DIFFUSION OF RADIATION IN PLANETARY ATMOSPHERES

I. The Earth's Atmosphere: A Model

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Abstract. The problem of interaction of the solar radiation with the turbid Earth atmosphere, containing complicated polydispersive aerosol systems, is discussed in this paper. Equations for computing the angular functions of n-th order scattering are derived. On the basis of these functions the spectral radiance, radiation flows and radiation balance of the atmosphere in the short-wave spectral range are calculated. The relations obtained can be used to calculate the complex index of refraction, distribution function and other characteristics of the submicron aerosol fraction, by solving the inverse problems.

1. Nomenclature

A_λ	albedo of Earth's surface
α	azimuth
B	balance
$\beta_{\mathrm{m}}, \beta_{\mathrm{a}}, \beta_{\mathrm{o}}$	molecular, aerosol and ozone extinction coefficients
Γ	scattering function
h, h_1	altitudes
$i_n^{\pm}, j_n^{\pm}, i_{an}^{\pm}$	radiance functions for n-th order scattering
I_h, I_{h,h_1}	transfer functions
J	integral flux of radiation
$J_\lambda^{\pm}$	spectral fluxes of radiation
μ, μ_0	optical air masses
p	transmittance
$P_{\mathrm{m}}, P_{\mathrm{a}}$	molecular and aerosol phase functions
$\theta^{\pm}$	scattering angles
ξ, ξ_0	zenith angles

2. Introduction

One of the methods of investigation of planetary atmospheres is the study of their radiation. This method makes it possible to obtain data on the chemical composition, photochemical processes and dynamics of the atmospheres. The space probes have already collected relevant information about the majority of planets of the solar system in this way.

In addition to the spectroscopic methods (Krasnopolskij, 1987), the measurements of atmosphere radiances and fluxes of direct and diffuse solar radiation

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(Minin, 1988) play an important role. Structural characteristics of the atmosphere at different heights, its gas composition, temperature, pressure, concentration of individual gas components and their stratification can be determined or expressed more precisely by optical methods. Optical methods also help in investigating the submicron aerosol fraction. This fraction is most active in the visible spectrum, and represents an important component of the atmosphere contributing to the instability of its optical characteristics. Particles of this fraction are difficult to identify by direct methods (e.g., by particle detectors) without damaging their structure, and that is why the use of indirect optical methods for identifying them is most reasonable. Indirect optical methods use the results of physical theories of interaction of radiation with particles. The theoretical basis is Mie's theory of radiation scattering by a spherical homogeneous particle (Mie, 1908). Mie's functions, whether in already modified (Van de Hulst, 1961; McCartney, 1979; Bohren and Haufmann, 1986) or original form (Mie, 1908), are used also for determining the optical parameters of interplanetary dust particles (McDonnell, 1978; Wickramasinghe, 1967) and help to solve the complicated integral equations appearing in the inverse problems for the determination of the structure and quality of these microparticles.

This paper concentrates on a theoretical analysis of the transfer and diffusion of radiation in the Earth's atmosphere, with emphasis on the interaction of solar radiation with the submicron fraction of particles in the upper atmosphere. Analytical relations for the calculation of radiance functions for n-th order scattering are derived, the flows of radiation and radiation balance of atmospheric layers are calculated. These relations can be used for solving the inverse problems in the calculation of the complex refraction index (Wendisch and Hoyningen-Huene, 1991, 1992), vertical stratification and size distribution function of microparticles not only in the upper atmosphere, but also in its lower layers.

3. First Order Scattering

The character of radiation scattering in a disperse environment depends upon the type of inhomogeneities and their distribution in space. The whole Earth's atmosphere in the optical spectrum represents the type of atmospheres of small optical thickness (Minin, 1988).

Scattering by a system of particles depends on the mutual distribution determining the character of superposition of the scattered electromagnetic waves. If the inhomogeneities (e.g., particles) are distributed in space randomly, i.e., form an environment in which the Poisson statistic is applicable, then the calculations, with or without the consideration of the wave characteristics of the radiation, yield the same result. The total intensity is given by the additive sum of intensities, produced by individual scatterers. If the distances between the particles are much larger than their dimensions, then in the majority of the real objects (e.g., the turbid atmosphere, clouds), the inhomogeneities are distributed in space at random and radiation scattering can be described by the intensities (Ivanov and Lojko and Dik, 1988). By enlarging the space concentration of particles up to a certain limit, no principal changes in the rules of radiation scattering appear, and all processes can be analysed with the help of energetic quantities. This is also the case of the Earth's atmosphere and other planetary atmospheres with densities less than the order of the water density (Ivanov and Lojko and Dik, 1988).

