CALCULATION OF THE GLOBAL SYSTEM OF THE FIELD-ALIGNED CURRENTS ON THE DAYSIDE OF THE MAGNETOSPHERE

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Abstract. Using the well-known equation for the normal component of the current which exist near the tangential discontinuity in the plasma in the case of the frozen-in magnetic field, and supposing that the current closes in the ionosphere in the auroral oval in the region 1, one calculates and compares with the data of observations the dependence of the density of the field-aligned current at the level of the ionosphere on the local time.

1. Introduction

In a previous paper (Samokhin, 1992) we considered the closing in the ionosphere of the normal component of the current which exists near the magnetopause as the tangential discontinuity in the plasma with the ideal frozen-in magnetic field in the case of the plasma flow across the magnetic field. However, there the field-aligned current was calculated approximately by multiplying the normal component of the current in the plane of the equator by the area of the magnetopause. In this paper we make the calculation more accurately with the aid of the integration along the magnetic field lines in the supposition that the field-aligned currents flow in the thin layer near the magnetopause. The form of the magnetopause sets approximately, the dependence of the density on the distance is suggested to be inversely to the cube of the geocentric distance for the best consent with the data of observations.

In Section 2 one derives the equation for the field-aligned current in the supposition that it flows in the thin layer near the magnetopause. In Section 3 this current is calculated for some model of the magnetopause, for the dipole magnetic field and for the plasma density varing inversely to the cube of the geocentric distance. In Section 4 the results of the calculation are compared with the data of observations (Iijima and Potemra, 1976).

2. Derivation of the Equation for the Field-Aligned Current

In the paper by Samokhin (1992) we have derived the equation for the normal component of the current near the tangential discontinuity in the case of the ideal frozen-in magnetic field and of the plasma velocity perpendicular to the magnetic field

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$$j_n = -\frac{c}{4\pi}(B \times \nabla \ln B^2/\rho)_n\,, \tag{1}$$

where j, B stands for the current and the magnetic field, ρ for the mass density, c for the light velocity and n for the inside normal.

As in that paper we shall suppose the magnetopause to be the tangential discontinuity and the normal component of the current (1) to exist on its inside which closes in the ionosphere as a field-aligned current flowing in the thin layer with thickness Δz near the magnetopause.

Let us derive the equation for the field-aligned current. Let us introduce three unit vectors (n, s, h), where the unit vector h is directed along the magnetic field, the unit vector s is parallel to the plasma velocity. Then it stands $n \times h = s$ and the next equation follows Equation (1) as

$$j_n = -\frac{cB}{4\pi}\frac{\partial}{\partial s}\ln\frac{B^2}{\rho}\,. \tag{2}$$

The equation of the continuity of the current $\operatorname{div} j = 0$ gives

$$\operatorname{div} j_{\|} = -\operatorname{div} j_{\perp}\,, \tag{3}$$

where the subscripts $\|$ and $\perp$ mean the components of the vector along and across the magnetic field. But as it is known,

$$\operatorname{div} j_{\|} = B\frac{\partial}{\partial h}\frac{j_{\|}}{B}\,. \tag{4}$$

For the simplification of calculations we shall suppose the cross current to turn into field-aligned one in a thin layer near the magnetopause where the field-aligned current distributes approximately homogeneously. Then one can write

$$j_{\perp} = j_n f(z)\,, \tag{5}$$

where

$$f(z) = \frac{1}{\Delta z}(z - z_0)\,; \tag{6}$$

z standing for the co-ordinate along the normal to the magnetopause, the subscript 0 refers to the magnetopause, the thickness Δz is small in comparison with the characteristic scale of the magnetopause.

Finally the equation

$$\frac{\partial}{\partial h}\frac{j_{\|}}{B} = -\frac{1}{B}\frac{j_n}{\Delta z} \tag{7}$$

yields Equations (3)–(6); we find from them that

$$j_{\|} = -B \int \frac{j_n}{B} \frac{\mathrm{d}h}{\Delta z} . \tag{8}$$

The integration is made from the equatorial plane where the field-aligned current is equal to zero to the current point.

