ON THE ROTATION OF RIGID VENUS

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Abstract. According to A.A. Khentov Venus' rotation is in the quasi-stationary state as a result of the balance interaction of the solar tidal torque with the aerodynamical torque of the rotating Venus' atmosphere. In case of the nonconservative forces are negligible and the solar attraction is the stabilizing factor, the rotation of the rigid Venus may be assumed as the first approximation. The theory of the rotation of the rigid Venus in the coordinates μ, ν, π had been constructed. It have been found that Venus rotates almost uniformly and the libration harmonics are negligible.

Key words: Venus' rotation, libration

1. Introduction

One of the important results of Venus' radar ranging observations was the discovery of its retrograde rotation. Earth-based radar observations used to perform near an inferior conjunction of the planet. It was discovered from the outset that during the inferior conjunctions, the same area of Venus' surface is observed. It was also found that the sidereal period of Venus' rotation, which is equal to 241.01 days, very close to the 243.167 days period, so at every conjunction the same Venus' meridian is nearly directed to the Earth's mass center.

Goldreich and Peale (1968) developed the second kind resonance rotation theory for the case when apart from the Sun other planet (the Earth) exerts the influence on the rotation of the planet (Venus). Later on the resonance rotation theory was developed by Beletskij and his collaborators (Beletskij, 1975), (Beletskij *et al.*, 1980). They have found that although the second kind resonance is possible, but the probability of the capture of Venus to the retrograde rotation synchronized with the orbital motion of the Earth is negligible small. Barkin (1987) has found that in case of unrestricted three-body problem there are periodic solutions of the first sort, describing Venus resonance motion mentioned above. In this problem the Sun and the Earth are considered as points and Venus is considered as a rigid body.

Analyzing expansion of Venus gravity potential made by Williams *et al.* (1983) Burša (1985) concluded that the Earth cannot control Venus' resonance. Zhang and Shen (1987) calculated the interaction of the torques of the body tides by the Sun and by the Earth with the atmospheric tides of Venus for non-rigid Venus model. They concluded that for some values of the $(B - A)/C$ and Q parameters the tidal torque exerted on Venus by the Earth can control Venus', even though atmospheric torques would be only a half of the Sun's torque.

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Khentov (1982, 1986, 1989) investigated evolution paths, which may lead to the resonance rotations of the planets and concluded that observed Venus' rotation cannot be considered as synchronized with the orbital motion of the Earth. So Venus is not captured into the resonance zone by tidal forces, its angular rate differs from resonance one and the observed synchronization is outgoing phenomenon. Khentov (1992) suggested a new theory of "aerodynamical" type. The action of the solar gravitational torque on the planet is trying to establish its direct rotation. But it is suppressed by the rapid retrograde rotation – the superrotation of lower atmosphere layers.

The torque stabilizing the retrograde rotation is expressed by the formulae $M_a = k(\omega_a - \omega)$, where k is the fixed coefficient, ω_a is the averaged angular velocity of the lower dense atmosphere, ω is Venus angular velocity. We observe the quasi-stationary rotation of Venus resulting from the balance interaction of tidal and aerodynamical torques.

The problem of retrograde rotation phenomenon almost coinciding with the velocity of resonance one is not a subject of this paper. We have considered the dynamics of the observed rotation motion is under assumption that the nonconservative forces are negligible, so in the first approximation the quasi-stationary rotation may be thought as the rigid rotation. Under this conditions the rotation theory of a rigid planet whose equator has a small inclination to the orbit have been constructed using μ, ν, π coordinates.

2. Observational Data. Aphroditocentric celestial sphere.

From the radar measurements we can obtain the geoequatorial coordinates of the venusian north pole (α_1, δ_1), the diurnal rotation rate (n) and the prime meridian position. According to IAU Report on Cartographic Coordinates (1991)

$$\alpha_1 = 272^\circ\!.76, \quad \delta_1 = 67^\circ\!.16 \qquad (J2000).$$

The angular distance of the prime meridian from the descending node Q of Venus' equator on the geoequator is equal to

$$w = 160^\circ\!.20 - 1^\circ\!.48813688d \qquad (J2000).$$

Knowing these fundamental values the positions of all basic points and lines of the aphroditocentric celestial sphere may be found (Fig.1). So the ecliptic longitude λ_1 and latitude β_1 of the north rotate pole are equal to

