EVIDENCE FOR CONVECTION IN PLANETARY INTERIORS FROM FIRST-ORDER TOPOGRAPHY*

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Abstract. Center of mass-center of figure offsets are known for the Earth, Moon, Mars and Venus. Such an offset requires a density distribution asymmetric about the center of mass. Observational evidence indicates that the terrestrial, lunar and Martian offsets result from crusts of variable thickness rather than lateral density inhomogeneities and that the thickness variations are more likely caused by internal convection than impact.

1. Introduction

Displacements of the center of mass from the center of figure have been reported for a number of planetary bodies. The Apollo 15 laser altimeter has confirmed such an offset for the Moon. The approximate equatorial projection of this offset is 2 km with the center of mass displaced from the center of figure toward the Earth in a direction 35° E of the Earth–Moon line (Kaula *et al.*, 1972). It has been known for some time that a center of mass–center of figure offset exists for the Earth (e.g. Jeffreys, 1962). From a spherical harmonic analysis of the Earth's topography with the oceans replaced by an equivalent mass of rock, Balmino *et al.* (1972) find the center of mass displaced by 1.1 km from the center of figure in the direction 145° W long and 42° S lat. A Martian equatorial offset of about 1 km with the center of mass displaced in the direction ~120° W long. has been reported (Schubert and Lingenfelter, 1972) on the basis of terrestrial radar observations (Pettengill *et al.*, 1971) of Martian surface elevations. Similar Earth-based radar observations of the equatorial topography of Venus have yielded an offset of ~1.5 km with the center of mass displaced approximately toward the Earth at inferior conjunction (Smith *et al.*, 1970).

Considerable geophysical interpretation has been based on comparisons of second and higher order terms in harmonic analyses of topography and gravity. The significance of a first order term in the topography, which reflects an offset, seems not to have been generally appreciated. We shall show in this paper that observation of an offset strongly suggests that a planetary body has differentiated a crust, lighter than the subcrustal material and asymmetrically distributed over the surface. From a consideration of the terrestrial, lunar and Martian offsets, such asymmetric crustal distributions seem to have been produced by internal convective processes rather than by a few large impacts.

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2. Center of Mass – Center of Figure Offset

The definition of the center of mass of a body is well known. If the origin of coordinates is at the center of mass

$$\int_V \varrho \mathbf{r} \, \mathrm{d}V \equiv 0, \tag{1}$$

where ϱ is the density, $\mathbf{r}$ the position vector, $\mathrm{d}V$ the element of volume and the integration extends over the entire body. Equation (1) may be rewritten as a surface integral

$$\int_S \hat{\mathbf{r}} \, \mathrm{d}\Omega \left\{ \int_0^R \mathrm{d}r \, r^3 \varrho \right\} = 0, \tag{2}$$

where $\hat{\mathbf{r}}$ is the unit vector $\mathbf{r}/r$, $\mathrm{d}\Omega$ is the element of solid angle and the integration extends over the entire surface of the body. For a uniform density, Equation (2) simplifies to

$$\int_S \mathbf{r} \, \mathrm{d}\Omega \, (r^3) = 0. \tag{3}$$

By the center of figure of a body we mean that origin of coordinates with respect to which the spherical harmonic expansion of the surface elevations contains no first order terms

$$\int_S \mathbf{r} \, \mathrm{d}\Omega \equiv 0. \tag{4}$$

Equation (4) may be rewritten as the volume integral

$$\int_V \frac{\mathbf{r}}{r^3} \, \mathrm{d}V = 0. \tag{5}$$

A comparison of either Equations (1) and (5) or Equations (2) and (4) shows that in general the centers of figure and mass are distinct points. Equation (1) states that the moments, about the center of mass, of the density distribution are zero. On the other hand, Equation (3) locates the center of figure as that point about which the moments of the function r^{-3} are zero. It is obvious from Equations (3) and (4) that even for a body of uniform density, the centers of figure and mass do not in general coincide.

