$

a_0, e_0 are osculating orbital elements (semimajor axis, eccentricity) (f_0 – true anomaly) of the parent body at the instant of releasing the particle with zero velocity (Equations (30)–(31) in Klačka 1992d). Quantities q_{in}/q_0 and Q_{in}/Q_0 for two values of β are shown in Figures 1–2 for the case $e_0 = 0.85$ (comet Encke).

4. Example

Let us have a particle released at perihelion of a comet. Cometary eccentricity is e_0. Let the particle is ejected with zero velocity with respect to the comet and let the particle moves in parabolic orbit after ejection. This case corresponds to the situation $e_{in} = 1$ in Equation (7) and one can easily verify that physical properties of the particle are characterized by the condition

$$\beta = (1-e_0)/2. \tag{8}$$

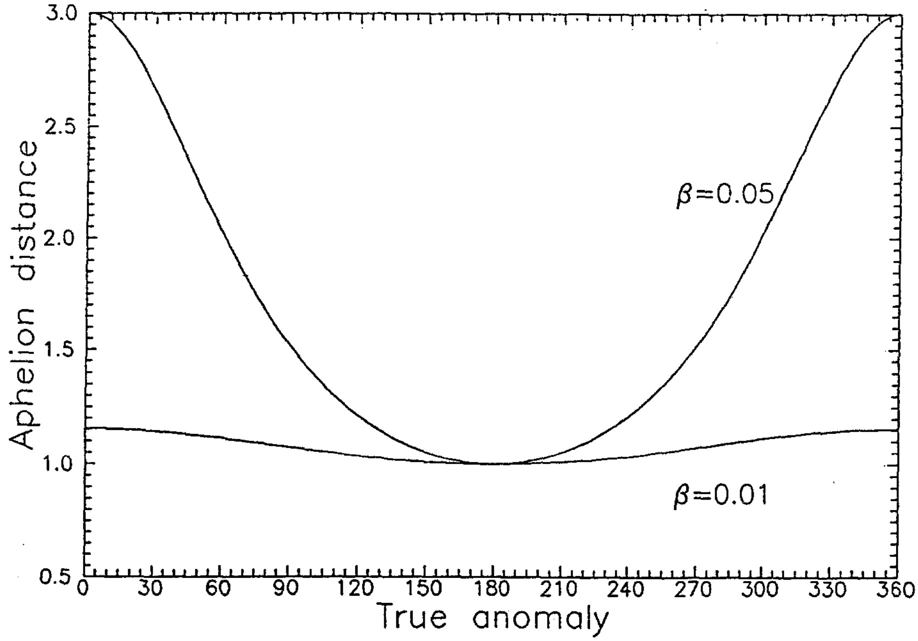

Fig. 2. Initial aphelion distance (Q_{in}/Q_0) for dust particle released from a large parent body (aphelion distance Q_0) as a function of positions on the orbit of the parent body. Figure depicts situation for $e_0 = 0.85$.

Conic section is characterized by the well-known relation

$$r = p/(1 + e\cos f) \tag{9}$$

and during the ejection process the semi-latus rectum is suddenly changed from the initial value p_0 to the value p, given by

$$p = p_0/(1 - \beta) \tag{10}$$

(see Equation (27) in Klačka, 1992d).

The perihelion distance of the cometary orbit is, according to Equation (9),

$$q_0 = p_0/(1 + e_0) \tag{11}$$

and perihelion distance of the parabolic orbit of the particle is ($e = 1$ in Equation (9))

$$q_P = p/2 \tag{12}$$

(suffix 'P' denotes ejection at perihelion of the cometary orbit). By virtue of Equations (8) and (10)–(11) the last relation can be rewritten in the form

