THE MARTIAN MEAN MOMENT-OF-INERTIA AND THE SIZE OF THE MARS' CORE

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(Received 9 November 1993)

Abstract. The mean moment-of-inertia ratio, I/MR^2, of Mars cannot be derived from its precessional constant because the exact value of the Martian axial precession is unknown presently. Using the known geodetic parameters of Mars as the constrained condition, we constructed nine Martian internal structure models (see Table II). We can then estimate the nonhydrostatic components of the principal moment-of-inertia for these models. The interplanetary comparison suggests that the reasonable range of the mean moment-of-inertia ratio, I/MR^2, of Mars is 0.350 ~ 0.360, and the range of the corresponding radius of Mars' core is 1520 ~ 1850 km. The two parameterically simple models recommended in this paper (see Table IV) can be used for reference in the future theoretical researches.

1. Introduction

As the Soviet spacecraft mission to Phobos (1988/07) and the US Mars Observer mission (1992/09/25) were achieved in succession, there has been renewed interest recently in researches on the dynamics of Martian rotation. Lately, there are a series of papers dealing with the problems of the mean moment-of-inertia ratio, I/MR^2, of Mars (Bills, 1989, 1990; Kaula *et al.*, 1989; Kaula and Asimow, 1991), the constructions of the Martian internal structure models (Bills, 1990; Zharkov *et al.*, 1991; Severova, 1992), and the theory of the Mars' pole motion (Hilton, 1990, 1991), etc.

The mean moment-of-inertia ratio, I/MR^2, is one of the important parameters of a planet. When we study the problems on the dynamics of Martian rotation, it is necessary to adopt the correct value of I/MR^2.

The major objective of the paper is to construct a set of internal structure models of Mars by comparing Mars with the Earth, the Moon, and the terrestrial planets, then to discuss the likely range of the size of Mars' core, and to check whether the adopted value of I/MR^2 for Mars is reasonable.

2. Two-Layer Model of Density Distribution

The density in the Mars' interior is assumed to have the form (cf. Zhang and Shen, 1988)

$$\rho(x) = \begin{cases} \rho_0(1 - c_0 x^2), & 0 \leq x \leq x_c, \\ \rho_m, & x_c \leq x \leq 1\,. \end{cases} \tag{1}$$

where ρ_0 and c_0 are two positive constants; ρ_0 is the central density of Mars, ρ_m

Earth, Moon, and Planets **64**: 117–124, 1994.

the density of the mantle and considered as a parameter in the following discussions; $x = r/R$ is the relative radius, and $x_c = r_c/R$ is the relative radius of the core.

In this case, it is easy to see that the mean density and mean moment-of-inertia ratio I/MR^2 have the following expression, respectively

$$\left.\begin{aligned} \bar{\rho} &= 3\int_0^{x_c} \rho_0(1 - c_0x^2)x^2\,dx + 3\int_{x_c}^1 \rho_m x^2\,dx, \\ \frac{I}{MR^2} &= \frac{2}{\bar{\rho}}\left[\int_0^{x_c} \rho_0(1 - c_0x^2)x^4\,dx + \int_{x_c}^1 \rho_m x^4\,dx\right]. \end{aligned}\right\} \tag{2}$$

Let us assume that the density of the core-mantle bound is ρ_{CMB}. Obviously $\rho_{CMB} \geqslant \rho_m$ hold true. Introducing the quantity $\Delta\rho = \rho_{CMB} - \rho_m \geqslant 0$, we derive easily from Equation (2)

$$\frac{I}{MR^2} = \frac{2}{5}\frac{1}{\bar{\rho}}\left[\frac{5}{7}x_c^2(\bar{\rho} - \rho_m) + \frac{2}{7}x_c^5\,\Delta\rho\right] + \frac{2}{5}\frac{\rho_m}{\bar{\rho}}, \tag{3}$$

or

$$\frac{I}{MR^2} = \frac{2}{35}\frac{1}{\bar{\rho}}x_c^5[2(\rho_0 - \rho_m) + 5\Delta\rho] + \frac{2}{5}\frac{\rho_m}{\bar{\rho}}. \tag{4}$$

Equations (3) and (4) can be used for estimating the range of the value of I/MR^2 for Mars. In the calculations, the mean density $\bar{\rho}$ is taken as 3.933497 g cm^{-3}, and the accepted value of $\Delta\rho$ is equal to 4.5 g cm^{-3}.

Figure 1 illustrates the dependence of I/MR^2 on x_c (Figure 1a) and on ρ_0 (Figure 1b) respectively for a set of the parameters ρ_m (3.20 ~ 3.50 g cm^{-3}). We took x_c = 0.5 in the calculation of the variation of I/MR^2 with ρ_0. The curves in Figure 1 are labeled according to the corresponding value of ρ_m.