Nonetheless for simple objects of uniform density such as spheres and ellipsoids, both centers are indeed coincident. Moreover, the centers also coincide in the case of a body with a density distribution which is an arbitrary function $\varrho(r)$ of radial distance from the center of mass and with an arbitrary but linear topography (i.e. surface elevation differences are very much less than the mean radius of the body). This can be readily seen from the following simple argument. With the surface defined by

$R_0 + R_1(\hat{\mathbf{r}})$, where R_0 is the mean radius and the magnitude of the topography $|R_1| \ll R_0$, the equation for the center of mass (2) can be written as

$$\int_S \hat{\mathbf{r}}\, \mathrm{d}\Omega \left\{ \int_0^{R_0} \mathrm{d}r \left[r^3 \varrho(r) + R_1(\hat{\mathbf{r}}) R_0^3 \varrho(R_0) + \cdots \right] \right\} = 0 . \tag{6}$$

Since the integral of the first term vanishes by symmetry, then for the linear case Equation (6) becomes

$$\int_S \hat{\mathbf{r}}\, \mathrm{d}\Omega R_1 \approx 0 , \tag{7}$$

which is identical to the definition of the center of figure, see Equation (4).

The topography of each of the planetary bodies considered in this paper is clearly linear. Thus the observation of a center of mass–center of figure offset requires that the density distribution is not simply a function of radius but is asymmetric about the center of mass. Contrary to the suggestion of O'Keefe and Cameron (1962) such an offset is not evidence of the existence of isostasy. There is by definition no first order term in the gravitational potential about the center of mass. Thus it is meaningless to talk about isostasy in connection with the first order components of the topography, since the absence of the corresponding gravitational component does not imply that any process of compensation has occurred.

Two simple two-layer models for interpreting an offset are shown in Figure 1. One has an outer layer of constant density ϱ but varying thickness $t + \Delta t \cos\theta$ and the other an outer layer of varying density $\varrho + \Delta\varrho \cos\theta$ but constant thickness t. In both models the inner boundary of the outer layer is defined by a sphere of radius a.

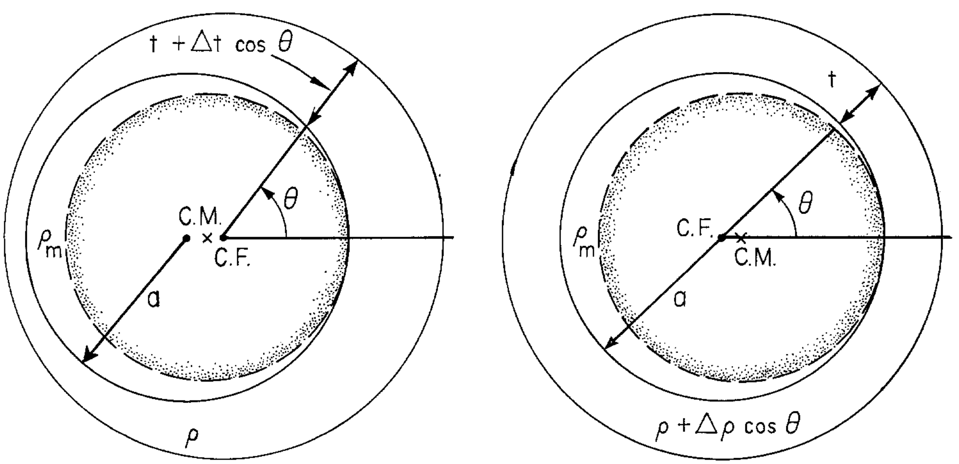

Fig. 1. Two simple two-layer models, one relying on variations in thickness of a crustal layer and the other on lateral density variation, to explain the center of mass–center of figure offset. The thickness of the outer layer is exaggerated for clarity.

Beneath the outer layer, but above the largest possible inner sphere about the center of mass, the material has a constant density ϱ_m. Within this inner sphere the density can be an arbitrary function of the distance from the center of mass. We consider these models only in the linear case, where the offset is much less than the radius of the body. In this case both bodies are spheres of the same radius and mass.