$$\lambda_1 = 30^\circ\!.19, \qquad \beta_1 = 88^\circ\!.76.$$

The longitude of the descending node K of the Venus' equator on the ecliptic is

$$\lambda_k = 120^\circ\!.19.$$

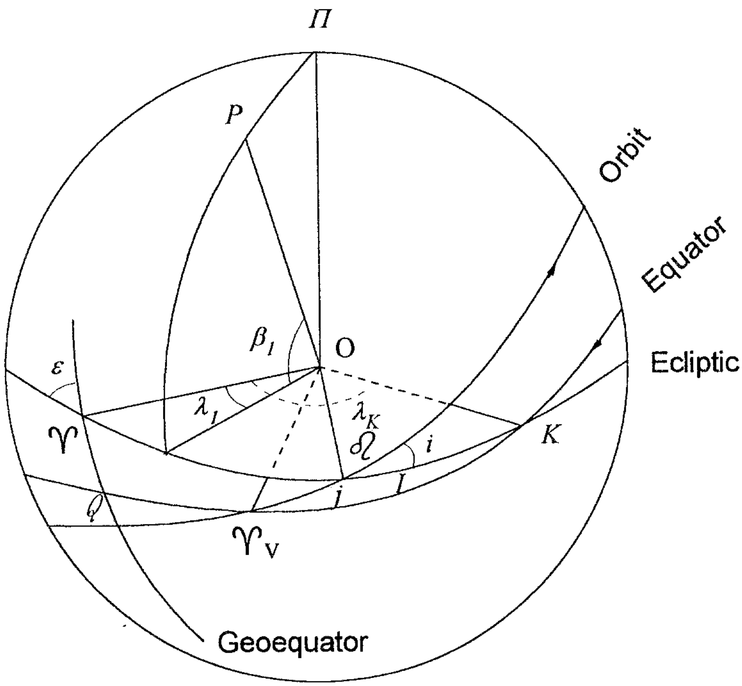

Fig. 1. Aphroditocentrical celestial sphere. Π – ecliptic north pole; ϒ – the Earth equinox; P – Venus north pole; Υ_V – Venus vernal equinox; λ_1, β_1 – ecliptic coordinates of P; Ω ascending node of the orbit on the ecliptic; j – the inclination of Venus' equator to the orbit; Q – descending node of Venus equator on the Earth equator; K – descending node of Venus equator on the ecliptic.

The inclination of the equator to the ecliptic is

$$I = -1^{\circ}.24.$$

The angle between the vernal equinox of Venus Υ_V and the ascending node of the orbit on the ecliptic Ω is

$$\smile \Omega\Upsilon_V = 18^{\circ}.89.$$

The inclination of Venus' equator to the orbit is

$$j = -2^{\circ}.64.$$

The distance of the prime meridian from the node K is

$$w_k = 104^{\circ}.92 - 1^{\circ}.4813291d.$$

And the inclination of Venus' orbit to the ecliptic is

$$i = 3^{\circ}.39.$$

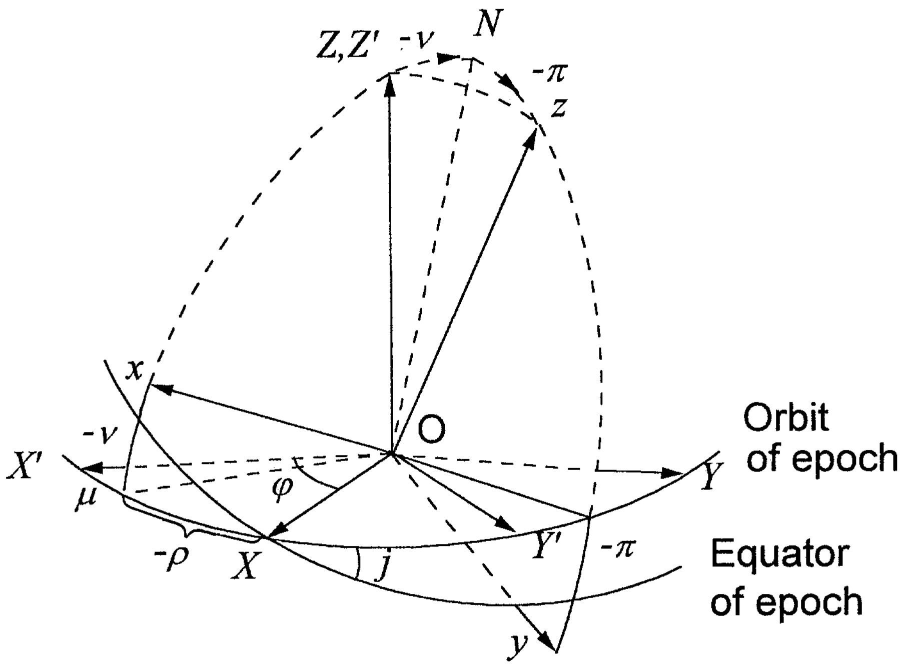

Fig. 2. Frame of reference. $OXYZ$ – inertial frame; $OX'Y'Z'$ – coordinate system rotating at the orbit plane of epoch; xyz – principal axes of the planet; $-\rho$, $-\nu$, $-\pi$ – the angles determining the position of $Oxyz$ relative to inertial system $OXYZ$; μ, $-\nu$, $-\pi$ – the angles determining the position of $Oxyz$ relative to moving system $OX'Y'Z'$.

3. Rotation of the planet with small obliquity of the equator to the orbit plane

System of reference. Let us introduce the next planetocentric rectangular coordinate systems (Fig. 2):

1. $OXYZ$ is the inertial coordinate system with axes OX and OY are at the standard epoch orbit plane of the planet. OX axis is directed to the ascending node of the orbit on the equator Υ_p. OY axis forms the right-hand system and OZ axis is directed to the north pole of the orbit.
2. $OX'Y'Z'$ is the system uniformly rotating at the orbit plane. Let us put the rate of this rotation n equal to the mean rotation rate of the planet. The rotation angle of the system would be counted inverse from the equinox point Υ_p: $\varphi = -nt + \varphi_o$.
3. $Oxyz$ is the mobile system, which is fixed in the body of the planet and coincides with the principle axes of inertia A, B, C, where $A < B < C$.

μ, ν, π - angles. The position of the $Oxyz$ frame relative to the fixed system $OXYZ$ will be determined by the next clockwise rotation angles: by $-\rho$ around OZ axis, by ν around OY axis in the new position and by π around OX in the new position again. And the position of $Oxyz$ relative to the flowing system $OX'Y'Z'$ will be determined by the angles μ, $-\nu$, $-\pi$, where $\mu = \varphi - \rho$.