It is concluded from Figure 1 that the influence of the variations of x_c on I/MR^2 is obvious (when ρ_m is given), and the influence of the variations of ρ_0 on I/MR^2 is slight (when ρ_m and x_c are given). When x_c is given, it is necessary to adjust simultaneously the values of ρ_0 and ρ_m in order that the value of I/MR^2 may be changed between 0.345 and 0.365.

3. Estimation of the Physical Parameters of Mars

Let us assume that the Martian interior (core and mantle) is in the hydrostatic equilibrium state, and the bulk modulus $K(r)$ and the pressure $p(r)$ satisfy the following linear relation

$$K(r) = K_0 + bp(r), \tag{5}$$

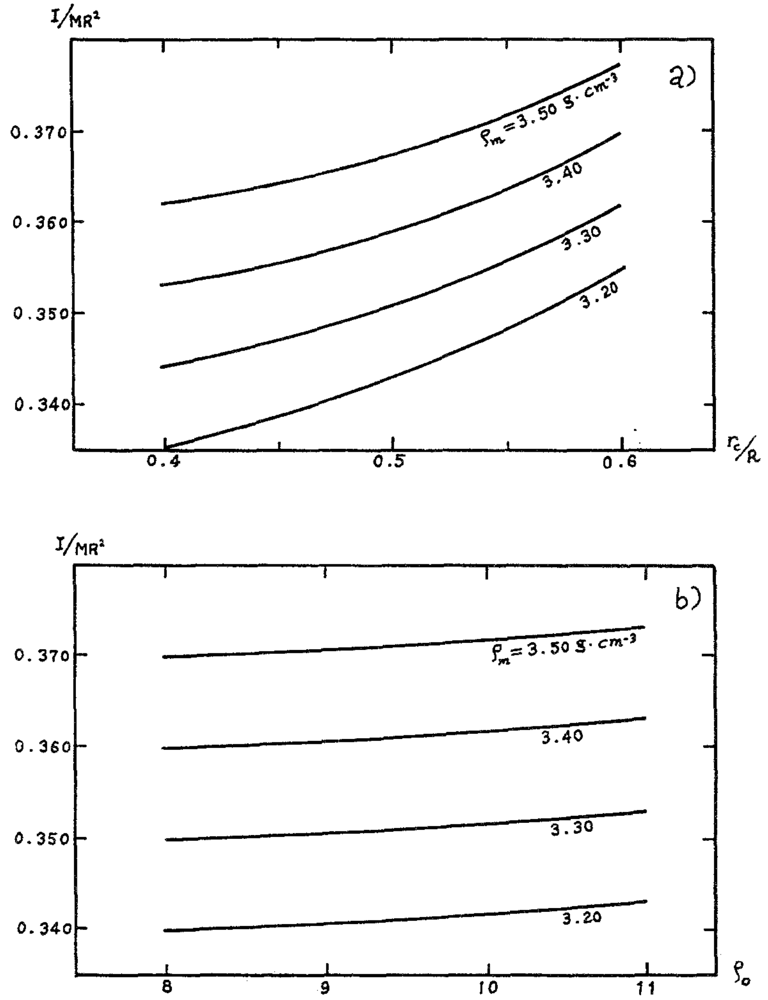

Fig. 1. Dependence of the I/MR^2 value on r_c/R (a) and on ρ_0 (b).

where both K_0 and b are two positive constants. We refer to those of the Earth's and lunar models (cf. Zhang, 1992) and adopt K_0 and b values for Mars. The values of K_0, b, and σ (Poisson's ratio) used in construction of the Martian models are listed in Table I.

Using the results of simplified two-layer model in the preceding section and the known geodetic parameters of Mars, we constructed nine Martian internal structure models (see Table II) by solving the Emden equation (e.g. Shen and Zhang, 1988; Zhang, 1992). In these models the thickness of the Martian crust is taken as 180 km.

The nine Martian models may be divided into two groups. The first group

TABLE I

Values, K_0, b, and σ, used in construction of Martian models

Zone	K_0 (10^3 kbar)	b	σ
Core	2.0	4.0	0.34
Mantle	2.3	3.25	0.30

consists of five models, *MARS* 93-01 ~ 93-05, in which the central density is taken as 7.76 g cm^{-3}. When x_c is taken from 0.55 to 0.46 and the mean mantle density, $\bar{\rho}_m$, increases from 3.25 g cm^{-3} to 3.64 g cm^{-3}, the value of I/MR^2 increases from 0.346 to 0.365, and the relative mass, M_c/M, of the core decreases from 0.321 to 0.182. The second group comprises *MARS* 93-05 ~ 93-09, in which the relative radius, x_c, of the core is about equal to 0.45. When the central density, ρ_0, increases from 7.76 g cm^{-3} to 11.00 g cm^{-3} and $\bar{\rho}_m$ is taken from 3.64 g cm^{-3} to 3.43 g cm^{-3}, the value of I/MR^2 decreases from 0.365 to 0.345, and M_c/M increases from 0.182 to 0.229.