The center of mass–center of figure offset Δz for the model with variable thickness is

$$\Delta z = \frac{\Delta t\,(\varrho_m - \varrho)\,a^3}{(\varrho_m - \varrho)\,a^3 + \varrho\,(a + t)^3}, \tag{8}$$

and for the model with variable density it is

$$\Delta z = \frac{\frac{1}{4}\Delta\varrho\,[(a + t)^4 - a^4]}{(\varrho_m - \varrho)\,a^3 + \varrho\,(a + t)^3}. \tag{9}$$

In the case where $t \ll a$ Equation (8) reduces to

$$\Delta z \approx \frac{\Delta t\,(\varrho_m - \varrho)}{\varrho_m}, \tag{10}$$

and Equation (9) to

$$\Delta z \approx \frac{\Delta\varrho t}{\varrho_m}. \tag{11}$$

Equations (10) and (11) are qualitatively similar in that the offsets can be viewed as a surface loading of a homogeneous sphere, with the mass per unit area of the loading given by either $\Delta t(\varrho_m - \varrho)\cos\theta$ or $\Delta\varrho t\cos\theta$ (see for example, Jung, 1956). In the variable thickness model, the topographic low is associated with the thinnest part of the outer layer, while in the variable density model the low occurs above the densest part of the outer layer. We will consider the outer layer of the variable thickness model to be a thin variable thickness crust of lighter material differentiated from the subcrustal region. The outer layer of the variable density model could be either a crust of uniform thickness with lateral density variations or a layer which extends deeper than a crust to include the lateral density variations of subcrustal or mantle material.

One can find elements of both these mathematical models in the example of the terrestrial offset which is clearly associated with the asymmetric distribution of continents and oceans (Cook, 1777). There is a wealth of geologic, seismic and gravimetric evidence (Kaula, 1968) that the continental crust is lighter and thicker than the oceanic crust. The variable thickness model would attribute the terrestrial offset to the greater thickness of continental crust while ignoring the density difference between continental and oceanic crust. Similarly, if only lateral density variations in the crust were considered, the variable density model could attribute the offset to the difference between sialic and simatic densities rather than to differences in thickness between continental and oceanic crusts. Alternatively the variable density model might suppose lateral density variations in the subcrustal material, correlated with the oceanic and continental crustal regions, to produce the offset. Attributing an offset to crustal thickness or density variations is analogous to the Airy (1855) and Pratt (1855) views of isostasy.

In the following section we discuss which of these possibilities is the most likely explanation for each of the terrestrial, lunar and Martian offsets. Though there is no unique model of the internal density distribution implied by an offset, other geophysical evidence strongly suggests that the offsets are best understood in terms of models with lower density crusts of variable thickness.

3. Interpretation of the Offset

Observation of an offset on the Earth, Moon, Mars and Venus requires that each of these bodies is asymmetrically differentiated to some extent. In this section we shall consider the nature of that asymmetry in terms of the two models discussed above.

Simple calculations show that the terrestrial offset can be caused by the difference in continental and oceanic crustal thicknesses but not by the difference between sialic and simatic crustal densities, contrary to the suggestion of O'Keefe and Cameron (1962). The amplitude of the first order crustal thickness variation Δt is about 7 km, taking into account the difference between continental and oceanic crustal thicknesses of about 30 km and the relative surface areas of the continents and oceans. With $\varrho_m - \varrho$ about 0.5 g cm^{-3} and ϱ_m about 3.3 g cm^{-3}, we find that Equation (10) predicts an offset of about 1.1 km, in precise agreement with the measured terrestrial offset. On the other hand, the amplitude of the first order crustal density variation $\Delta\varrho$ is about 0.025 g cm^{-3} considering the difference between sialic and simatic densities of about 0.1 g cm^{-3} and the relative surface area of oceans and continents. With an average crustal thickness t of about 10 km Equation (11) gives an offset of only about 0.075 km, an order of magnitude too small to account for the observed offset.