We consider the case when the planets equator has a small inclination to the orbit plane. Since the rotation rate of $OX'Y'Z'$ is assigned to be equal to the mean rotation rate of the planet ($Oxyz$ triad) the angles μ, ν, π would be small. Under these conditions the transformation matrix is represented as:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} \tag{1}$$

where

$$\begin{array}{lll} \beta_{11} = 1 - \frac{1}{2}\mu^2 - \frac{1}{2}\nu^2 & \beta_{12} = \mu & \beta_{13} = \nu \\ \beta_{21} = -\mu + \nu\pi & \beta_{22} = 1 - \frac{1}{2}\nu^2 - \frac{1}{2}\pi^2 & \beta_{23} = -\pi \\ \beta_{31} = -\nu - \mu\pi & \beta_{32} = \pi - \mu\pi & \beta_{33} = 1 - \frac{1}{2}\nu^2 - \frac{1}{2}\pi^2 \end{array}$$

Uniform rotation of the rigid planet. Let us suppose that a planet (the $Oxyz$ triad) rotates uniformly around the axis of inertia C (the Oz axis). Let the rate of the rotation would be equal to n, and the inclination of the rotation axis would be equal to j. The position of the planet would be determined by the rotation angle φ around the Oz axis counted inverse from the nodes line OX (Fig.2). Then the rotation of the planet will be expressed in terms of ρ, ν, π by the equations:

$$\begin{aligned} \tan\rho &= +\cos j \tan\varphi, \\ \sin\nu &= +\sin j \sin\varphi, \\ \tan\pi &= +\tan j \cos\varphi, \\ \varphi &= -nt + \varphi_0. \end{aligned}$$

For small values of arguments we have

$$\begin{aligned} \rho &= +\varphi - j^2 \sin 2\varphi + \cdots, \\ \nu &= +j \sin\varphi + \frac{1}{8} j^3 \sin\varphi + \frac{1}{24} j^3 \sin 3\varphi + \cdots, \\ \pi &= +j \cos\varphi - \frac{1}{4} j^3 \cos\varphi + \frac{1}{12} j^3 \cos 3\varphi + \cdots. \end{aligned} \tag{2}$$

Note that in general case of rotation there is

$$\mu(t) = \frac{1}{4} j^2 \sin 2\varphi + \tau(t),$$

where $\tau(t)$ is the libration in longitude .

Kinematic Euler equations. In the $Oxyz$ frame we can write the Euler equations in terms of $-\rho, \nu, \pi$ angles in the form:

$$\begin{aligned} \omega_x &= -\dot\rho \sin\nu - \dot\pi, \\ \omega_y &= \dot\rho \cos\nu \sin\pi - \dot\nu \cos\pi, \\ \omega_z &= -\dot\rho \cos\nu \cos\pi - \dot\nu \sin\pi. \end{aligned} \tag{3}$$

And in the fixed system $OXYZ$ the same equations will have the form

$$\begin{aligned}
\omega_X &= -\dot{\nu}\sin\rho - \dot{\pi}\cos\rho\cos\nu, \\
\omega_Y &= \dot{\nu}\cos\rho + \dot{\pi}\sin\rho\cos\nu, \\
\omega_Z &= -\dot{\rho} - \dot{\pi}\sin\nu.
\end{aligned} \tag{4}$$

Canonical variables. We will solve the problem using the canonical variables. We take the angles ρ, ν, π as the generalized coordinates and P_ρ, P_ν, P_π as canonical impulses, which together forms the system of the canonical variables. The formulas for the impulses are written in the form:

$$\begin{aligned}
P_\rho &= A\omega_x\sin\nu - B\omega_y\cos\nu\sin\pi - C\omega_z\cos\nu\cos\pi, \\
P_\nu &= B\omega_y\cos\pi - C\omega_z\sin\pi, \\
P_\pi &= A\omega_x.
\end{aligned} \tag{5}$$

In order to construct the Hamiltonian it is necessary to represent the kinetic energy T and the force function U in terms of canonical variables.

Kinetic energy. From the system of equations (5) we can define $\omega_x, \omega_y, \omega_z$ and substitute them into the expression for the kinetic energy

$$2T = A\omega_x^2 + B\omega_y^2 + C\omega_z^2,$$

So we obtain

$$\begin{aligned}
2T = &\left(\frac{1}{B} - \frac{1}{C}\right) P_\rho^2 \sec^2\nu\sin^2\pi + \frac{1}{C}P_\rho^2\sec^2\nu + \left(\frac{1}{B} - \frac{1}{C}\right) P_\nu^2\cos^2\pi + \\
&+\frac{1}{C}P_\nu^2 + \left(\frac{1}{B} - \frac{1}{C}\right) P_\pi^2\sin^2\pi\tan^2\nu + \frac{1}{C}P_\pi^2\tan^2\nu \\
&-\left(\frac{1}{B} - \frac{1}{C}\right) P_\rho P_\nu\sec\nu\sin 2\pi - 2\left(\frac{1}{B} - \frac{1}{C}\right) P_\rho P_\pi\tan\nu\sec\nu\sin^2\pi \\
&-2\frac{1}{C}P_\rho P_\pi\tan\nu\sec\nu + \left(\frac{1}{B} - \frac{1}{C}\right) P_\nu P_\pi\tan\nu\sin 2\pi + \frac{1}{A}P_\pi^2
\end{aligned}$$

Let us expand the right part of this equality as the power series in ν, π, P_ν, P_π and retain the next terms:

$$T = \frac{P_\rho^2}{2C} + \frac{P_\nu^2}{2B} + \frac{P_\pi^2}{2A} + \frac{P_\rho^2}{2C}\nu^2 - \frac{P_\rho}{C}\nu P_\pi + \kappa_1\frac{P_\rho^2}{2C}\pi^2 - \kappa_1\frac{P_\rho}{C}P_\nu\pi + \cdots,$$

where $\kappa_1 = (C - B)/B$.

