FORMATION OF SATELLITE AND RING SYSTEMS : COMPARATVE ASPECTS

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Abstract. There are four systems of a massive central body with a regularily structured satellite system in the Solar system: the planetary system and the satellite systems of Jupiter, Saturn and Uranus. Comparable structures in these four systems can be understood as indications for comparable processes of origin and formation. It is the aim of this paper to describe comparable properties, and to discuss possible physical processes in pre-satellite disks which can be the cause for this comparability.

1. The Four Systems

Distances from the central body, orbital inclinations and radii of the satellites are given with Tables 1 - 4 for the satellite systems of the Sun, Jupiter, Saturn and Uranus (the data for Tables 1-5 were taken from K.R.Lang, 1992).

It is interesting to note that there are comparable subgroups in these systems. There is an inner group of small bodies at low inclination orbits, a group of the "principal satellites" (described by Table 5), containing most of the mass of the satellite system, and an outer group of smaller bodies at high inclination (including retrograde) orbits. The outer group may consist of captured (external) bodies and of bodies, belonging originally to the system and being scattered gravitationally outward from the more massive inner regions. The satellites of the inner group may have formed there, or they have been scattered gravitationally inward from the more massive region of the principal satellites.

As an additional clue to understand processes in the early solar nebula also some of the observed structures in planetary rings have to be taken into account (Lissauer, Cuzzi,1985).

Throughout this paper, the principal satellites and their comparable properties shall be discussed mainly. They contain most of the mass of the satellite systems, and their formation was the key process in the early evolution of these systems.

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TABLE 1
Jovian Satellite System

Satellite	Distance from Planet Center (10^6 m)	Inclination (degrees)	Radius (10^3m)
J14 Adrastea	128	ª0	20±5
J16 Metis	128	ª0	20±5
J5 Amalthea	181	0.4	135×85×75
J15 Thebe	221	ª0	40±5
J1 Io	422	0.0	1815
J2 Europa	671	0.5	1569
J3 Ganymede	1 070	0.2	2631
J4 Callisto	1 880	0.2	2400
J13 Leda	11 110	26.7	ª5
J6 Himalia	11 470	27.6	90±10
J10 Lysithea	11 710	29.0	ª10
J7 Elara	11 740	24.8	40±5
J12 Ananke	20 700	147	ª10
J11 Carme	22 350	164	ª15
J8 Pasiphae	23 300	145	ª20
J9 Sinope	23 700	153	ª15

TABLE 2
Saturnian Satellite System

Satellite	Distance from Planet Center (10^6m)	Inclination (degrees)	Radius (10^3m)
S17 Atlas	137.7	ª0	20×10
S16 Prometheus	139.4	ª0	70×50×40
S15 Pandora	141.7	ª0	55×45×35
S10 Janus	151.4	ª0	110×100×80
S11 Epimetheus	151.5	ª0	70×60×50
S1 Mimas	186	1.5	196
S2 Enceladus	238	0.0	250
S3 Tethys	295	1.1	530
S13 Telesto	295		17×14×13
S14 Calypso	295		17×11×11
S4 Dione	377	0.0	560
S12 Helena	377	0.2	18×16×15
S5 Rhea	527	0.4	765
S6 Titan	1 222	0.3	2 575
S7 Hyperion	1 481	0.4	205¥130¥110
S8 Iapetus	3 561	14.7	730
S9 Phoebe	12 954	150	110±10

TABLE 3
Uranian Satellite System

Satellite	Distance from Planet Center (10^6m)	Radius (10^3m)
U13 Cordelia	49.7	ª20
U14 Ophelia	53.8	ª25
U15 Bianca	59.2	ª25
U9 Cressida	61.8	ª30
U12 Desdemona	62.7	ª40
U8 Juliet	64.6	ª40
U7 Portia	66.1	ª40
U10 Rosalind	69.9	ª30
U11 Belinda	75.3	ª30
U6 Puck	86.0	85±5
U5 Miranda	129.9	236
U1 Ariel	190.9	579
U2 Umbriel	266.0	586
U3 Titania	436.3	790
U4 Oberon	583.4	762

Note that only Miranda has a notable inclination of 3.4°.