Description of the radiation scattering process in the Earth's atmosphere with respect to its properties can be simplified to a certain degree. Generally, the atmosphere not only attenuates the radiation (by the influence of scattering and absorption), but also radiates itself. The water vapours play a very important role in this process. The intensity of the real emission (thermal radiation) of the atmosphere decreases with increasing altitude as rapidly as the concentration of water vapours and other absorbing components. The submicron aerosol fraction is optically very active, in particular in the visible spectrum, which is also the optimum spectral range for monitoring of these particles. With respect to the fact that the influence of the real emission of the atmosphere is detectable only in the infrared part of the spectrum, it is not necessary to consider it in the calculation of atmospheric radiance in the visible spectral range – also because we are interested in the middle and upper atmosphere, without any cloudiness. Basing on these facts, the components of radiance downwards (+) and upwards (−) for the first-order scattering can be expressed by the formulae (Kocifaj, 1992a,b)

$$i_1^+(h_1, \xi_0, \xi, \alpha) = \mu \int_{h_1}^{\infty} \Gamma(h, \vartheta^+) I_h(\mu_0) I_{h_1,h}(\mu)\, \mathrm{d}h, \tag{1}$$

$$i_1^-(h_1, \xi_0, \xi, \alpha) = \mu \int_{h_{0n}}^{h_1} \Gamma(h, \vartheta^-) I_h(\mu_0) I_{h,h_1}(\mu)\, \mathrm{d}h, \tag{2}$$

where h_{0n} and h_1 are altitudes above sea level of the ground of the observing position, ϑ is the scattering angle, Γ is the scattering function of elementary scattering, μ and μ_0 are the optical air masses of the measured and solar horizontal and I_h and $I_{h_1,h}$ are the transfer functions. Scattering functions in the relations (1) and (2) can be for the Earth's atmosphere expressed by:

$$\Gamma(h, \vartheta) = \frac{P_\mathrm{m}(\vartheta)}{4\pi} \beta_m(h) + \frac{P_\mathrm{a}(\vartheta, h)}{4\pi} \beta_a(h), \tag{3}$$

where P_m and P_a are the phase functions for the molecular atmosphere and aerosols. The transfer functions are expressed by relations:

$$I_h(\mu_0) = I_0 \exp\left\{ -\mu_0 \int_h^{\infty} [\beta_a(z) + \beta_m(z) + \beta_0(z)]\, \mathrm{d}z \right\}, \tag{4}$$

$$I_{h_1,h}(\mu) = \exp\left\{ -\mu \int_h^{h_1} [\beta_a(z) + \beta_m(z) + \beta_0(z)]\, \mathrm{d}z \right\}, \tag{5}$$

where β_a, β_m and β_0 are coefficients of extinction for aerosol, molecular atmosphere and ozone. If notations ξ_0, ξ and α are used for the zenith angle of the sun, zenith angle and azimuth (measured from the sun) of a chosen element in the sky, then the scattering angles ϑ^+ and ϑ^- can be calculated from the relations:

$$\cos\vartheta^+ = \cos\xi_0\cos\xi + \sin\xi_0\sin\xi\cos\alpha, \tag{6}$$

$$\cos\vartheta^- = -\cos\xi_0\cos\xi + \sin\xi_0\sin\xi\cos\alpha. \tag{7}$$

The irradiation of an elementary volume of air by scattered radiation of the first order includes also the radiation reflected by the Earth's surface. If we suppose that the reflection by the Earth's surface corresponds to the reflection by a Lambert surface, the components of the first order scattering connected with the albedo of the Earth's surface A_λ can be written as follows:

$$\begin{aligned} i_{a1}^+(h_1,\xi_0,\xi) = {} & \frac{A_\lambda}{2\pi}\mu I_0 p^{\mu_0}(h_{0n})\cos\xi_0\int_{h=h_1}^{\infty}\times \\ & \times I_{h_1,h}(\mu)\int_{\xi'=0}^{\pi/2} I_{h_{0n},h}(\mu')\sin\xi'.\times \\ & \times\int_{\alpha'=0}^{2\pi}\Gamma(h,\vartheta_a^-)\,\mathrm{d}\alpha'\,\mathrm{d}\xi'\,\mathrm{d}h, \end{aligned} \tag{8}$$