$$q_P = p_0/(1 + e_0) = q_0. \tag{13}$$

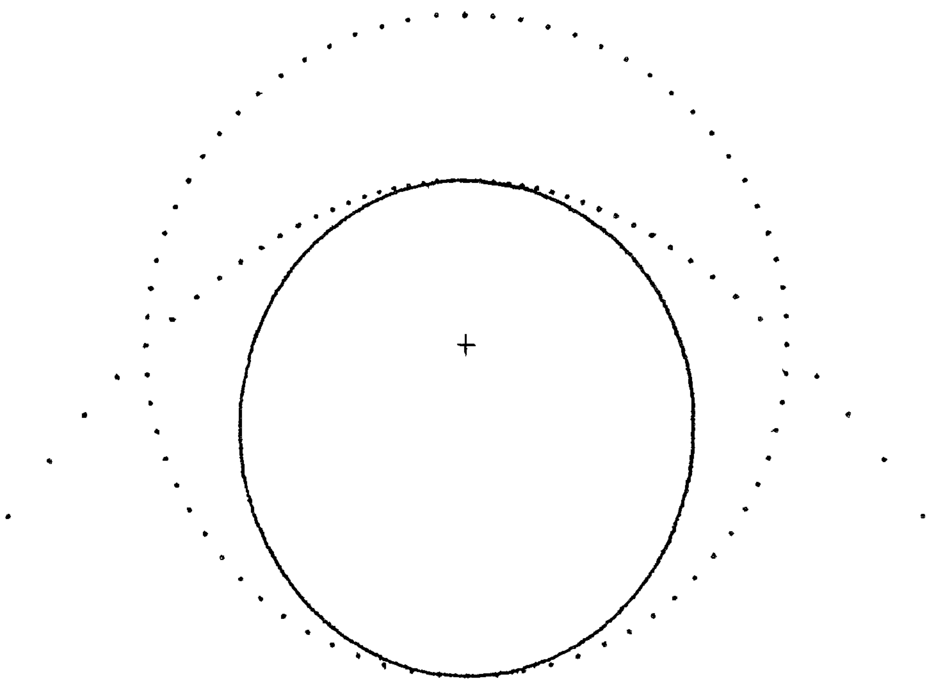

Fig. 3. Dust particle with $\beta = e_0 = 1/3$ moves in parabolic orbit if released at perihelion and in circular orbit if released at aphelion of cometary orbit (continuous ellipse) with eccentricity e_0.

Thus, the perihelion distance of the particle is unchanged after ejection. This is physically evident since the velocity vector is perpendicular to the radius vector only for perihelion of parabolic orbit.

Let a particle of the same physical properties, characterized by Equation (8), is ejected at aphelion of the cometary orbit. Equations (7) ($f_0 = \pi$) and (8) yield then

$$e = |1 - 3e_0|/(1 + e_0) . \tag{14}$$

The perihelion distance q_A (suffix 'A' – ejection at aphelion) of this new orbit is

$$q_A = \frac{p}{1+e} = \frac{2p_0}{1+e_0}\left(1 + \frac{|1-3e_0|}{1+e_0}\right)^{-1} , \tag{15}$$

if also Equations (9), (10), (8) and (14) are used. We can write, on the basis of Equation (11),

$$q_A = 2q_0\left(1 + \frac{|1-3e_0|}{1+e_0}\right)^{-1} . \tag{16}$$

For the case $e_0 = 0$ the last equation yields $q_A = q_0(=q_P)$ and this is physically correct: perihelion distance is the same as aphelion distance for circular orbit; compare this result with that represented by Equation (16) in Kresák (1976).

Finally, we want to mention the special interesting case. If $\beta = e_0$ and dust particle is released at aphelion of the cometary orbit, then Equation (7) yields $e = 0$ – *the particle moves in circular orbit. Moreover, if its physical properties are characterized by Equation* (8), *i.e.*, $\beta = e_0 = 1/3$, then:

1. the particle moves in circular orbit, if released at aphelion of the cometary orbit,
2. the particle moves in parabolic orbit, if released at perihelion of the cometary orbit.

The situation is illustrated also in Figure 3.

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