But the value $j_{\|} \Delta z$ is equal to $I_{\|}$ – to the current on the unit of the length, therefore,

$$I_{\|} = -B \, \Delta z \int \frac{j_{\|}}{B} \frac{\mathrm{d}h}{\Delta z} . \tag{9}$$

The value Δz changes along the field line and can not be taken out of the sign of the integral as it is the distance between the magnetopause and the neighbouring surface formed by the magnetic field lines.

Substituting for the value j_n in Equation (9) from Equation (2) we obtain

$$I_{\|} = \frac{cB}{4\pi} \Delta z \int \frac{\partial}{\partial s} \ln \frac{B^2}{\rho} \frac{\mathrm{d}h}{\Delta z} . \tag{10}$$

3. Calculation of the Field-Aligned Current for the Selected Models of the Magnetopause and of the Density Distribution

As in the paper by Samokhin (1992), let us set the form of the magnetopause in the equatorial plane with the aid of the equation

$$y = -z_0 + l \ln \frac{2}{1 + \cos x/l} , \tag{11}$$

where the origin of the co-ordinate (x, y) coincides with the geomagnetic dipole, the y-axis is directed from the Sun, the x-axis is directed to the dusk, it stands $r_0 = 10R_E$, $\pi 1 = 20R_E$, R_E is the radius of the Earth.

Let us suppose that out of the equatorial plane the magnetopause is formed by the dipole field lines which go through the line (11) in the equatorial plane: i.e.,

$$L \sin^2 \theta = r , \tag{12}$$

where L stands for the geocentric distance to the field line in the equatorial plane. Here we have introduced the spherical co-ordinate system with the polar axis directed to the North, with the centre of rectangular co-ordinate system and the azimuth angle φ counted from the x-axis. Then it stands $L \cos \varphi = x$, $L \sin \varphi = y$, and the equation of the magnetopause takes the form finally with taking into account Equations (11) and (12)

$$f(r, \theta, \varphi) = \frac{r \sin \varphi}{\sin^2 \theta} + r_0 - l \ln \frac{2}{1 + \cos(r \cos \varphi / l \sin^2 \theta)} = 0 . \tag{13}$$

The dipole magnetic field is known to be determined with the equation

$$B = -\frac{2M_g}{r^3}\cos\theta e_r - \frac{M_g}{r^3}\sin\theta e_\theta\,, \tag{14}$$

where M_g stands for the moment of the geomagnetic dipole, e_r and e_θ stand for the unit vectors of the spherical coordinate system and then the unit vector h introduced in the preceeding section can be written in the form

$$h = \frac{-2\cos\theta e_r - \sin\theta e_\theta}{\sqrt{1+3\cos^2\theta}}\,. \tag{15}$$

Then the interior normals

$$n = \nabla f/|\nabla f|\,, \tag{16}$$

where the function f is determined by Equation (13).

The calculations yield

$$\nabla f = \frac{1}{\sin^3\theta}\frac{1}{\cos(L\cos\varphi/2l)}\left[\sin\left(\varphi - \frac{L\cos\varphi}{2l}\right)(\sin\theta e_r - 2\cos\theta e_\theta) + \right.$$

$$\left. + \cos\left(\varphi - \frac{L\cos\varphi}{2l}\right)e_\varphi\,, \right. \tag{17}$$

$$|\nabla f| = \frac{1}{\sin^3\theta}\frac{1}{|\cos(L\cos\varphi/2l)}\left[1 + 3\sin^2\left(\varphi - \frac{L\cos\varphi}{2l}\right)\cos^2\theta\right]^{1/2}. \tag{18}$$

The dipole magnetic field is symmetrical on the azimuth. Supposing that the density of the plasma does not depend on φ too one can omit the derivative $\partial/\partial\varphi$ in the operator $\partial/\partial s = (n\times h)\cdot\nabla$ and then

$$(n\times h)\cdot\nabla = -\frac{\cos(\varphi - L\cos\varphi/2l)[(2\cos\theta/r)\,\partial/\partial\theta - \sin\theta\,\partial/\partial r]}{\sqrt{1+3\cos^2\theta}\sqrt{[1+3\sin^2(\varphi - L\cos\varphi/2l)\cos^2\theta]}}\,. \tag{19}$$

Let us suppose the plasma density to vary inversely to the cube of the geocentric distance, $\rho \sim r^{-3}$; then

$$\frac{\rho}{B^2}\frac{\partial}{\partial s}\frac{B^2}{\rho} = \frac{r^3}{1+3\cos^2\theta}\frac{\partial}{\partial s}\frac{1+3\cos^2\theta}{r^3}\,. \tag{20}$$