TABLE 4
The Solar System

Planet	Distance from the Sun (10^{11}m)	Inclination	Equatorial Radius (10^6m)
Mercury	0.579	7.0	2.439
Venus	1.082	3.39	6.051
Earth	1.496	0	6.378
Mars	2.279	1.85	3.397
Jupiter	7.783	1.31	71.492
Saturn	14.270	2.49	60.268
Uranus	28.696	0.77	25.559
Neptune	44.966	1.77	24.764
Pluto/Charon	59.00	17.15	1.123/0.56

Note that the satellites of Neptune were not taken into account. This system seems to have been modified essentially toward non-regular structures by processes, related to the probable capture of Triton.

2. Angular Momenta and Radial Extension

There is a remarkable discrepancy between the above mentioned three planetary satellite systems and the planetary system in the distribution of angular momenta beween the spin of the central body and the orbital angular momentum of the satellite systems.

While most of the angular momentum of the Solar system is in the orbital motion of the giant planets, the spin of the central mass dominates the angular momenta in the three planetary satellite systems.

Table 6 describes this more in detail. This difference in dynamic properties is sometimes interpreted as an indication for different origin and formation of these systems, and as a hint for a Laplace-type formation of the planetary

TABLE 5
Principal Satellites

Satellite	Distance (10^6m)	Radius (10^3m)	Mass (10^{20}kg)	Density (10^3kg/m^3)
		Jupiter		
J1 Io	422	1 815	892	3.55
J2 Europa	671	1 569	487	3.04
J3 Ganymede	1 070	2 631	1 490	1.93
J4 Callisto	1 880	2 400	1 075	1.83
		Saturn		
S1 Mimas	186	196	0.455	1.44
S2 Enceladus	238	250	0.74	1.13
S3 Tethys	295	530	7.55	1.20
S4 Dione	377	560	10.52	1.41
S5 Rhea	527	765	24.9	1.33
S6 Titan	1 222	2 575	1 346	1.88
(Iapetus	3 561	730	18.8	1.15)
		Uranus		
U5 Miranda	129.9	236	0.8	1.25
U1 Ariel	190.9	579	13.5	1.55
U2 Umbriel	266.0	586	12.7	1.58
U3 Titania	436.3	790	34.8	1.69
U4 Oberon	583.4	762	29.2	1.64

satellite systems from that mass, which was left behind from the rotationally unstable forming central bodies.

That this is not necessarily the case can be seen from the radially restricted "sphere of influence" of these planets. In other words, these satellite systems are restricted in their radial extension by the disturbing influence of the gravitation of the Sun. The scales of these limitations in extension of the planetary satellite systems are given in Table 7, indicating that bodies in the outer subgroup of satellites may be influenced over longer timescales by "disturbing" solar gravitation.

TABLE 6
Angular momenta

	Orbital momentum L of the satellite system ($kg\ m^2/s$)	Spin S of the central mass ($kg\ m^2/s$)	ratio L/S
Jupiter	$4.50\ 10^{36}$	$6.72\ 10^{38}$	$6.7\ 10^{-3}$
Saturn	$9.57\ 10^{35}$	$8.72\ 10^{37}$	$1.1\ 10^{-2}$
Uranus	$1.40\ 10^{34}$	$2.0\ 10^{36}$	$7\ 10^{-3}$
Sun	$3.15\ 10^{43}$	$1.7\ 10^{41}$	185.3

Consequently, the orbital angular momentum of the planetary satellites, which is proportional to the square root of the orbital radius, is restricted by the limited extension of stable regions around the planets. This constraint did not exist during the formation of the Solar system. The difference in angular momenta distribution is therefore not necessarily connected with a different origin of these systems.

TABLE 7
Spheres of Influence

$$\text{Hill sphere} \quad r_H = R_{orb}\left(\frac{m_{Pl}}{M_{Sun}}\right)^{1/3}$$

Sphere of gravitational equilibrium

$$r_E = R_{orb}\left(\frac{m_{Pl}}{M_{Sun}}\right)^{1/2}$$

Planet	r_H (m)	r_E (m)
Mercury	$3.19\ 10^{8}$	$2.37\ 10^{7}$
Venus	$1.46\ 10^{9}$	$1.69\ 10^{8}$
Earth	$2.16\ 10^{9}$	$2.59\ 10^{8}$
Mars	$1.56\ 10^{9}$	$1.09\ 10^{8}$
Jupiter	$7.66\ 10^{10}$	$2.40\ 10^{10}$
Saturn	$9.42\ 10^{10}$	$2.42\ 10^{10}$
Uranus	$1.01\ 10^{11}$	$1.90\ 10^{10}$
Neptune	$1.67\ 10^{11}$	$3.22\ 10^{10}$

3. Vertical Disk Structure

The equilibrium in a circumstellar or circum-protoplanetary disk is governed by thermal pressure of the disk and gravitation of the central mass. For the vertical "z-"components this can be described by

$$\frac{1}{\rho}\frac{dp}{dz} = -\gamma\frac{M_c}{r^2}\frac{z}{r} \qquad (1)$$

where M_c is the central mass, r the radial distance from the central body, and p and r are pressure and mass density, respectively. They can be related by an equation of state $p=\rho c^2$ with "c" as a velocity of sound.