$$i_{a1}^-(h_1,\xi_0,\xi) = \frac{A_\lambda}{2\pi}I_0 p^{\mu_0}(h_{0n})\cos\xi_0 I_{h_{0n},h_1}(\mu), \tag{9}$$

where

$$\cos\vartheta_{\mathrm{a}}^- = -\cos\xi'\cos\xi + \sin\xi'\sin\xi\cos(\alpha-\alpha'). \tag{10}$$

In the relations (8) and (9) $p_{h_{0n}}$ is the transmittance of the atmosphere measured from the Earth's surface and

$$p^{\mu_0}(h_{0n}) = \frac{I_{h_{0n}}(\mu_0)}{I_0} \tag{11}$$

The total radiance calculated for the first order scattering for the radiation downwards (+) and upwards (−), is

$$j_1^+(h_1,\xi_0,\xi,\alpha) = i_1^+(h_1,\xi_0,\xi,\alpha) + i_{a1}^+(h_1,\xi_0,\xi), \tag{12}$$

$$j_1^-(h_1,\xi_0,\xi,\alpha) = i_1^-(h_1,\xi_0,\xi,\alpha) + i_{a1}^-(h_1,\xi_0,\xi), \tag{13}$$

The scattering functions of the first order Equations (12) and (13) include components of Equations (1) and (2) corresponding to the process of single scattering in the atmospheric environment, and components of Equations (8) and (9) are connected with the reflection of radiation by the Earth's surface.

4. Higher Scattering Orders

When calculating the components of scattered radiation of higher orders (method of successive scattering), approximations are often used with respect to the difficulty of these calculations. Various methods of different accuracy have already been described, (e.g., in papers by Kattawar *et al.*, 1976; Lenoble, 1977; Liou, 1980; Box and Deepak, 1981; Wendish and Hoyningen-Huene, 1991). In the present paper, a step-by-step calculation of the components of higher order scattering was used. On the basis of known scattering functions of the n-th order scattering, the radiance functions for $(n+1)$th order scattering have been calculated. In the process of higher scattering orders the atmosphere itself is the source of radiation and the intensity of this radiation is described by the scattering functions of n-th order. Components j_n^+ and j_n^- calculated by the approach mentioned above can be expressed by the relations:

$$\begin{aligned} i_n^+(h_1,\xi_0,\xi,\alpha) = {} & \mu \int_{h=h_1}^{\infty} I_{h_1,h}(\mu) \int_{\xi'=0}^{\pi/2} \sin\xi' \times \\ & \times \int_{\alpha'=0}^{2\pi} \{\Gamma(h,\varphi_1'') j_{n-1}^+(h,\xi_0,\xi',\alpha') + \\ & + \Gamma(h,\varphi_2'') j_{n-1}^-(h,\xi_0,\xi',\alpha')\} \,\mathrm{d}\alpha' \,\mathrm{d}\xi' \,\mathrm{d}h, \end{aligned} \tag{14}$$

$$\begin{aligned} i_n^-(h_1,\xi_0,\xi,\alpha) = {} & \mu \int_{h=h_{0n}}^{h_1} I_{h_1,h}(\mu) \int_{\xi'=0}^{\pi/2} \sin\xi' \times \\ & \times \int_{\alpha'=0}^{2\pi} \{\Gamma(h,\varphi_2'') j_{n-1}^+(h,\xi_0,\xi',\alpha') + \\ & + \Gamma(h,\varphi_1'') j_{n-1}^-(h,\xi_0,\xi',\alpha')\} \,\mathrm{d}\alpha' \,\mathrm{d}\xi' \,\mathrm{d}h, \end{aligned} \tag{15}$$