Note that the function T does not depends on ρ and as will be seen below the force function does not depends on P_ρ, P_ν, P_π. Thus the Hamiltonian structure takes the form

$$H = T(\nu, \pi, P_\rho, P_\nu, P_\pi) - U(\rho, \nu, \pi, t). \tag{6}$$

With the canonical transformation of variables ρ, P_ρ to μ, P_μ by the formulas

$$\rho = -\mu + nt, \quad P_\rho = P_\mu - Cn$$

we obtain

$$\begin{aligned} H &= \frac{P_\mu^2}{2C} + \frac{1}{2}\left[\frac{P_\nu^2}{B} + \frac{P_\pi^2}{A} + Cn^2\nu^2 + \kappa_1 Cn^2\pi^2 + 2n\nu P_\pi + 2\kappa_1 n\pi P_\nu\right] \\ &\quad - U(\mu, \nu, \pi, t) \end{aligned} \tag{7}$$

Force function. The force function of the gravitational interaction between the planet and the Sun may be written in the form:

$$U = -\left(\frac{r}{a}\right)^{-3}(m_1 u_2^2 + m_2 u_3^2). \tag{8}$$

Here

$$m_1 = \frac{3}{2}\frac{GM_\odot}{a^3}(B - A), \quad m_2 = \frac{3}{2}\frac{GM_\odot}{a^3}(C - A), \quad m_3 = \frac{3}{2}\frac{GM_\odot}{a^3}(C - B),$$

$M_\odot$ is the mass of the Sun, r is the planet's radius-vector, G is the universal gravitational constant, a is the mean distance, u_2, u_3 are the direction cosines of the Sun's radius-vector relative to the Oy and Oz axes.

Planetocentric coordinates of the Sun relative to the rotating system $X'Y'Z'$ are:

$$X' = -r\cos(\psi + v), \quad Y' = -r\sin(\psi + v), \quad Z' = 0, \tag{9}$$

where v is the true anomaly, $\psi = \varphi + \Delta + \omega$, ω is the angular distance of the perihelion from the node Ω, $\Delta = (\Omega - \Upsilon_p)$ is the angular distance between the orbit nodes on the ecliptic and on the equator, $\varphi = -nt + \varphi_0$ is the rotation angle of the $OX'Y'Z'$ system.

The direction cosines of the Sun relative to the $Oxyz$ triad from the transformation (1) are:

$$u_1 = \frac{1}{r}(\beta_{11}X' + \beta_{12}Y'), \quad u_2 = \frac{1}{r}(\beta_{21}X' + \beta_{22}Y'),$$

$$u_3 = \frac{1}{r}(\beta_{31}X' + \beta_{32}Y').$$

Substituting last expressions to (8) we obtain:

$$\begin{aligned} U &= -\left(\frac{r}{a}\right)^{-3}[-\mu m_1 \sin(2\psi + 2v) + \mu^2 m_1 \cos(2\psi + 2v) + \\ &\quad + \nu^2 m_2 \cos^2(\psi + v) + + \pi^2 m_3 \sin^2(\psi + 2v) + \\ &\quad + \nu\pi m_3 \sin(2\psi + 2v)]. \end{aligned} \tag{10}$$

It is easy to expand the functions

$$\left(\frac{r}{a}\right)^{-3} \frac{\sin}{\cos}(2\varphi + 2v) \quad \text{and} \quad \left(\frac{r}{a}\right)^{-3} \frac{\sin^2}{\cos^2}(\varphi + v)$$

as the following trigonometric series with arguments multiplied of the mean anomaly g, i.e.:

$$\begin{aligned}\left(\frac{r}{a}\right)^{n} \frac{\sin}{\cos}(\beta + mv) &= \sum_{i=1} \frac{1}{2}(C_i^{n,m} + S_i^{n,m}) \frac{\sin}{\cos}(\beta + ig) + \\ & \sum_{i=1} \frac{1}{2}(C_i^{n,m} - S_i^{n,m}) \frac{\sin}{\cos}(\beta - ig),\end{aligned}$$

where coefficients $C_i^{n,m}$, $S_i^{n,m}$ are depends from the orbital eccentricity e. As a result of the expansion the next series has been derived:

$$\begin{aligned}\left(\frac{r}{a}\right)^{-3} \sin(2\psi + 2v) &= \sum_{i=1} a_i \sin(2\psi + ig), \\ \left(\frac{r}{a}\right)^{-3} \cos(2\psi + 2v) &= \sum_{i=1} a_i \cos(2\psi + ig), \\ \left(\frac{r}{a}\right)^{-3} \sin^2(\psi + v) &= -\frac{1}{2}\sum_{i=1} a_i \cos(2\psi + ig) + \frac{1}{2}\sum_{i=1} b_i \cos(ig), \\ \left(\frac{r}{a}\right)^{-3} \cos^2(\psi + v) &= \frac{1}{2}\sum_{i=1} a_i \cos(2\psi + ig) + \frac{1}{2}\sum_{i=1} b_i \cos(ig).\end{aligned} \tag{11}$$

which is necessary to substituted to (10).

