COMMENTS ON A GLOBAL POLYTROPIC MODEL FOR THE SOLAR AND JOVIAN SYSTEMS

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(Received 21 April, 1994)

Abstract. By comparing the predictions of a global polytropic model (developed by the first author) for the Solar and Jovian systems with other models and observational data it is shown that, even though this model has a minimal input, it produces interesting results concerning the masses and distances of planets and satellites.

1. The Model

The global polytropic model is based on the assumption of hydrostatic equilibrium and is described by the classical Lane–Emden equation. The first root of the Lane–Emden equation ξ_1 is at the star's (in this case the Sun's) radius. By integrating the equation beyond this first root using the "Complex Plane Strategy" – described in Geroyannis (1988), and applied in Geroyannis (1990), and Geroyannis (1992); also Dallas and Geroyannis (1993) use a simplified variation – we can calculate other solutions $\xi_1 < \xi_2 < \xi_3$ etc. The spherical polytropic shells S_i, i = 2, 3, . . . defined by pairs of radii ((ξ_1, ξ_2) for S_2, (ξ_2, ξ_3) for S_3, etc.) are appropriate places for a planet to exist. Therefore, by only describing the polytropic configuration of the Sun (or, for that matter, any star) we can have estimates for its planetary system. This procedure is described in Geroyannis (1993). The inverse problem, namely calculating the polytropic configuration of the star from its planetary system data, is examined in Geroyannis and Valvi (1993a). The application of the model to the Jovian satellite system is also discussed by Geroyannis and Valvi (1993b).

In our results for the solar system we use a polytropic index n = 3.23 (for details see Geroyannis, 1993, §2). This value is appropriate for representing solar-type stars (Russel, 1939; Geroyannis and Valvi, 1986). Nonrotating polytropic configurations with $n > 3$ are intrinsically unstable due to a negative binding energy (Tooper, 1964, §3), but this is not critical. Many investigators have studied polytropes with $n > 3$ (for example: Anand, 1968, Chandrasekhar and Lebovitz, 1962; Geroyannis, 1988; Geroyannis and Valvi, 1985; Hachisu, 1986; Horedt, 1983; Martin, 1970; Monaghan and Roxburg, 1965). In addition, unstable configurations have been examined in respect to planetary formation before (Cameron, 1978; §12).

Since the model is in hydrostatic equilibrium, its results effectively demonstrate not the current data, but those corresponding to an earlier epoch, even though we compare our results to observations (from Kaufmann, 1991).

Earth, Moon and Planets **65**: 15–19, 1994.