The solution is

$$\rho = \rho_0 \exp\{-\frac{z^2}{2H^2}\} \tag{2}$$

where ρ_0 is the midplane mass density. The "scale height" H is given by

$$H = \frac{c}{\Omega_K} \tag{3}$$

and Ω_K is the Keplerian angular velocity

$$\Omega_K = \gamma \frac{M_c}{r^3} \tag{4}$$

with the constant of gravitation γ. Consequently, the ratio of scale height and radial distance is given by

$$\frac{H}{r} = \frac{c}{\Omega_K r} = \frac{c}{v_\varphi} \tag{5}$$

where ϖ_φ is the Keplerian azimutal orbital velocity.

Fig. 1 gives this ratio for the four different systems under consideration.

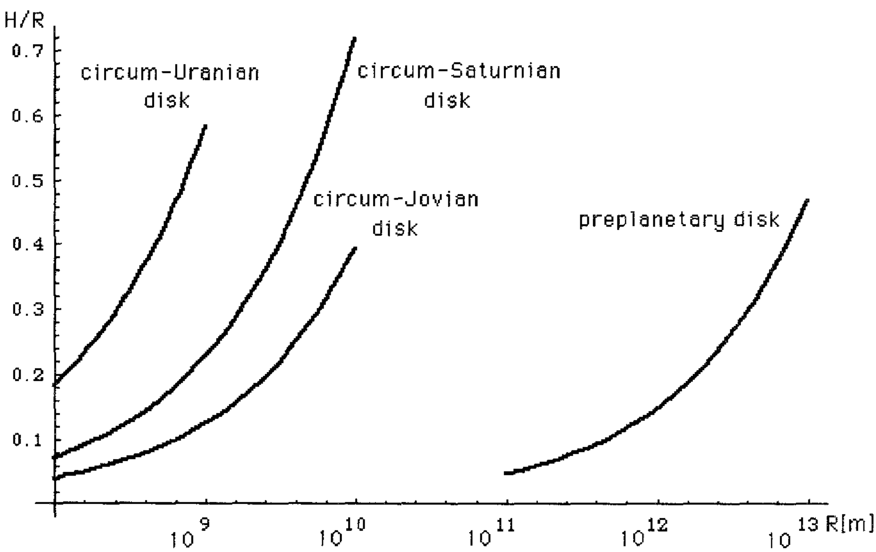


Fig. 1 Relative disk scale heights

Obviously, disk scale heights are comparable in these systems, they are of the order of a few tenths of the radial distance. For further computations, a value of H=0.1 r will be used throughout this paper. The corresponding volume of a disk is then $V=\pi R^2 H \cong 0.3\ R^3$, where R is the radial extension of the disk.

4. Gravitational Stability of Disks

It has been discussed intensively in the literature (Cameron,1978) that the self-gravitation of a preplanetary or pre-satellite disk might be able to cause local gravitational instabilities which might be trigger mechanisms for the growth of solid bodies in the km- to 10 km scale, called "planetesimals". It can be shown from corresponding dispersion relations, derived by linearized perturbation theory, that a condition for self-gravitation to overcome the disrupting action of the Keplerian shear motion, described by the Keplerian angular velocity, is given by

$$\omega_g^2 >> \Omega_K^2 \tag{6}$$

where the frequency for effects due to disk self-gravitation is given by $\omega_g^2 = 4\pi\gamma\rho$ with the disk mass density ρ. With a disk mass m_d, a density $\rho=m_d/V$ and the above given disk volume $V=0.3\ R^3$ it follows as a necessary condition for self-gravitation to overcome disrupting effects of the gravitation of the central body that

$$\frac{m_d}{M_c} \geq \frac{1}{40} \tag{7}$$

It can be seen from Table 8 that this condition is not fulfilled in the four systems under consideration.