It is also clear that since deep lateral density differences on a global scale are unlikely to exceed that between sialic and simatic crustal materials, a variable density model would require lateral density differences correlated with oceanic and continental surface regions to extend to depths greater than 100 km, in order to account for the terrestrial offset. Though the lithosphere extends to such depths there is at present no observational evidence for systematic lateral variations in lithospheric density. Moreover since the lithosphere beneath the oceans is younger and hence presumably hotter than that under the continents, one might expect the lower density lithospheric material to be under the oceans, whereas the direction of the offset requires that the material beneath the Pacific Ocean basin be of higher density. Similarly, shallow mantle convection might produce even deeper lateral density inhomogeneities, but since the material below the Pacific Ocean basin would have to be denser it should be a region of downwelling. This is contrary to the evidence presented by the East Pacific Rise, indicating that if shallow mantle convection is correlated with the Pacific basin it must be a region of upwelling.

We conclude therefore that the terrestrial offset is produced by an asymmetric distribution of variable thickness crust. Jeffreys (1962) understood the terrestrial offset as implying that the rocks under the Pacific were heavier than those under the continents. This view is correct only in that the oceanic crust is thinner than the continental one. We cannot conclude from the offset that sima is denser than sial, or that even deeper density contrasts exist.

The lunar offset is also most likely due to a difference in crustal thickness, thinner on the nearside and thicker on the farside. Contrary to the suggestion of O'Keefe and Cameron (1962), the most obvious variable density model, the asymmetric distribution of heavier mare material, cannot be responsible for the offset. The density difference of ~ 0.4 g cm^{-3} between highland and mare material would give an amplitude of the density variation $\Delta\varrho \sim 0.1$ g cm^{-3}, considering the surface distribution of mare material. The density ϱ_m is about 3 g cm^{-3}. Thus the observed offset of ~ 2 km would require the frontside mare material to have an average thickness of about 60 km. However, observations of the so-called 'ghost' craters which appear not to have been completely filled by mare material, suggest that the average thickness of Procellarum and other irregular maria is only a few kilometers. Since the surface area of the ringed maria is only a fifth that of all frontside maria, they would have to be filled with mare material to a depth of 300 km to account for the offset.

We might also consider the possibility that the lunar offset is associated with lateral density variations in a deep convective layer. Since the direction of the topographic low is toward the frontside this would have to be the region of higher density and should therefore be a region of downwelling convective motions. This however, seems to be contrary to the appearance of the frontside maria, which is suggestive of massive flooding by low viscosity material above an upwelling region.

The variation of the lunar crustal thickness implied by the offset can be deduced from Equation (10). Assuming $\varrho \approx 2.9$ g cm^{-3} and $\varrho_m \approx 3.3$ g cm^{-3}, we see that along the direction of the offset the farside crust is about 30 km thicker than that on the nearside. Such thickness variations are consistent with the absence of positive gravity anomalies in the highlands with respect to the maria (Muller and Sjogren, 1968).

Similarly the Martian offset of ~ 1 km implies that the planet has differentiated a crust, lighter than the subcrustal material and thicker under the Tharsis highlands ($\sim 120°$W) than under the region between Hellas and Syrtis Major. For a difference of less than 10% between the densities of the crustal and subcrustal rock, the difference in crustal thickness must be greater than 20 km. Variations of this magnitude are reasonable in light of the partial compensation implied by a comparison of the low order components of the Martian gravity field (Lorell *et al.*, 1972) and the topography.

The Martian surface, unlike that of the Earth and Moon, reveals no hemispheric asymmetries in albedo which might suggest large scale lateral variations in the density of surface material, Thus there would seem to be no basis for invoking near surface lateral density variations to explain the Martian offset. The possibility of internal density variations in a deeper convective layer producing the offset would appear to be inconsistent with the type of volcanism found in the Tharsis region. This is in fact the site of the major volcanism on Mars. These volancoes show a rough alignment and an apparent stratocone structure similar to those associated with downwelling convective regions on the Earth. However, the direction of the offset would require this to be a lower density, upwelling region if deep internal density variations were the cause of the offset.