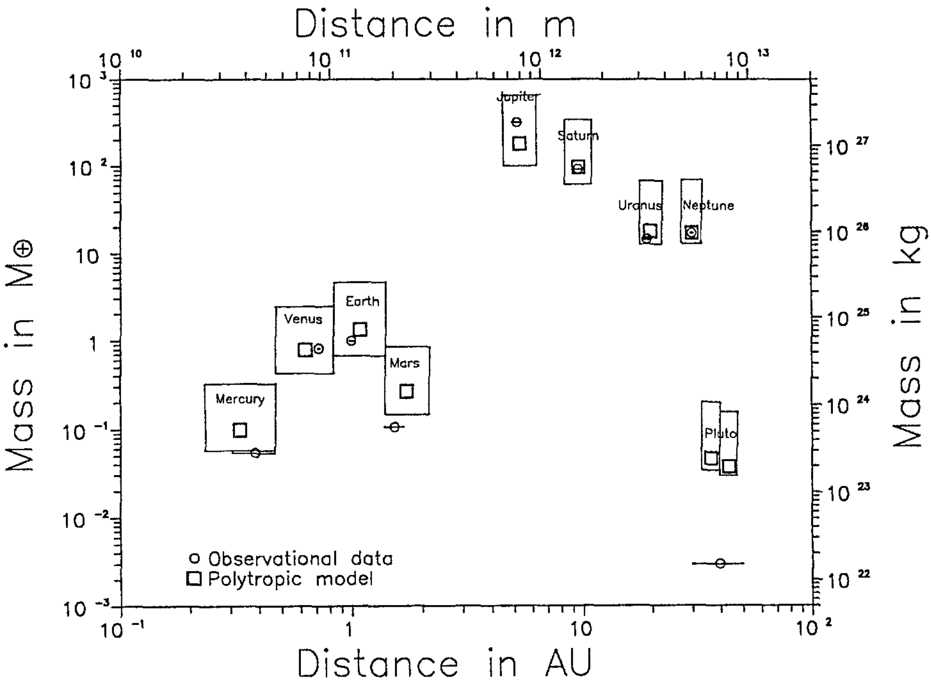

Fig. 1. Comparison between the current observational data (A_p, M_p) for the planets (circles), and the predictions (a_p, m_p) of the global polytropic model (squares). The bars that run through the circles show the perihelion and aphelion distance of each planet. The large rectangles show the limits on the planets' positions ($a_i < a_p < a_0$) and masses ($m_{pav} < m_p < m_{pmx}$) imposed by the global polytropic model.

2. The Results

Benz *et al.* (1988) show that a proto-Mercury with mass 2.25 times the present value colliding with a body one sixth that size can account for the chemical composition of Mercury. This solves the mass difference predicted by the global polytropic model and explains the eccentric orbit of Mercury. Shells below S_5 are not occupied, either because they are within the Sun's Roche limit, or because of evaporation processes due to the Sun's proximity, similar to the ones proposed for Mercury (Cameron, 1985). Results for Venus fit the observational data adequately.

Calculations for Earth predict a mass 1.3 $M_\oplus$ but this difference can be accounted for. In particular, the results of Benz *et al.* (1987) show that a collision between the proto-Earth with an impactor of 0.12 $M_\oplus$ can create the Moon. Such a collision process could account for the missing mass and the difference in the orbital data. It is worth noting that various n-body simulations of the inner planets (Cox and Lewis, 1980; Lecar and Aarseth, 1986; Beaugeé and Aarseth, 1990) predict too many final bodies, with large differences in both their formation sites and masses. On the other hand, the global polytropic model predicts only four planets with satisfactory accuracy to their positions and masses.