Table 8
Disk/Central body mass ratio

System	m_d/M_c
Sun	$1.34\ 10^{-3}$
Jupiter	$2.08\ 10^{-4}$
Saturn	$2.48\ 10^{-4}$
Uranus	$1.05\ 10^{-4}$

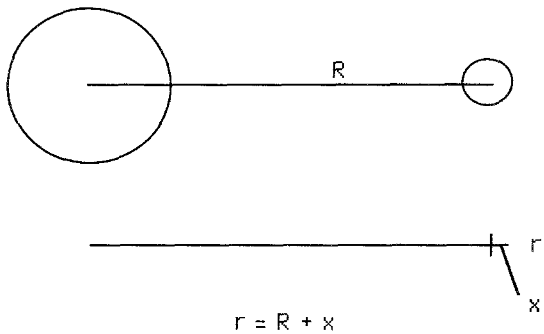
Consequently, self-gravitation was not essential as a large scale structuring process or a trigger mechanism for planetesimal formation in the presatellite disks under consideration, or, most of the original mass of the disk was lost after the formation of solid bodies and satellites.

5. Tidal Forces

Tidal forces can be essential for the evolution of disks. It can be seen from Table 9 that these forces can have been much more effective in the small presatellte disks than in the planetary system. The consequences of the action of tidal forces are at one side the increased potential for destruction of local structures in the disk, and on the other side, an increased outward angular momentum transport. These processes can be essential for the evolution of a disk above a central mass. They can be a cause for the different properties of the preplanetary and the presatellite disks.

Table 9
Tidal forces

$$f = -\gamma \frac{m\,M_c}{r^2} + \Omega_K^2\, r = -\gamma \frac{mM_c}{r^2} + \gamma \frac{mM_c}{R^3}\, r$$



$$f(x) = 2\gamma \frac{mM_c}{R^3}\, x + \gamma \frac{mM_c}{R^3}\, x = 3\,\gamma \frac{mM_c}{R^3}\, x$$

System	$3gM_c/R^3$
Sun	$1.18\ 10^{-13}$ (Earth)
Jupiter	$3.80\ 10^{-10}$ (10^9m)
Saturn	$1.14\ 10^{-10}$ (10^9m)
Uranus	$1.40\ 10^{-10}$ ($5\ 10^8$m)

6. Compositional Gradients

The composition of the planets in the Solar systems follows a clear radial gradient with refractory matter of higher density in the inner parts and volatiles and condensates in the outer parts of the system. The inner planets are stony with a high iron content, while the giant planets are gas-dominated and the outer planets seem to "ice-planets". This "composition-gradient" is related to the temperature in the preplanetary disk, and insofar, a similar picture can be estimated to exist in the planetary satellite systems. Table 10 describes the composition in these systems.

A trend, similar to that in the planetary system can be found again in the Jovian system, but not in the others, indicating also for the Jovian system a high temperature inner disk. This leads to a very interesting conclusion and question. Are the satellites of the inner group of the Jovian satellites made of refractories? This should be assumed in analogy to the terrestrial planets. A positive answer would be a verification of the theoretical approach of hot inner disks. On the other hand, there is a further question. Why is there no compositional gradient in the satellite systems of Saturn and Uranus? Was the luminosity of these protoplanets not sufficient to heat up the disks sufficiently? This seems to be a challenge for future theoretical approaches.

Table 10
Collisional gradients

Satellite	Density ($10^3 kg/m^3$)	Compositional Features
Io	3.55	Rock
Europa	3.04	Rock+100km H_2O-layer
Ganymede	1.93	60% rock , 40% ice
Callisto	1.83	60% rock , 40% ice
Mimas	1.44	~40% rock, 60% ice
Enceladus	1.13	"
Tethys	1.20	"
Dione	1.41	"
Rhea	1.33	"
Titan	1.88	60% rock, 40% ice+N_2, CH_4
Miranda	1.25	icy ?
Ariel	1.55	icy ?
Umbriel	1.58	icy ?
Titania	1.69	Rock-ice ?
Oberon	1.64	Rock-ice ?

7. Disk Temperatures

Energy sources for preplanetary and pre-satellite disks are the friction caused dissipation in the Keplerian shear motion and the heating from the growing protosun or protoplanet. At the outer edge of the disk, energy is lost by radiation, as described by the Stefan-Boltzmann law with the radiation constant σ.

Neglecting any additional internal heating, caused by the opacity of disk matter, the disk temperature T can be estimated by equating the radiative energy loss by the dissipation generated heat, which is proportional to the kinematic viscosity ν :

$$\frac{9}{4}\nu\,\Omega_K^2 = \frac{2}{\Sigma}\sigma T^4 \qquad (7)$$

where Σ is the surface mass density of the disk.