The use of the operator standing in the divisor of Equation (19) yields

$$\left(\frac{2\cos\theta}{r}\frac{\partial}{\partial\theta} - \sin\theta\frac{\partial}{\partial r}\right)\frac{1+3\cos^2\theta}{r^3} = \frac{3\sin^3\theta}{r^4}\,. \tag{21}$$

We find finally with help of Equations (19)–(21) that

$$\frac{\rho}{B^2}\frac{\partial}{\partial s}\frac{B^2}{\rho} = -\frac{1}{r}\frac{3\sin^3\theta}{(1+3\cos^2\theta)^{3/2}} \times$$
$$\times \frac{\cos(\varphi - L\cos\varphi/2l)}{\sqrt{[1+3\sin^2(\varphi - L\cos\varphi/2l)\cos^2\theta]}}. \tag{22}$$

The element of the length along the dipole field line in accordance with Equation (14) is

$$\mathrm{d}h = \sqrt{\mathrm{d}r^2 + r^3\,\mathrm{d}\theta^2} = \sqrt{(B_r/B_\theta)^2 + 1}\,r\,\mathrm{d}\theta = \frac{r\sqrt{1+3\cos^2\theta}\,\mathrm{d}\theta}{\sin\theta}. \tag{23}$$

For the calculation of the integral (10) determining the field-aligned current one needs to calculate also Δz. These calculations are more complex.

The value Δz is an element of the length of the arc on the normal to the magnetopause. Therefore

$$\mathrm{d}z = \sqrt{\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2}, \tag{24}$$

where

$$\frac{\mathrm{d}r}{n_2} = \frac{r\,\mathrm{d}\theta}{n_\theta} = \frac{r\sin\theta\,\mathrm{d}\varphi}{n_\varphi}. \tag{25}$$

Substituting in Equation (24) with taking into account (25) for the components of the normal in accordance with Equations (16)–(18) we find finally that

$$\Delta z = \frac{\sqrt{[1+3\sin^2(\varphi - L\cos\varphi/2l)\cos^2\theta]}}{2\cos\theta\sin(\varphi - L\cos\varphi/2l)}\,r\,\Delta\theta. \tag{26}$$

Since we are on the definite field line when we calculate Equation (10) it is convenient to introduce in Equation (26) the value which defines the field line for example the colatitude of the point of the cross of the field line with the Earth surface θ_0. Then the equation of the field takes the form

$$\sin^2\theta = r\sin^2\theta_0, \tag{27}$$

where the geocentric distance is measured in the Earth's radii R_E.

Differentiating Equation (27) we find that

$$\Delta r\sin^2\theta_0 - 2\sin\theta\cos\theta\,\Delta\theta = -2r\sin\theta_0\cos\theta_0\,\Delta\theta_0. \tag{28}$$

In this equation the values Δr and $\Delta\theta$ are connected with the equation of the normal (25): i.e.,

$$2\cos\theta\,\Delta r + r\sin\theta\,\Delta\theta = 0. \tag{29}$$

From Equations (28) and (29) one can find $\Delta\theta$ as

$$\Delta\theta = \frac{4\sin\theta\cos\theta\,\mathrm{ctg}\,\theta_0}{1+3\cos^2\theta}\Delta\theta_0\,. \tag{30}$$

Then we find finally with the aid of Equations (26), (27) and (30)

$$\Delta z = \frac{\sqrt{[1+3\sin^2(\varphi - L\cos\varphi/2l)\cos^2\theta]}\,\sin^3\theta}{\sin(\varphi - L\cos\varphi/2l)(1+3\cos^2\theta)}\,\frac{2\cos\theta_0\,\Delta\theta_0}{\sin^3\theta_0}\,. \tag{31}$$

Introducing the quantity

$$k^2 = \sin^2(\varphi - L\cos\varphi/2l)\,, \tag{32}$$

one can write the integral (10) with the aid of Equations (14), (22), (23) and (31) in the form

$$I_{\|} = -\frac{3cM_g}{4\pi R_E^3}\sin^3\theta\cos\left(\varphi - \frac{L\cos\varphi}{2l}\right)\frac{\sqrt{1+3k^2\cos^2\theta}}{\sqrt{1+3\cos^2\theta}}\times$$
$$\times\int_0^{\pi/2}\frac{\mathrm{d}\theta}{(1+3k^2\cos^2\theta)\sin\theta}\,. \tag{33}$$