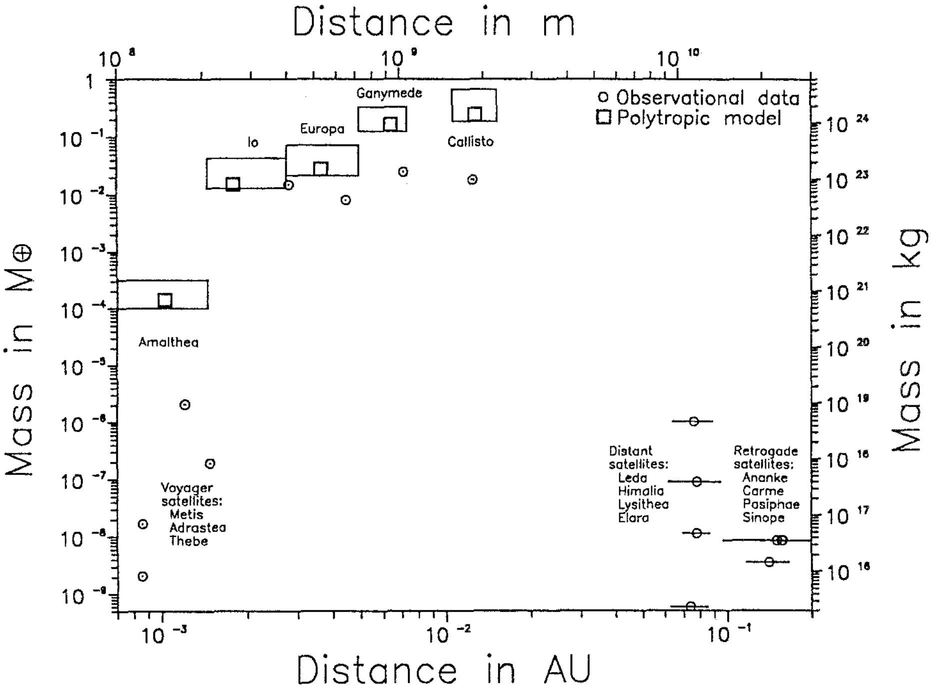

Fig. 2. Comparison between the current observational data (A_p, M_p) for the Jovian satellites (circles), and the predictions (a_p, m_p) of the global polytropic model (squares). The bars that run through the circles show the minimum and maximum distance of each satellite from Jupiter. The large rectangles show the limits on the satellites' positions ($a_i < a_p < a_0$) and masses ($m_{pav} < m_p < m_{pmx}$) imposed by the global polytropic model.

The Mars and Jupiter cases exhibit significant variations from the observations. The small size of Mars, the absence of a planet in the asteroidal belt, and the large size of Jupiter are interrelated. Jupiter was created early and its size increased quickly (Weidenschilling, 1987; Ward, 1989). As a result it scattered large planetesimals inward, thus causing collisional destruction of many of the minor bodies that would contribute to the growth of Mars and the asteroidal region planet (Cameron, 1988, §7). This process can account for the missing planet in S_{12} of the global model, too. Contrary to the small bodies among the outer planets, asteroids in S_9 and S_{10} are stable, especially if they are moving on eccentric and inclined orbits (Flogaitis *et al.*, 1991). Therefore, shells S_9 and S_{10} are occupied, while S_{12}, S_{14}, S_{15}, and S_{17} are not.

Predictions for Saturn and Neptune fit very well with the data, showing that these regions evolved under calm processes. The discrepancy for Uranus can be accounted for by adopting the calculations (Safronov, 1969; Korycansky *et al.*, 1990) that an impact with a mass of 1 to 2 $M_\oplus$ could explain the tilted Uranus axis. The missing planets in the shells S_{14}, S_{15}, S_{17} are accounted for by the tidal effects of Saturn, Uranus and Neptune. Small bodies in these shells move in largely unstable orbits and would not survive the lifetime of the solar system (Gladman and Duncan, 1990); they would have collided with the giant planets, ejected outwards or scattered inwards to highly eccentric orbits. There is a possibility that the asteroids 944 Hidalgo and 2060 Chiron, that occupy the polytropic shells S_{12} and S_{14} and move in highly eccentric and chaotic orbits respectively, are leftovers from these tidal effects.

Pluto has the biggest error from any other planet, but it is such an exceptional case that it is not really important. There has been a heated debate on whether Pluto was formed by the stellar nebula, or it is an ejected Neptunian satellite (see Whyte, 1980 and references therein for the different opinions). It may even have been created in the region between Uranus and Neptune and forced by gravitational interaction with the major planets outwards, to its present highly eccentric orbit, which is located in a dynamically stable region (according to the processes described and simulated in Gladman and Duncan, 1990, §5). In addition, there is a trend to avoid discussion of Pluto in planetary formation models. Both shells S_{19} and S_{20} are within the orbit of Pluto, with S_{20} giving a better mass estimate. The polytropic model also predicts that the 10th planet, if it exists, should occupy a shell from S_{21} upwards.

The predictions of the global polytropic model for the system of Jovian satellites ($n = 2.45$; for details see Geroyannis and Valvi, 1993b, §2) are successful from the order-of-magnitude point of view. Since the satellite formation was strongly influenced by the rest of the solar system, this is to be expected. However, it seems that the global polytropic character of the Jovian system has been maintained due to mild processes. The results are acceptable for the Galilean satellites (Io, Europa, Ganymede, Callisto) and plausible for the inner group (Metis, Adrastea, Amalthea, Thebe), if we accept that the calculated value for Amalthea represents the cumulative of all four bodies. No calculations were performed for the small outer satellites.

3. The Verdict

We do not claim that the global polytropic model is a complete alternative to other elaborate models and simulations used to describe the origin and evolution of the solar system. However, we do feel that the favorable results it produces, despite its simplicity, can be useful if combined with more sophisticated methods.

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