Estimating the viscosity from the friction caused inward motion v_r of the disk matter via v_r=-3ν/2r and using the equation of continuity in the form $2\pi r\Sigma(-v_r) = \dot{M}$, there follows

$$T^4 = (3\gamma / 8\pi\sigma)M\dot{M} / r^3 \qquad (8)$$

where $\dot{M}$ is the growth rate of the central body. Note that additional heating by the luminosity of the central body and heating due to opacity were not taken into account. These should have been essential additional heat sources, at least in the case of the Solar system and the Jovian system. Values, following with $\dot{M}=M_c/10^6$years are given in Fig. 2 (with T_e=T). For more details, the reader is referred to Lin and Papaloizou (1985).

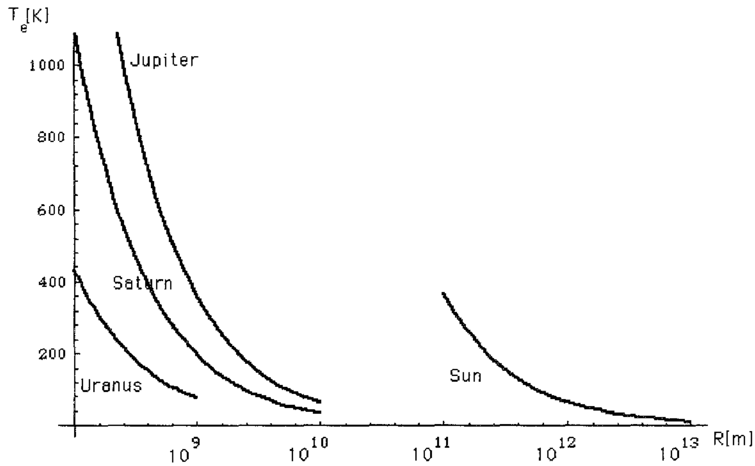


Figure 2. Disk Temperatures

It is an interesting result of Fig. 2 that the temperatures in the disks of the four systems under consideration are in comparable ranges.

8. Disk Surface Mass Densities

A key parameter to understand disks around massive cental bodies is the (height integrated) surface mass density. Using the value of $\zeta = m_{dust}/m_{gas} = 0.0034$, as it was proposed by Podolak and Cameron, there follows m_{disk}ª$300 m_{dust}$ª$300 m_{sat}$, where m_{sat} is the mass of the present satellite system. So, and in the sense of an estimation, the pre-satellite disk surface mass density can be related to known values via $\Sigma = m_{disk}/pr^2$. Table 11 gives the corresponding values for the four systems under consideration.

Here, the midplane central pressure is mentioned too, as it follows from $p = \Sigma \, \Omega_K c / 4$. It is interesting to note that the pressure in the pre-satellite disks is much higher than in the preplanetary disk. This might have been an essential difference between these disks (note that fluid phases seem to be possible under these conditions; This might be essential for growth processes). To estimate the relative importance of self-gravitation, as discussed in Chapter 4, the ratio of the two characteristic frequencies is given too with Table 11. It can be seen, that self-gravitation was too weak to be a dominating process.

Table 11
Physical parameters

System	Surface mass density [kg/m^2]	Mass density [kg/m^3]	Central pressure [mbar]	w_g^2/W_K^2
Sun (<4.5 10^{12}m) (at Jupiter)	1.25 10^4	8.03 10^{-8}	8.93 10^{-4}	0.24
Jupiter (Europa)	1.04 10^7	7.7 10^{-2}	7.32 10^2	0.15
Saturn (Rhea)	7.86 10^6	7.5 10^{-2}	4.43 10^2	0.24
Uranus (Umbriel)	2.42 10^6	4.5 10^{-2}	1.49 10^2	0.12

It can be seen from Table 11 that the mass densities in the pre-satellite systems were greater than in the preplanetary disk. But it has to be noted, that all these estimations are based on the model of a more or less stable disk. Very probably, these disks have to be seen also as a transient phenomenon, governed in its time scales by internal dissipation, solidification and clearing and by the inflow of fresh matter. Therefore, the timescales for the evolution of these disks have to be studied.