One supposes in Equation (33) that there is the considered point on the magnetopause. The contribution to the field-aligned current (33) gives only the cross current on the magnetopause therefore when one calculates the field-aligned current at the level of the ionosphere one must restrict the lower limit of the integration with the value θ_1 for which the magnetic field line turns from the magnetopause to the Earth. For θ_1 we shall take the root of the equation

$$n_x = 0\,. \tag{34}$$

In fact, for the used model of the magnetopause the normal turns to be perpendicular to the axis dawn–dusk at the high latitudes.

Let us find the root taking into account that

$$e_x = (e_r\sin\theta + e_\theta\cos\theta)\cos\varphi - e_\varphi\sin\varphi\,. \tag{35}$$

Taking into account Equations (16)–(18) we obtain from Equation (34)

$$(1-\cos^2\theta)\sin\left(\varphi - \frac{L\cos\varphi}{2l}\right)\cos\varphi - \cos\left(\varphi - \frac{L\cos\varphi}{2l}\right)\sin\varphi = 0\,, \tag{36}$$

from which

$$\cos\theta_1 = (1/\sqrt{3})\sqrt{[1 - \mathrm{tg}\,\varphi\,\mathrm{ctg}(\varphi - L\cos\varphi/2l)]}\,. \tag{37}$$

4. Results of Calculations and the Comparison with the Data of Observations

As in the paper by Samokhin (1992) we shall suppose the field-aligned current generated on the magnetopause by the normal component of the current to close in the polar boundary of the auroral oval creating the global system of field-aligned currents in the region 1 (Iijima and Potemra, 1976) and in the corresponding closed circuit of the current there to be the reverse field-aligned current and e.m.f. on the second boundary of the layer of the antisolar convection – on the boundary of the reverse of the convection.

For finding the density of the current at the level of the ionosphere the integral (33) is calculated at first which gives the field-aligned current on the unit of the length of the aurora oval. The density of the field-aligned current is determined by the equation

$$j_{\|} = I_{\|}/R_E\,\Delta\theta\,, \tag{38}$$

where the width of the oval $\Delta\theta$ as in the paper (Samokhin, 1992) is taken to be 2°.

The azimuth angle φ on the magnetopause we find with help of the equation

$$\varphi = \arc\sin(y/r)\,, \tag{39}$$

then the local time in hours is determined by the equation

$$\vartheta = \frac{24}{360^\circ}\left(270^\circ + \frac{180^\circ}{\pi}\arc\sin\frac{y}{r}\right). \tag{40}$$

For the calculation it is convenient to take x as a function of y from (11)

$$x = l\arc\cos\left[2\exp\left(-\frac{y+z_0}{l}\right) - 1\right]. \tag{41}$$

For finding the dependence of $j_{\|}$ on the local time one took the y-values in the interval from $-10R_E$ to $50R_E$ and calculated the field-aligned current with the aid of Equations (33), (37)–(39) at the level of the ionosphere.

Let us note that in Equation (33) the dimensional value is $(B_E = 2M_g/R_E^3)$

$$\frac{3cB_E}{8\pi}\left(\frac{r_0}{R_E}\right)^{3/2} = 2.25A/M\,; \tag{42}$$

and it is convenient to write the integral in the form

$$\int_{\theta_1}^{\pi/2} \frac{\mathrm{d}\theta}{(1+3k^2\cos^2\theta)\sin\theta} = \int_0^{\cos\theta_1} \frac{\mathrm{d}x}{(1+3k^2x^2)(1-x^2)}\,. \tag{43}$$

In Figure 1 the density of the field-aligned current which is calculated theoretically is compared with the data of observations (Iijima and Potemra, 1976) for $2- \subseteq K_p \subseteq 4+$. Yet the last results are presented by the different way: the current