$$\begin{aligned} i_{an}^+(h_1,\xi_0,\xi) = {} & \frac{A_\lambda}{2\pi} \left\{ \int_{h'=h_1}^{\infty} I_{h_1,h'}(\mu) \int_{\xi'=0}^{\pi/2} I_{h_{0n},h'}(\mu') \sin\xi' \times \right. \\ & \left. \times \int_{\alpha'=0}^{2\pi} \Gamma(h',\vartheta_a^-) \,\mathrm{d}\alpha' \,\mathrm{d}\xi' \,\mathrm{d}h' \right\} \\ & \times \int_{\xi''=0}^{\pi/2} \int_{\alpha''=0}^{2\pi} j_{n-1}^+(h_{0n},\xi_0,\xi'',\alpha'') \sin\xi'' \cos\xi'' \,\mathrm{d}\xi'' \,\mathrm{d}\alpha'', \end{aligned} \tag{16}$$

$$\begin{aligned} i_{an}^-(h_1,\xi_0,\xi) = {} & \frac{A_\lambda}{2\pi} I_{h_{0n},h_1}(\mu) \int_{\xi''=0}^{\pi/2} \int_{\alpha''=0}^{2\pi} \times \\ & \times j_{n-1}^+(h_{0n},\xi_0,\xi'',\alpha'') \sin\xi'' \cos\xi'' \,\mathrm{d}\xi'' \,\mathrm{d}\alpha''. \end{aligned} \tag{17}$$

Then,

$$j_n^+(h_1, \xi_0, \xi, \alpha) = i_n^+(h_1, \xi_0, \xi, \alpha) + i_{an}^+(h_1, \xi_0, \xi), \tag{18}$$

$$j_n^-(h_1, \xi_0, \xi, \alpha) = i_n^-(h_1, \xi_0, \xi, \alpha) + i_{an}^-(h_1, \xi_0, \xi). \tag{19}$$

Scattering angles φ and ϑ_a in the relations (14–17) can be calculated from the equations

$$\cos\varphi_1'' = \cos\xi\cos\xi' + \sin\xi\sin\xi'\cos(\alpha - \alpha'), \tag{20}$$

$$\cos\varphi_2'' = -\cos\xi\cos\xi' + \sin\xi\sin\xi'\cos(\alpha - \alpha'), \tag{21}$$

$$\cos\vartheta_a^- = -\cos\xi'\cos\xi + \sin\xi'\sin\xi\cos(\alpha - \alpha'). \tag{22}$$

The above relations for the calculation of higher order scattering can be used for $n \geq 2$. Functions of atmospheric radiance for the layer below the level h_1 (height of measurement) were calculated for positive values of ξ, and the fact that it concerns negative values, was included into the calculation of scattering angles ϑ^-. The total radiance of a given element of the celestial sphere in the Earth's atmosphere can be written as the sum

$$j^+(h_1, \xi_0, \xi, \alpha) = \sum_{n=1}^{N\to\infty} j_n^+(h_1, \xi_0, \xi, \alpha), \tag{23}$$

$$j^-(h_1, \xi_0, \xi, \alpha) = \sum_{n=1}^{N\to\infty} j_n^-(h_1, \xi_0, \xi, \alpha). \tag{24}$$

In general it is necessary to handle the sums of Equations (23) and (24) as infinite series. However, it appears that for the calculation of radiation scattering in the Earth's atmosphere, it is sufficient to limit the number of members by a certain value N. In the cloudless atmosphere j_1 is the dominant component and values of j_n for $n > 1$ decreases rapidly with the order of scattering. In the visible spectrum we can substitute the infinite sums of Equations (23) and (24) by finite sums with $N = 3$ or 4 with a satisfactory accuracy (Kocifaj, 1992). With increasing turbidity of the atmosphere the maximum shifts progressively to the scattering components of higher order, and in that case it is necessary to adopt a higher value of N. In the majority of the real situations in the cloudless atmosphere, when the increase of turbidity can be caused only by the increase of concentration of particles and by the change of their structural and qualitative features, the component j_1 is still dominant and values j_3, j_4 are still many times smaller than j_1, resp. j_2. In the cloudy atmosphere the maximum passes to $n > 2$, and depends on the mass and optical characteristics of the cloud layers; with the value $n = N$ it is possible to finish the calculation. In a cloudless atmosphere the majority of particles is concentrated in the lower troposphere. With the increasing altitude of the measurement h_1, the importance of the component j_1 steeply increases. Already at the heights of $h_1 > 5$ km, the sums of Equations (23) and (24) can be substituted by the sum of the components j_1 and j_2. Above the layer in which the clouds are usually present (approximately at the height of 10–13 km, i.e., at the upper limit between the troposphere and tropopause), $N = 1$ can be used.