9. Sedimentation and Particle Growth

The frictional force between gas and a dust particle in a gas with a free mean path larger than the particle size is given by the Epstein law, describing the frictional acceleration as

$$a = \frac{\rho_g}{\rho_p}\frac{\overline{v}}{r_p}\Delta v \tag{9}$$

where ρ_p and ρ_g are the mass densities of the gas and the dust particle, $\overline{v}$ is the mean thermal speed of the gas particles, ρ_p is the radius of the dust particle and Δv is the velocity of the gas relative to the dust particle. This leads to an equation of (vertical) motion

$$\ddot{z} = -\frac{\rho_g}{\rho_p}\frac{\overline{v}}{r_p(t)}\dot{z} - \Omega_K^2 z \tag{10}$$

where the growth of the dust particle by colliding and sticking (sticking probability w_{stick}) can be described by

$$\frac{dr_p}{dt} = \frac{w_{stick}}{4}\frac{\rho_{dust}}{\rho_p}|\dot{z}| \tag{11}$$

This gives

$$r = r_0 + \left(1 - \frac{z}{z_0}\right)\frac{w_{stick}\Sigma}{8\rho_p} \tag{12}$$

for a particle, starting with a radius $r(0)=r_0$ at the scale height $z(0)=z_0=H$. Here, $\Sigma=2H\rho$ has been used. It has been shown by Weidenschilling that dust particles can grow by collisional aggregation towards the 10cm to meter scales in size (Weidenschilling and Cuzzi, 1993), if not only vertical settling but also collisional growth during inward motions in the midplane are taken into account. But it has to be mentioned here that the sticking probability is a yet unknown parameter in these calculations. The assumption $w_{stick}\approx 1$ is probabely far from reality. Laboratory experiments are necessary to get more reliable parameters for these interactions.

According to Hayashi et al. (1985) the corresponding sedimentation time for vertical settling can be derived from the above given equations, leading to

$$t_{sed} = t_K \frac{4}{\pi^{3/2}} \frac{1}{w_{stick}\varsigma(1+\frac{8\rho_p}{w_{stick}\Sigma})r_0} \ln\frac{z_0 r}{z r_0} \tag{13}$$

where t_K is the Keplerian orbital period. Using the above given parameters for the planetary and the satellite systems, it can be derived that the typical sedimentation time is of the order of some 10^3 years for the planetary system, but it is only of the order of about 50 years for the satellite systems. The small sedimentation time in these systems is caused mainly by the assumed higher densities and the smaller scales in these systems. This time scale would be prolonged, if not a stable and more dense but a transient and thinner disk is assumed for these systems. The characteristic life-time of these transient pre-satellite disks is determined then by that of the preplanetary disk, feeding the pre-satellite disks. But, in every case, there forms a thin midplane-subdisk of dust. The above mentioned processes of self-gravitation may have become effective in the small scales in these dust subdisks, supporting the growth of larger bodies from the dust.

It is interesting to note in this context that growth processes are limited by the tidal action of the central body within the so-called "Roch-limit". Here, only small bodies can form and survive. Their interaction and erosion is probably an essential cause of the ring phenomenon, observed in the planetary satellite systems. The other source of single ring phenomena are probably decay processes of satellites. For more details, related to the origin of planetary rings the interested reader is referred to Harris (1984).

10. Collisional accretion towards larger bodies

Following the original ideas of Schmidt and Safronov (Safronov, 1972), the growth of larger bodies via the phase of km-sized planetesimals is assumed to be dominated by collisional accretion. The time scale of these processes can be estimated from the inverse of the collision frequency of colliding particles of cross section Q, number density N and average velocity $\overline{v}$

$$t_{acc} = \frac{1}{NQ\overline{v}} \tag{14}$$

With $H = \overline{v} / \Omega_K$, $\Sigma = 2H\rho$ and

$$N = \frac{\rho}{m} = \frac{\Sigma}{2mH} = \frac{\Sigma\Omega_K}{2m\overline{v}} \tag{15}$$

there follows for the characteristic time of accretional growth towards a planet of density ρ_{Pl} and Radius R_{Pl}

$$t_{acc} = \frac{4\rho_{Pl} R_{Pl}}{3\pi \Sigma} t_K \tag{16}$$

To form a body of 1000km in radius in the inner planetary system, a time of about 10^7 years seems to be typical, but problems appear in the outer parts of the system, where this timescale increases over the age of the Solar system. Some processes of equipartitioning of energy between the larger and the smaller bodies have been discussed, and it has been shown by Wetherill (1991) that a runaway growth is able to shorten the longer time scales.

For the satellite systems, the above derived timescale is drastically shorter, it ranges between some hundred and thousand years (if the above given values for pre-satellite disks with relatively high density are used). So, collisional accretion can be assumed to be the basic growth mechanism for the larger bodies in all the four systems under consideration.

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