Finally, the observed offset of ~1.5 km on Venus also requires that it is asymmetrically differentiated. By analogy to the Earth, Moon and Mars it seems likely that the offset is caused by variations of ~30 km in crustal thickness, but lateral density variations cannot be ruled out.

4. Convective Origin of the Offset

In this section we shall consider the origin of the crustal thickness variations which are apparently responsible for the center of mass–center of figure offsets observed on the Earth, Moon, Mars and Venus. We conclude that asymmetric crustal distributions most likely result from large scale internal convection during or after the period of crustal differentiation. Convective motions of subcrustal material would lead to an accumulation of lighter crustal material above regions of downwelling, making the crust thicker there and thinner in the region of upwelling. This does not necessarily imply a single convection cell of global scale but only that the effect of all convection cells is some net transport on a global scale into a particular hemisphere. Nothing is implied about the depth of the convection cells; they might be quite shallow. A less likely mechanism for producing asymmetric crustal distributions is throwout of crustal material from a few massive impact craters.

In the case of the Earth it now seems apparent that convection, not impact, is responsible for the present distribution of the continental and oceanic crusts. Moreover continental material is apparently still accumulating at downwelling convective regions along the island arcs. That some form of convection was responsible for the accumulation of continental material was first suggested by Hills (1934, 1947).

The apparent asymmetry in lunar crustal thickness, implied by the offset, also seems likely to be due to some internal convective process rather than impacts. If the asymmetry were due to impacts, the location of the thinnest crust should be related to the positions of the largest impact features, This, however, is not the case. The three largest impact features are Imbrium, Orientale, and the recently discovered (Kaula *et al.*, 1972) Imbrium-sized basin on the lunar farside. These lie 50°, 130°, and 215° W of the location of the thinnest crust at about 35° E long. as implied by the offset (Kaula *et al.*, 1972). Obviously there is no correlation between the locations of the largest impacts and the direction of the offset. If, on the other hand, convective processes were responsible for the asymmetry, the region around the topographic low, at 35° E long. should be a region of upwelling and the antipodal region one of downwelling where the thickest crust accumulates. The appearance of the frontside maria, as discussed above, is in fact suggestive of massive flooding by low viscosity material above an upwelling region.

The occurrence of the major Martian volcanoes in the Tharsis region also suggests (Schubert and Lingenfelter, 1972) an internal convective origin for the apparent asymmetric crustal distribution on Mars. Thus we would expect that the Tharsis region is the site of downwelling convective motions with the ensuing accumulation of a relatively thick, light crust. The associated volcanism could then result from frictional heating along local fracture zones stressed by the downward motion. The morphology (McCauley *et al.*, 1972) and partial alignment of the Martian volcanoes in the Tharsis region is, in fact, qualitatively similar to that of the composite, or strato, volcanoes associated with downwelling regions on the Earth, although the analogy is crude at best. It should be noted that the morphology of the Martian volcanoes is quite inconsistent with the massive flooding that would be expected in a region of upwelling. The Mariner 9 imagery (Masursky *et al.*, 1972; McCauley *et al.*, 1972) also revealed other evidence of extensive volcanic and tectonic modifications of the Martian surface which might reflect deep internal processes. The location of the Hellas basin near the direction of the thinnest crust, as implied by the offset, might suggest that an impact produced the asymmetric crustal distribution. But such an impact alone could not account for the volcanism at Tharsis, although it could have influenced a pattern of internal convection. That the latter might be the case may be indicated by the nearly diametrically opposite positions of the Tharsis volcanic region and the Hellas basin.

In summary then, we have seen that a center of mass–center of figure offset in a body requires some form of asymmetric density distribution in that body. Furthermore, from a consideration of the Earth, Moon and Mars we have seen that most likely form of such an asymmetric distribution is a variable thickness crustal layer rather than deeper internal density variations. For these same bodies we have also found that convective rather than impact processes are the most likely cause of variations in the crustal thickness. Thus we conclude that the observation of a center of mass–center of figure offset is a strong indication of past or present internal convection.

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