Figure 2 illustrates the relation between the relative mass, M_c/M, of the core and the relative radius, r_c/R, of the core for the Martian models. In Figure 2 the symbol "⊕" represents the corresponding values of the Earth's model *PREM*.

In short, with the reduction of the value of I/MR^2 , the mean mantle density, $\bar{\rho}_m$, certainly decreases (cf. Zharkov *et al.*, 1991).

If $A < B < C$ denote three principal moment-of-inertia of Mars, and δA, δB, δC are, respectively, the minimum, intermediate, and maximum nonhydrostatic components of the moment-of-inertia, the observed value of J_2, one of the Stokes coefficients of degree two, can be divided into two parts, namely,

$$J_2 = J_2^{(0)} + \delta J_2 \, , \tag{6}$$

where $J_2^{(0)} = 1/MR^2[C - \frac{1}{2}(A + B)]$ is the hydrostatic value of J_2, and δJ_2 is the non-hydrostatic component. Hence (cf. Zharkov *et al.*, 1991),

$$\delta J_2 = \frac{1}{MR^2}\left[\delta C - \frac{1}{2}(\delta A + \delta B)\right], \tag{7}$$

$$J_{22} = \frac{1}{4MR^2}(\delta B - \delta A) \, , \tag{8}$$

where J_{22} is another of the Stokes coefficients of degree two.

Following Bills (1989), we introduce the quantity

$$\delta f = \frac{\delta B - \delta A}{\delta C - \delta A} \, , \tag{9}$$

and may derive the expression,

TABLE II

Parameters for the Martian models 93-01–09

Zone	Parameter	Model								
		93-01	93-02	93-03	93-04	93-05	93-06	93-07	93-08	93-09
Core	ρ_0 (g cm^{-3})	7.760	7.760	7.760	7.760	7.760	8.900	9.710	10.370	11.00
	p_0 (kbar)	470.00	465.00	450.00	445.00	410.00	460.00	500.00	542.00	580.00
	r_c (km)	1875.93	1855.08	1750.87	1730.02	1563.27	1542.43	1521.59	1521.59	1500.74
	$\rho(r_c)$ (g cm^{-3})	7.242	7.005	7.197	6.760	6.987	7.871	8.491	8.876	9.971
	M_c/M	0.321	0.304	0.260	0.242	0.182	0.198	0.206	0.218	0.229
Mantle	$\rho(r_c)$ (g cm^{-3})	3.368	3.425	3.549	3.617	3.792	3.689	3.639	3.585	3.537
	$\rho(r_m)$ (g cm^{-3})	3.172	3.222	3.325	3.385	3.526	3.426	3.374	3.324	3.274
	$\bar{\rho}_m$ (g cm^{-3})	3.254	3.307	3.418	3.482	3.636	3.550	3.515	3.461	3.426
Shell	ρ_{S2} (g cm^{-3})	3.150	3.200	3.320	3.381	3.400	3.400	3.350	3.300	3.150
	ρ_{S1} (g cm^{-3})	2.850	2.950	2.906	2.955	2.900	2.900	2.900	2.900	2.850
Total	I/MR^2	0.34653	0.34996	0.35501	0.35909	0.36549	0.35904	0.35489	0.35102	0.34528
	$\bar{\rho}$ (g cm^{-3})	3.93342	3.93347	3.93346	3.93348	3.93346	3.93345	3.93348	3.93344	3.93350

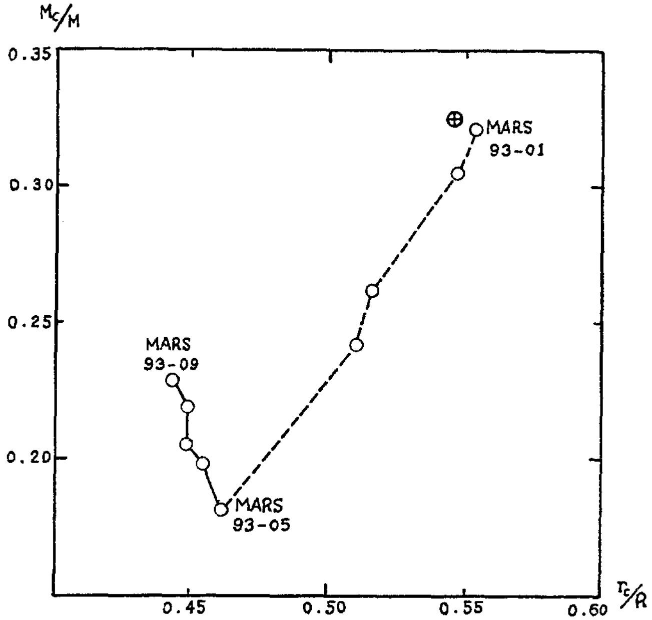

Fig. 2. Relation between the relative mass M_c/M and the relative radius r_c/R of the core for the Martian models.

$$\delta f = \frac{4J_{22}}{\delta J_2 + 2J_{22}}, \tag{10}$$

from Equations (7) ~ (9).