Hamiltonian. After all substitutions the function (7) may be written in an expanded form:

$$\begin{aligned} H = & \frac{P_\mu^2}{2C} - \mu m_1 \sum_{i=1} a_i \sin(2\psi + ig) + \mu^2 m_1 \sum_{i=1} a_i \cos(2\psi + ig) + \\ & + \frac{1}{2}\left[\frac{P_\nu^2}{B} + \frac{P_\pi^2}{A} + Cn^2\nu + \kappa_1 Cn^2\pi^2 + 2n\nu P_\pi + 2\kappa_1 n\pi P_\nu\right] + \\ & + \frac{1}{2}\nu^2 m_2 \left(\sum_{i=0} b_i \cos ig + \sum_{i=0} a_i \cos(2\psi + ig)\right) + \\ & + \frac{1}{2}\pi^2 m_3 \left(\sum_{i=0} b_i \cos ig - \sum_{i=1} a_i \cos(2\psi + ig)\right) + \\ & + \nu\pi m_3 \sum_{i=1} a_i \sin(2\psi + ig) + \cdots \end{aligned} \tag{12}$$

The obtained expression represents the first approximation in the expansion of the Hamiltonian (6). In this approximation the Hamiltonian splits at two independent parts:

$$H = F(\mu, P_\mu) + \Phi(\nu, \pi, P_\nu, P_\pi)$$

Hence the canonical equations which are constructed using these parts will be solved independently of one another.

μ coordinate. With the Hamiltonian

$$F(\mu, P_\mu) = \frac{P_\mu^2}{2C} - \mu m_1 \sum a_i \sin(2\psi + ig) + \mu^2 m_1 \sum a_i \cos(2\psi + ig)$$

we can built the equations of the second order:

$$\frac{d^2\mu}{dt^2} = \frac{m_1}{C} \sum a_i \sin(2\psi + ig) + -2\mu \frac{m_1}{C} \sum a_i \cos(2\psi + ig).$$

In general case these equations may be used as the generalized Hill's equations. In the planet problem the factor m_1/C is represented as the value of order 10^{-6} – 10^{-8} and as result the second term in the right hand of the equation gives the negligible contribution to the solution. Thus it is quite sufficient to integrate the equation in the linear approximation, i.e.:

$$\mu = (c_1 t + c_2) + \frac{m_1}{C} \sum \frac{a_i}{(2\dot{\psi} + i\dot{g})^2} \sin(2\psi + ig) \tag{13}$$

ν and π coordinates. The solutions for unknowns ν and π we shell seek by the construction of an asymptotic series by the Deprit-Hori method (Deprit, 1969). To do this a number of canonical transformations of the Hamiltonian $\Phi(\nu, \pi, P_\nu, P_\pi)$ are required.

At first we transform (ν, P_ν, π, P_π) to (x_2, X_2, x_3, X_3) by the formulas

$$\begin{aligned} \nu &= g_{22}X_2 + g_{23}X_3, \quad P_\nu = h_{22}x_2, \\ \pi &= g_{32}x_2 + g_{33}x_3, \quad P_\pi = h_{32}X_3, \end{aligned} \tag{14}$$

where

$$g_{22} = \frac{1}{n\sqrt{C}}, \quad g_{23} = \frac{1}{n\sqrt{\kappa_2 C}}, \quad g_{32} = \frac{1}{\sqrt{C}}, \quad g_{33} = -\sqrt{\frac{\kappa_2}{C}},$$

$$h_{22} = -n\sqrt{C}, \quad h_{32} = -\sqrt{\frac{C}{\kappa_2}}, \quad \kappa_2 = \frac{(C - A)}{A}.$$

Then we have

$$\Phi = \frac{1}{2}\left(n^2 x_2^2 + X_2^2\right) + \frac{1}{2}\left(n^2 \kappa_1 \kappa_2 x_3^2 + X_3^2\right) +$$

$$+\frac{1}{2n^2}\frac{m_2}{C}\left(X_2^2+\frac{X_3^2}{\sqrt{\kappa_2}}+2\frac{X_2X_3}{\sqrt{\kappa_2}}\right)\left(\sum b_i\cos ig+\sum a_i\cos(2\psi+ig)\right)+$$
$$+\frac{1}{2}\frac{m_3}{C}\left(x_2^2+\kappa_2x_3^2-2\sqrt{\kappa_2}x_2x_3\right)\left(\sum b_i\cos ig-\sum a_i\cos(2\psi+ig)\right)-$$
$$-\frac{1}{n}\frac{m_3}{C}\left(x_2X_2-\sqrt{\kappa_2}x_3X_2+\frac{x_2X_3}{\sqrt{\kappa_2}}-x_3X_3\right)\sum a_i\sin(2\psi+ig).$$

The transformation (x_kX_k) to $(\eta_k\xi_k)_{k=2,3}$ we perform using:

$$\begin{aligned} x_2 &= \frac{1}{n}\sqrt{2\xi_2}\cos\omega_2\eta_2, \quad X_2=\sqrt{2\xi_2}\sin\omega_2\eta_2,\\ x_3 &= \frac{1}{n\sqrt{\kappa_1\kappa_2}}\sqrt{2\xi_3}\cos\omega_3\eta_3, \quad X_3=\sqrt{2\xi_3}\sin\omega_3\eta_3, \end{aligned}\tag{15}$$

where $\omega_2=n$, $\omega_3=n\sqrt{\kappa_1\kappa_2}$.