Relations (1–2) and (14–19) determine the components of diffusive radiation in the Earth's atmosphere at an arbitrary height and for an arbitrary configuration of its optical parameters. With respect to the strong connection of these functions with the structural and qualitative characteristics of microparticles, it is possible to use these relations and functions also for solving the inverse problems. In that case these relations are represented by difficult integral equations (Schmeidler, 1955). Their solution, however, results in the determination of, for example, the complex refraction index of microparticles $m(\lambda)$, their concentration and size distribution function $f_a(r)$ (r is a radius of particle). In such calculations Mie's theory (Mie, 1908) or some of its approximations are adopted (Hulst, 1961; McCartney, 1979). According to Mie's relations the functions P_a and β_a are calculated and they are directly connected with the above functions $m(\lambda)$ and $f_a(r)$.

An important part of the research on planetary atmospheres is also the study of the energetic the processes occuring in the individual atmospheric layers. Relations (1–2) and (14–19) determine the radiative balance of the Earth's atmosphere, on the basis of which the various energetic and photochemical processes can be studied. By the integration of radiance the fluxes of radiation on the boundary of two layers are obtained. Thus for the total atmospheric volume the energy escaping from the Earth's atmosphere into space can be calculated. Spectral fluxes of radiation at the height h_1 directed to the Earth's surface (+) and to the space (−) are

$$J_\lambda^+(h_1, \xi_0) = \int_{\xi=0}^{\pi/2} \int_{\alpha=0}^{2\pi} j^+(h_1, \xi_0, \xi, \alpha) \cos\xi \sin\xi \, \mathrm{d}\xi \, \mathrm{d}\alpha + I_{h_1}(\mu_o) \cos\xi_0, \quad (25)$$

$$J_\lambda^-(h_1, \xi_0) = \int_{\xi=0}^{\pi/2} \int_{\alpha=0}^{2\pi} j^-(h_1, \xi_0, \xi, \alpha) \cos\xi \sin\xi \, \mathrm{d}\xi \, \mathrm{d}\alpha. \quad (26)$$

The net flux on the boundary at the height h_1 will be obtained at the difference of the radiation flux directed to the Earth's surface and that escaping into the space,

$$J_\lambda(h_1, \xi_0) = J_\lambda^+(h_1, \xi_0) - J_\lambda^-(h_1, \xi_0). \quad (27)$$

Relations (1–27) describe the process of interaction of monochromatic radiation with the atmosphere. However, the total balance B of a given atmospheric layer situated between the boundary h_1 and $h_1 + \Delta h_1$ is given by the complete range of the wavelengths. If the integral flux in the certain interval of the wavelengths from λ_1 to λ_2 is denoted by $J(h_1, \xi_0)$

$$J(h_1, \xi_0) = \int_{\lambda=\lambda_1}^{\lambda_2} J_\lambda(h_1, \xi_0)\, \mathrm{d}\lambda, \tag{28}$$

then the balance of the layer for this spectral range will be

$$B(h_1, h_1 + \Delta h_1, \xi_0) = J(h_1 + \Delta h_1, \xi_0) - J(h_1, \xi_o). \tag{29}$$

Therefore, from Equation (29) the amount of the radiative energy absorbed or emitted by the layer can be determined.

Modelling of the radiation transfer according to the above relations yields the possibility to calculate the response of the change of the structure and quality of the environment in its energetic characteristics.

5. Conclusion

In this paper the problem of the transfer and diffusion of radiation in a turbid aerosol environment of the atmosphere is theoretically worked out. Scattering functions make it possible to calculate the intensity of the n-th order scattered radiation. The relations obtained can be used for solving the inverse problems, such as the calculation of the complex refraction index $m(\lambda)$, vertical stratification and distribution function of microparticles. These functions and methods for their calculation will be the subject of further studies using the results of this paper. On the basis of the relations for the radiance of atmosphere, the radiation fluxes and radiative balance in atmospheric layers were defined, too. According to this, the amounts of radiative energy absorbed or emitted by a given layer can be calculated. Modelling of the transfer of radiation according to the presented relations will be used in further studies for monitoring of the response of structural and quality changes of particles in radiative characteristics.

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