In the nine Martian models we choose five models – 93-01, 93-03, 93-05, 93-07, and 93-09, and the profile of the flattening, $e(r)$, in the Mars' interior for these models can be obtained from solving numerically the Clairaut equation (cf. Zhang, 1993). Then the values of $J_2^{(0)}$ and δf can be determined. In the calculations, the accepted values of J_2 and J_{22} are equal to 1960.45×10^{-6} and 63.098×10^{-6}, respectively (Balmino *et al.*, 1982). The results obtained are listed in Table III.

At present, the values of δf are well known for the Moon (0.361), the Earth (0.579), and Venus (0.490) (Bills, 1989). If the nonhydrostatic components of the second degree gravity field of Mars stand the range between those of the Earth and Venus, it seems that the reasonable range of the Martian mean moment-of-inertia ratio, I/MR^2, is 0.345 ~ 0.350.

4. Discussions and Suggestions

(1) The data in Table II obviously show that the central density, ρ_0, and central pressure, p_0, for the model *MARS* 93-09 are too high, whereas the mean mantle density, $\bar{\rho}_m$, for the model *MARS* 93-01 is too low. This deviates from the knowl-

TABLE III

Inferred properties of the Martian models

Quantity	Martian model				
	93-01	93-03	93-05	93-07	93-09
J_2 (10^{-6})	1600.52	1708.19	1816.84	1687.81	1578.06
δJ_2 (10^{-6})	359.83	252.26	143.61	272.64	382.39
δf	0.519	0.667	0.935	0.633	0.496
I/MR^2	0.3465	0.3550	0.3655	0.3549	0.3453
e	1/213.75	1/207.21	1/199.33	1/207.24	1/214.53
e (CMB)	1/274.27	1/267.90	1/258.48	1/302.18	1/339.00

edge of the chemical composition in Mars' interior. Therefore, if the value of I/MR^2 equals 0.345, this seems inconsistent with some known facts.

(2) If we adopt $I/MR^2 \simeq 0.355$, the central density, central pressure, radius of the core, and the mean mantle density for two relevant models (namely, 93-03 and 93-07) are all reasonable. We consider that the two parameterically simple

TABLE IV

Two parametric models of Mars

Region	Radius (km)	$\rho(x)$ $(\mathrm{g\,cm^{-3}})$	$K(x)$ (kbar)
MARS 93-03			
Core	0.0–1750.9	7.7600 −2.1103*x^2	3799.21 + 32.26*x −4191.71*x^2 + 847.99*x^3
Mantle	1750.9–3209.9	3.7928 −0.4622*x −0.0355*x^3	3597.87 −1340.80*x + 32.72*x^2
Shell	3209.9–3329.9 3329.9–3389.9	3.320 2.906	1150 550
MARS 93-07			
Core	0.0–1521.6	9.7100 −6.0526*x^2	3999.06 + 44.24*x −6245.71*x^2 +1542.80*x^3
Mantle	1521.6–3209.9	3.8592 −0.4848*x −0.0303*x^3	3868.99 −2518.58*x +1720.53*x^2 − 792.12*x^3
Shell	3209.9–3329.9 3329.9–3389.9	3.350 2.900	1150 550

models (see Table IV) can be used for reference in the future theoretical researches.

(3) The results obtained in this paper indicate that the reasonable range of the Martian mean moment-of-inertia ratio, I/MR^2, is $0.350 \sim 0.360$, meantime, the range of the radius of Mars' core is $1520 \sim 1850$ km.

(4) It is necessary for us to be engaged in ground-based astrometric observations of the Martian satellites with the *CCD* detector. This will certainly allow detection of the exact value of the Martian axial precession (cf. Sinclair, 1989) and will directly derive the mean moment-of-inertia ratio, I/MR^2, of Mars. In addition, it seems that the new periodic space exploration will have to include the programme for detection of the size of Mars' core and its character. This will allow improvement of the internal structure models of Mars and will help us to check the adopted value of I/MR^2 of Mars.

Acknowledgments

The author is grateful to Associate Professor Yifei Xia for helping to improve the English. This research was supported by the National Natural Science Foundation of China.

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