In new variables the Hamiltonian assumes an easy to use form:

$$\begin{aligned} \Phi &= \xi_2+\xi_3+\varepsilon\Big\{\xi_2\sum A^{k_1k_2k_3}\cos(k_12\omega_2y_2+k_22\psi+k_3(ig))+\\ &+\xi_3\sum B^{k_1k_2k_3}\cos(k_12\omega_3y_3+k_22\psi+k_3(ig))+\\ &+\sqrt{\xi_2\xi_3}\sum C^{k_2k_3}\cos(\omega_2y_2\mp\omega_3y_3+k_22\psi+k_3(ig))\Big\}, \end{aligned}\tag{16}$$

where: $\varepsilon=m_2/C$, $k_1=0,+1$, $k_2=0,+1$, $k_3=-1,0,+1$. The factors A^{k_1,k_2,k_3}, B^{k_1,k_2,k_3}, C^{k_1,k_2,k_3} depends from dynamical parameters $(C-A)/A$, $(C-A)/C$, $(C-B)/B$, $(C-B)/C$, $(B-A)/C$ and also from the orbital motion parameters e and $D=\frac{3}{2}GM_\odot/a^3$.

The obtained form represent the zero and the first terms of the Hamiltonian expansion

$$\Phi=H_0+\varepsilon H_1+\cdots,$$

where $H_0=\xi_1+\xi_2$, and H_1 is the function enclosed in braces.

Deprit method. In this method properties of the Lie-series are used. The transformation of the variables (η_k,ξ_k) to (y_k,Y_k) by the formulas

$$\eta_k=y_k+\sum_{n=1}^{\infty}\frac{\varepsilon^n}{n!}y_k^{(n)}(y_k,Y_k),$$
$$\xi_k=Y_k+\sum_{n=1}^{\infty}\frac{\varepsilon^n}{n!}Y_k^{(n)}(y_k,Y_k)$$

is canonical one. It performs the generating function W, which is also represented by the series:

$$W(y_k,Y_k,\varepsilon)=\sum_{n>0}^{\infty}\frac{\varepsilon^n}{n!}W_{n+1}=W_1+\varepsilon W_2+\frac{\varepsilon^2}{2!}W_3+\cdots.$$

In new variables the differential equations retain the canonical form:

$$\frac{dy_k}{dt} = \frac{\partial K}{\partial Y_k}, \qquad \frac{dY_k}{dt} = -\frac{\partial K}{\partial y_k}, \tag{17}$$

where

$$\begin{aligned} K &= K_0 + \varepsilon K_1 + \frac{\varepsilon^2}{2!}K_2 + \cdots, \\ K_0 &= [H_0], \\ K_1 &= [H_1] + L_{W_1}[H_0] - \frac{\partial W_1}{\partial t}. \end{aligned} \tag{18}$$

The square brackets denote that in the functions H_0 and H_1 the variables η_k, ξ_k were replaced by y_k, Y_k. L_{W_P} is the Poisson's operator:

$$L_{W_P} = \sum_k \left(\frac{\partial}{\partial y_k} \cdot \frac{\partial W_P}{\partial Y_k} - \frac{\partial}{\partial Y_k} \cdot \frac{\partial W_P}{\partial y_k} \right)$$

Let us $K_1 = 0$. Then we obtain from (18) the partial differential equation with respect to W_1

$$\frac{\partial W_1}{\partial y_2} + \frac{\partial W_1}{\partial y_3} + \frac{\partial W_1}{\partial t} = [H_1].$$

This equation has the following partial integral:

$$\begin{aligned} W_1 &= Y_2 \sum \frac{A_i^{k_1,k_2,k_3}}{k_1 2\omega_2 + k_2 2\dot{\psi} + k_3(i\dot{g})} \sin(k_1 2\omega_2 y_2 + k_2 2\psi + k_3(ig)) + \\ &+ Y_3 \sum \frac{B_i^{k_1,k_2,k_3}}{k_1 2\omega_3 + k_2 2\dot{\psi} + k_3(i\dot{g})} \sin(k_1 2\omega_3 y_3 + k_2 2\psi + k_3(ig)) + \\ &+ \sqrt{Y_2 Y_3} \sum \frac{C_i^{k_2,k_3}}{\omega_2 \pm \omega_3 k_2 2\dot{\psi} + k_3(i\dot{g})} \sin(\omega_2 y_2 \pm \omega_3 y_3 + k_2 2\psi + k_3(ig)). \end{aligned} \tag{19}$$

Deprit method has one remarkable property. If $f(\eta, \xi, t)$ is the differentiable function of its arguments, then using generating function W this function may be expanded as series in y, Y, t:

$$f(\eta, \xi, t) = f^{(0)}(y, Y, t) + \varepsilon f^{(1)}(y, Y, t) + \frac{\varepsilon^2}{2!} f^{(2)}(y, Y, t) + \cdots, \tag{20}$$

where

$$f^{(0)} = f(y, Y, t); \quad f^{(1)} = L_{W_1} f^{(0)}; \quad f^{(2)} = 2L_{W_1} f^{(1)} + L_{W_2} f^{(0)}.$$

Let us use this property for construction of desired solutions. According to (15) the variables x_k, X_k are the functions of η_k, ξ_k and they may be represented as series (20):

$$x_k(\eta_k \xi_k) = x_k^{(0)} + L_{W_1} x_k^{(0)}, \quad X_k(\eta_k \xi_k) = X_k^{(0)} + L_{W_1} X_k^{(0)}, \tag{21}$$

where

$$\begin{aligned} x_2^0 &= \frac{1}{n}\sqrt{2Y_2}\cos\omega_2 y_2, \quad x_3^0 = \frac{1}{n\sqrt{\kappa_1\kappa_2}}\sqrt{2Y_3}\cos\omega_3 y_3, \\ X_2^0 &= \sqrt{2Y_2}\sin\omega_2 y_2, \quad X_3^0 = \sqrt{2Y_3}\sin\omega_3 y_3, \end{aligned} \tag{22}$$

$$\begin{aligned} L_{W_1} x_k^0 &= -\sqrt{Y_2}\sum \mathcal{L}_{ki}^{k_2,k_3}\cos(\omega_2 y_2 + k_2 2\psi + k_3(ig)) - \\ &\quad -\sqrt{Y_3}\sum \mathcal{M}_{ki}^{k_2,k_3}\cos(\omega_3 y_3 + k_2 2\psi + k_3(ig)), \\ L_{W_1} X_k^0 &= +\sqrt{Y_2}\sum \mathcal{N}_{ki}^{k_2,k_3}\cos(\omega_2 y_2 + k_2 2\psi + k_3(ig)) + \\ &\quad +\sqrt{Y_3}\sum \mathcal{P}_{ki}^{k_2,k_3}\cos(\omega_3 y_3 + k_2 2\psi + k_3(ig)). \end{aligned} \tag{23}$$