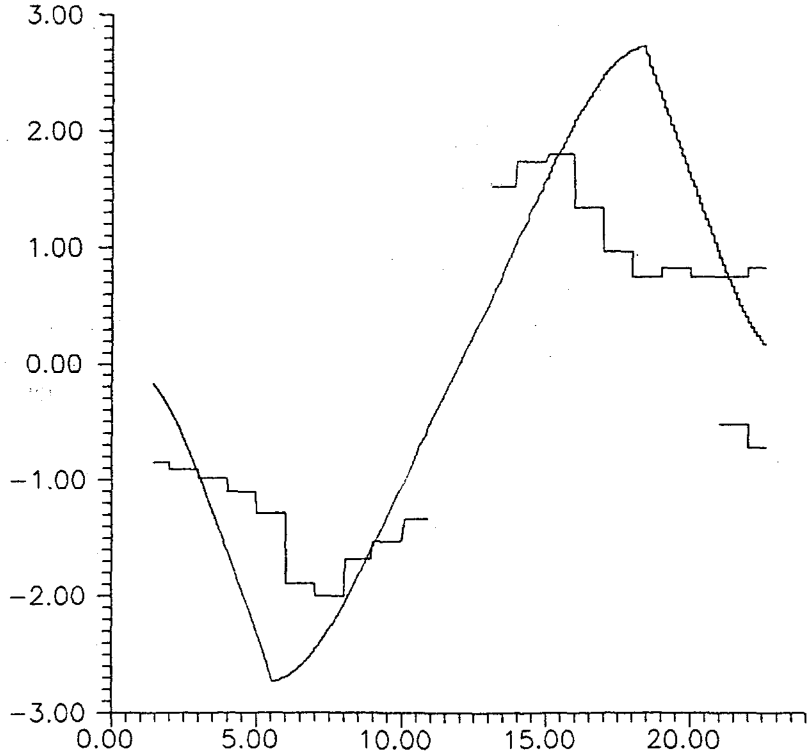

Fig. 1. The comparison of the theoretically calculated density of the field-aligned currents with the data of observations (Iijima and Potemra, 1976) in the region 1 in the dependence on the local time. The last results are presented by the different way: the current is considered to be positive if it flows from the ionosphere and is considered to be negative if it flows into the ionosphere. The local time is layed along the axis of abscissae. The density of the current in mkA m^{-2} is layed along the axis of ordinate. The unbroken line is the result of the calculations.

is considered to be positive if it flows out of the ionosphere and the current is considered to be negative if it flows into the ionosphere.

The comparison of these results shows the satisfactory consent. As it was pointed out in Samokhin (1992) on the value of the field-aligned current the component B_z of IMF can have influence owing to the origin of the normal component of the current on the outside of the magnetopause, in the magnetosheath, which closes in the ionosphere as a field-aligned current. Besides the parallel mechanism works for the B_y component but in this case the maximum of the field-aligned current exists at another local time what can explain the difference between the positions of the maxima for the calculation and for the data of observations. The supplementary calculations show that the dependence of the distribution of the plasma density on r and θ has the essential influence on the value of the field-aligned current. For some other distributions $\rho(r, \theta)$ the density of the field-aligned current at the level of the ionosphere can increase on the order of the magnitude and the sign of the normal component of the current on the magnetopause can change in the dependence on the latitude.

We suggest that the resutls of this calculation are approximately fit for the day side of the magnetosphere since on this side the magnetic field lines are quasidipolar and closed and in the layer of the antisolar convection the plasma flows fundamentally across the magnetic field. The particles oscillates between the mirror points along the magnetic field and the mean field-aligned velocity is equal to zero. On the night side both the magnetic field lines stretch and open and the plasma receives the great component of the velocity along the magnetic field. Therefore the reliability of the calculations for the night side is doubtful although the calculations are made and compared with the data of observations. This circumstance is reflected in the title of the paper.

Therefore, in the case of the ideal frozen-in magnetic field and the flow of the plasma across the magnetic field in the layer of the antisolar convection the component of the current is generated which is perpendicular to the magnetopause as the tangential discontinuity. This component of the current closes in the ionosphere at the boundaries of the layer of the anti-solar convection what produces the global system of the field-aligned currents. According to the above-mentioned calculation in the case of the dependence of the plasma density on the geocentric distance $\rho \sim r^{-3}$ this system of the currents is satisfactorily consistent on the value, on the direction and on the distribution with the local time with the data of Triad which correspond to the weekly distributed geomagnetic conditions.

References

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