Let turn our attention to equations (17). At $K_1 = 0$ they give the solutions:

$$y_2 = t + c_3, \quad Y_2 = c_4, \quad y_3 = t + c_5, \quad Y_3 = c_6.$$

Thus in the functions (22) and (23) Y_k are the constant values and the variables $\omega_2 y_2$ and $\omega_3 y_3$ are the linear functions:

$$\omega_2 y_2 = nt + \bar{c}_3, \quad \omega_3 y_3 = n\sqrt{\kappa_1\kappa_2}t + \bar{c}_5.$$

If we substitute functions (21) to (14) then we obtain the solution for ν and π as the series:

$$\begin{aligned} \nu &= g_{22}\sqrt{2Y_2}\sin\omega_2 y_2 + g_{22}L_{W_1}X_2^{(0)} + \\ &\quad + g_{23}\sqrt{2Y_3}\sin\omega_3 y_3 + g_{23}L_{W_1}X_3^{(0)}, \\ \pi &= g_{32}\frac{1}{n}\sqrt{2Y_2}\cos\omega_2 y_2 + g_{32}L_{W_1}x_2^{(0)} \\ &\quad + g_{33}\frac{1}{n\sqrt{\kappa_1\kappa_2}}\sqrt{2Y_3}\sin\omega_3 y_3 + g_{33}L_{W_1}X_3^{(0)}. \end{aligned} \tag{24}$$

Formulas, describing the planet rotation. In the construction of the numerical theory of the planets rotation it is necessary to use despite of mentioned above parameters, the values of the mean rotation rate n and the mean inclination of the equator to the orbit j. as an initial data of the problem. They can be determined from the observations of the rotational motion of the planets.

It was noted above that in case of small inclination of the equator to the orbit the uniform rotation of the rigid body (or of the xyz triad) may be written by φ, ν, π in form (2). Using this property let now determine the constants Y_2, c_3, Y_3 by the comparison of main harmonics in the solution (24), which have the frequencies equal to n, with the formulas (2) obtained from observations.

$$\begin{aligned} \frac{1}{n\sqrt{C}}\sqrt{2Y_2}\sin\omega_2 y_2 &= -j\sin\varphi, \\ \frac{1}{n\sqrt{C}}\sqrt{2Y_2}\cos\omega_2 y_3 &= -j\cos\varphi. \end{aligned}$$

Hence

$$\sqrt{\frac{2Y_2}{C}} = -jn, \quad \omega_2 y_2 = nt + \bar{c} = \varphi, \tag{25}$$

where also $Y_3 = 0$. If now we substitute the numerical values of Y_2 and $\omega_2 y_2$ to (24) we obtain the solution for ν and π for the planet, which has the mean rotation rate n and the inclination j.

The formulas for μ, ν, π obtained in the first approximation are given below:

$$\begin{aligned}
\mu &= (c_1 t + c_2) - \frac{m_1}{C} \sum \frac{A_i}{(2\dot{\psi} + i\dot{g})} \sin(2\psi + ig), \\
\nu &= -j \sin\varphi - jn \left[\sum_{i=0} Q_{1i}^{\pm} \sin(\varphi \pm ig) + \sum_{i=1} Q_{2i}^{\pm} \sin(\varphi \pm (2\psi + ig)) \right], \quad (26) \\
\pi &= -j \cos\varphi - jn \left[\sum_{i=0} R_{1i}^{\pm} \cos(\varphi \pm ig) + \sum_{i=1} R_{2i}^{\pm} \cos(\varphi \pm (2\psi + ig)) \right].
\end{aligned}$$

4. Rotation of Venus

Let now consider Venus' rotation with regard to the $X'Y'Z'$ system, which rotates with the rate equal to the mean observed rate:

$$n = 2.5855665 \cdot 10^{-2} rad/d.$$

To obtain the formulas of the rotation motion the next numerical values of the parameters and of the constants were taken:

- angular distance between the nodes Ω and Υ_V on the orbit $\Delta = 18^\circ.89$;
- inclination angle $j = 2^\circ.64$;
- exact value of the dimensionless moment of inertia C/MR^2 is unknown. There are theoretical computations, which were made by the construction of the models of Venus interior structure. For some models Kozlovskaya (1966) obtained the values of C/MR^2 in the range from 0.321 to 0.360, and Shen and Zhang (1988) — in the range from 0.321 to 0.350. We have taken

$$C/M_V R_V^2 = 0.340;$$

- Stokes parameters C_{20}, C_{22} relative to the axes of inertia A, B, C were obtained by Williams *et al.* (1983). The following values correspond to these parameters:

$$\frac{B-A}{C} = 1.0706 \cdot 10^{-5}, \quad \frac{C-B}{C} = 1.2618 \cdot 10^{-5},$$
$$\frac{C-A}{C} = 2.3324 \cdot 10^{-5};$$

– putting $D = \frac{3}{2}GM_\odot/a_V^3 = 1.17284 \cdot 10^{-3}$ we calculated

$$\frac{m_1}{C} = D\frac{B-A}{C} = 1.2556 \cdot 10^{-8},$$
$$\frac{m_2}{C} = D\frac{C-A}{C} = 2.7355 \cdot 10^{-8},$$
$$\frac{m_3}{C} = D\frac{C-B}{C} = 1.4799 \cdot 10^{-8};$$

– eccentricity $e = 6.772 \cdot 10^{-3}$;

– diurnal rate of the mean anomaly $\dot{g} = 2.796252 \cdot 10^{-2} rad/d$.

Using these numerical values of parameters the following expressions for coordinates μ, ν, π were obtained from formulas (26):

$$\begin{aligned}
\mu &= (c_1 t + c_2) + \tan^2\frac{j}{2}\sin 2\varphi + 0.''0001\sin(2\varphi + 2w + g) + \\
&\quad +0.''0241\sin(2\varphi + 2w + 2g) + 0.''0004\sin(2\varphi + 2w + 3g) + \cdots \\
\nu &= -(j + \frac{1}{8}j^3)\sin\varphi - 0.''3042\sin\varphi - 0.''0015\sin(\varphi + g) + \\
&\quad +0.''0304\sin(\varphi - g) + 0.''0004\sin(\varphi + 2w + g) + \\
&\quad +0.''0411\sin(\varphi + 2w + 2g) + 0.''0005\sin(\varphi + 2w + 3g) - \\
&\quad -0.''0010\sin(\varphi - 2w - 2g) + \frac{1}{24}j^3\sin 3\varphi + \\
&\quad +0.''0035\sin(3\varphi + 2w + 2g) + \cdots \\
\pi &= -(j - \frac{1}{4}j^3)\cos\varphi - 0.''3042\cos\varphi + 0.''0038\cos(\varphi + g) - \\
&\quad -0.''0472\cos(\varphi - g) + 0.''0004\cos(\varphi + 2w + g) + \\
&\quad +0.''0343\cos(\varphi + 2w + 2g) + 0.''0025\cos(\varphi + 2w + 3g) - \\
&\quad -0.''0020\cos(\varphi - 2w - 2g) + \frac{1}{12}j^3\cos 3\varphi - \\
&\quad -0.''0007\cos(3\varphi + 2w + 2g) + \cdots \\
\varphi &= -nt + \varphi_0; \qquad w = \Delta + \omega.
\end{aligned}$$

5. Conclusion

The obtained results shows that Venus rotates nearly uniformly with the rate n and we can conclude that the radar observations are not sensitive to the libration of its rotation. For the rigid body model this phenomenon is explained by the fact, that the dynamical figure of Venus is very close to the spherical one and that the values of parameters of its flattening are small – of order to $m_1/C \sim 10^{-8}$. Venus orbit is almost circular: $e \sim 10^{-3}$. And finally the inclination of Venus' equator to the orbit plane is also small. All these factors together cannot produce noticeable perturbations on the steady-state quasi-stationary rotation of Venus.

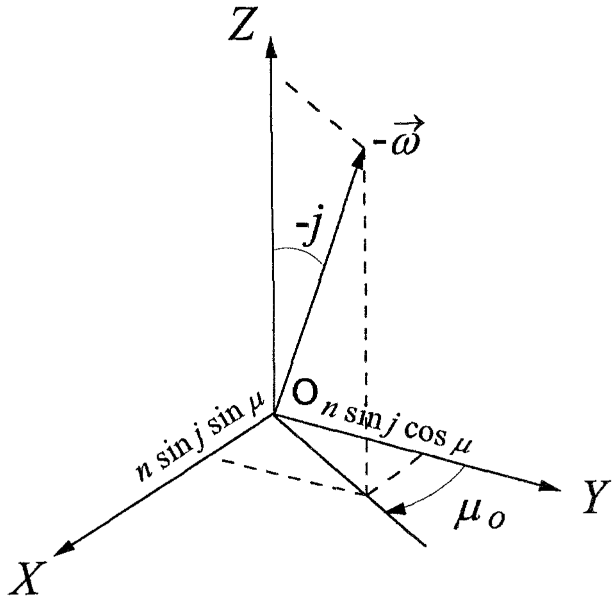

Fig. 3. Precession of the mean rotational axis relative to inertial frame of reference.

The longitudes component of the rotation $\rho = \varphi - \mu$ is affected to the smallest oscillations. But the secular term $\mu_0 = c_1 t + c_2$ presents the regular precession of the rotation axis. It became obvious if we express the components of rotation vector ω with regard to the fixed coordinates system XYZ. Let us substitute the expressions $\nu \simeq j \sin\varphi$ and $\pi \simeq j\cos\varphi$ to (4). Then we obtain:

$$\begin{aligned}
\omega_x &\simeq nj\sin\mu_0 \simeq n\sin j\sin\mu_0,\\
\omega_Y &\simeq nj\cos\mu_0 \simeq n\sin j\cos\mu_0,\\
\omega_Z &\simeq -n(1-\frac{1}{2}j^2) - \dot{\mu} \simeq -n\cos j - \dot{\mu}_0.
\end{aligned}$$

Hence the rotation axis lies at the meridian plane, distant from the coordinate plane to the angular distance μ_0 (Fig.3). The precession angle μ_0 increases or decreases uniformly depending on the sign of the precession constant c_1. In this case the precession of equinox point may be superposed on the precession of the axis of the proper rotation. But at the moment these theoretical discussions have no big practical sense since the modern observations methods do not allow to detect thin effects in the rotation of Venus.

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